+0.5 for clean HW

18/18

BME 118 MATH HW 1

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1 Problem 1

1.1 part a

• additive inverse $-V+V=0$

1.5/2

Here, we state the axioms related to scalars for a vector space:

(These are sourced from wikipedia)

Compatibility of scalar multiplication with field multiplication :

$$a(b\mathbf{v}) = (ab)\mathbf{v}$$

Identity element of scalar multiplication :

$$1\mathbf{v} = \mathbf{v}$$

Distributivity of scalar multiplication with respect to vector addition :

$$a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}$$

Distributivity of scalar multiplication with respect to field addition :

$$(a + b)\mathbf{v} = a\mathbf{v} + b\mathbf{v}$$

1/1

1.2 part b

Out of the options presented:a group, a ring, a field, and the real numbers, only a field satisfies the 4 axioms listed above.

A ring does not necessarily have a multiplicative identity and so it is ruled out. Real numbers could certainly serve as scalars, but they do not include other possible scalars such as the rational numbers. Thus, a field is the appropriate choice here.

2 Problem 2

2.1 part a

2/2

From simple algebra, why do these two hyperplanes, $\underline{H}$ in n dimensions and H in n + 1 dimensions, represent the same firing rule for the neuron?

We have:

, explain more

$$\sum_{i=1}^{n} w_i x_i = -w_0 x_0$$

which is equivalent to:

$$\sum_{i=0}^{n} w_i x_i = 0$$

1/1

2.2 part b

Assume $w_0 \neq 0$. What is the special property of H that $\underline{H}$ lacks?

Using H allows us to look at a plane which always passes through the origin, whereas $\underline{H}$ will only pass through the origin if $w_0 = 0$.

2/2

2.3 part c

Suppose $w_0, w_1, ..., w_n$ are all multiplied by 5. Does either hyperplane change? Suppose $w_0, w_1, ..., w_n$ are all multiplied by -5.

Does either hyperplane change?

No, multiplying an equation by a scalar of any quantity will not change the equation.

2.4 part d

2/2

Does either firing rule change?

The firing rule, on the other hand, is not an equation. Rather, it is an inequality. Hence, when it is multiplied by a positive scalar, it is unaffected, but when it is multiplied by a negative scalar, the inequality is flipped around, or reversed.

2.5 part e

1/1

Suppose $w_n \neq 0$ and $w_0, w_1, ..., w_n$ are all divided by w_n. Note that this reduces the number of free parameters needed to specify the hyperplane by 1, because $\frac{w_n}{w_n} = 1$

Does either hyperplane change under this "reduced parameter representation"?

Once again, multiplying or dividing the equations of either plane by a scalar will not mathematically change their equations at all.

2.6 part f

1/1

Does either firing rule change?

According to reasoning from part d, the firing rule will change if w_n is negative.

3 Problem 3

Problem 3. In the lecture we talked about the maximum number of regions that you can divide n-space into using m hyperplanes. These hyperplanes do not need to include the origin. Let's call this number $\Xi(n, m)$.

3.1 part a

2/2

Give an explicit formula for $\Xi(1,m)$ that you obtain by doing some examples and generalizing.

If we are in 1-space, we are dealing with a line. This means that our hyperplanes will be points on this line. Thus, the maximum number of regions for m hyperplanes in 1-space is $m+1$.

$$\Xi(1,m) = m+1$$

3.2 part b

1/1

Geometrically, why is this formula right?

Re-iterating what we have said above, if we place m points on a line in such a way that they are not on top of each other, we will have $m-1$ finite regions in between the points, and 2 infinite regions on either end of the furthest points in each direction. $m-1+2 = m+1$

3.3 part c

3/3

In class we gave the formula $\Xi(3,m) = \binom{m}{0} + \binom{m}{1} + \binom{m}{2} + \binom{m}{3} \approx m^3$, and the general formula $\Xi(n,m) = \binom{m}{0} + \binom{m}{1} + ... + \binom{m}{n} \approx m^n$. Why is this approximation $\approx m^n$ ok for m a lot greater than n when all we care about is a so-called "big Oh" approximation? In other words, why is $\Xi(n,m) = \binom{m}{0} + \binom{m}{1} + ... + \binom{m}{n} = \mathcal{O}(m^n)$ for fixed n and variable m?

Generally, we have

$$\Xi(n,m) = \binom{m}{0} + \binom{m}{1} + ... + \binom{m}{n}$$

$$= \frac{m!}{0!m!} + \frac{m!}{1!(m-1)!} + \frac{m!}{2!(m-2)!} + ... + \frac{m!}{(n)!(m-n)!}$$

If we say that $m >> n$, then we can neglect $0!, 1!, 2!...n!$. This yields the approximation:

$$\approx \frac{m!}{m!} + \frac{m!}{(m-1)!} + \frac{m!}{(m-2)!} + ... + \frac{m!}{(m-n)!}$$

$$= 1 + m + m(m-1) + ... + m(m-1)(m-2)...(m-(n-1))$$

Using the same $m >> n$ reasoning, we approximate further by saying that $(m-1), (m-2), ...(m-(n-1)) \approx m$. Once again, we say this because m is *so* much greater than n. Then our equation becomes

$$\approx 1 + m + m^2 + ... + m^n$$

and because $m^n >> 1+m+m^2+...+m^{n-1}$, we finallize our approximation as

$$\Xi(n, m) \approx m^n$$