# BME 118 Math Homework

17/18

## Problem 1

**Part a:** 1.5/2

- Distributivity of scalar multiplication over scalar addition: For all ~~a, scalar~~ $a, b \in F$ and all $\underline{v} \in V$ we have: $(a+b).\underline{v} = a.\underline{v} + b.\underline{v}$

• forgot inverse element $-v+v=0$

- Distributivity of scalar multiplication over vector addition: For all $a \in F$ and all $\underline{v}$ and $\underline{w}$ in $V$. we have: $a.(\underline{v}+\underline{w}) = a.\underline{v} + a.\underline{w}$.

- Associativity of scalar multiplication: For all $a, b \in F$ and all $\underline{v} \in V$, we have: $(ab).\underline{v} = a.(b.\underline{v})$.

- 1 is an identity ~~over~~ for scalar multiplication: For all $\underline{v} \in V$ we have $1.\underline{v} = \underline{v}$.

**Part b:** 1/1

The scalars need to form (3) a ~~field~~ "field" to be ~~vectoring~~ scalars of a vector space. To be scalars of the vector space, the vectors need both to be closed under addition and multiplications, and for the addition and multiplication operations must satisfy the distributive property. Option (4), "the real numbers" also satisfies this, as well as the required multiplicative and additive identities and inverses, but is also a field. It is worth noting that other fields also satisfy these properties - for instance, the set of all complex numbers, so field is the answer.

2a From simple Algebra, I will show that these two hyperplanes, $\underline{H}$ in $n$-dimensions and $H$ in $(n+1)$-dimensions, represent the same firing rule for the neuron.

2/2

$\underline{H}$ represents the neuron that fires on inputs $X$ where $x_0 = 1$ and $\sum_{i=1}^{n} w_i x_i > T$, where $T = -w_0$.

$H$ represents the neuron that fires on inputs $X$ where $\sum_{i=0}^{n} w_i x_i > 0$, where $x_0 = 1$.

Here, given $x_0 = 1$, we can expand this sum, and update the inequality:

$$\sum_{i=1}^{n} w_i x_i + w_0 x_0 > 0$$

$$\sum_{i=1}^{n} w_i x_i + w_0 (1) > 0$$

$$\sum_{i=1}^{n} w_i x_i > -w_0$$

where $T \stackrel{\text{def}}{=} -w_0$

Now, we have reached the $n$-dimensional firing rule. Hence, these two hyperplanes represent the same firing rule for the neuron.

2/2

2b. If $w_0 \neq 0$, H has the special property that $\underline{H}$ lacks: H passes through the origin, while $\underline{H}$ does not.

2c. If $w_0, w_1 \dots w_n$ are multiplied by 5, the position and orientation of each hyperplane does not change, but the distance of each from the origin scales by 5.

2/2

Neither of the firing rules change, because the comparison with the thresholds remain the same after changing the weights. For $\underline{H}$: $x_0 = 1$ and $\sum_{i=1}^{n} w_i x_i > T$ becomes $x_0 = 1$ and $\sum_{i=1}^{n} 5 w_i x_i > 5T$.
For H, the n+1 dimensional firing rule also remains unchanged because both sides are multiplied by 5.

2d. If $w_0, w_1, \dots w_n$ are multiplied by $-5$, the hyperplanes don't change, but the firing rules change. For $\underline{H}$, the sign of the T threshold changes. ~~for the n+1 dimension~~ For H, the sign of the $\sum_{i=0}^{n} w_i x_i > 0$ inequality changes, becoming $\sum_{i=1}^{n} w_i x_i < 0$.

2/2

2e. Hyperplane $\underline{H}$'s position and orientation will not change in this "reduced parameter representation" - only the distance from the origin will change.

1/1

For Hyperplane H, the position and orientation will also not change.

how about when $-w_n$ is negative?

.5/1

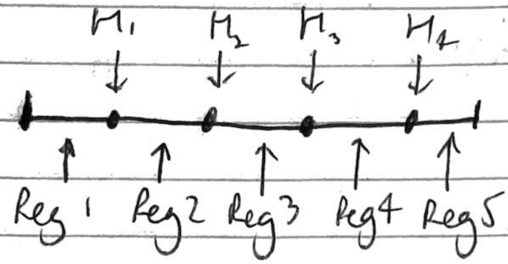
However, each of the firing rules will not change - the comparison with the threshold remains the same after dividing the weights by $w_n$.

2/2

3a In 1-dimension, a hyperplane is a point.

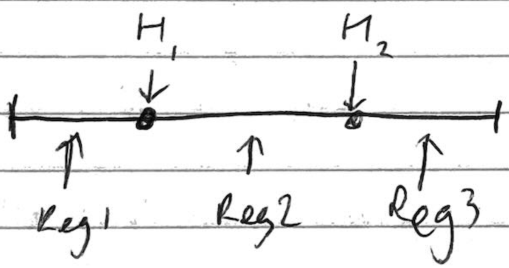
Some examples:

$\Xi(1, m=4) = 5$




= Total 5 regions

$\Xi(1, m=2) = 3$




= Total 3 regions

Generalising, this gives $\underline{\underline{\Xi(1, m) = m+1}}$

1/1

b It is quite intuitive: if we have a line and add a hyperplane (a point), this splits the line into 2 regions → the rule is still true: $m=1$, $R = 2 (= 1 +$
If we add another hyperplane, this will split another of the region into 2, resulting in 3 regions, and so forth.

~~Q~~ 3c

3/3

$\binom{m}{0} + \binom{m}{1} + \dots + \binom{m}{n} = O(m^n)$ for fixed $n$

and variable $m$ because when $m$ increases, the dominant term becomes $\binom{m}{n}$ – the largest term.

When $m$ is significantly larger than $n$,

$$\boxed{\frac{m!}{n!(m-n)!} \leq \frac{m^n}{n!}}$$ → show an intermediary step

Given $m > n$, this is approximately $= m^n$

Hence, the $\sum_{i=0}^{n} \binom{m}{i} = O(m^n)$ for fixed $n$ and variable $m$.