BME 218 Math HW1

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1

1a

1.5 / 2

We don't break the field properties, or order doesnt matter
$a(bu) = (ab)u$
We don't break the identity element (multiply by scalar 1 yields no change)
$1 \cdot u = u$
Distributive across both vector and field additions so
$a(u+v) = au + av$ and $(a+b)u = au + bu$

forgot additive inverse

$-v + v = 0$

1b

1/1

You can define a vector space over anything that obeys field axioms.

2

2a

2/2

, next time add justification

$\sum_1^n w_n \cdot x_n = -w_0 \Longleftrightarrow w_0 \cdot 1 + \sum_1^n w_n \cdot x_n = 0 \Longleftrightarrow w_0 \cdot x_0 + \sum_1^n w_n \cdot x_n = 0 \Longleftrightarrow \sum_0^n w_n \cdot x_n = 0$

2b

1/1

Passes through origin (linear rather than affine)

2c, d, e, f

A plane is fully specified by a (not unique) normal vector, and orthogonal vectors dot product to 0. The "firing rule" of our neuron checks if a vector is oriented above or below our plane.

c,d

2/2

$\underline{H}$

For $\underline{H}$ we have $\sum_1^n 5 \cdot w_n \cdot x_n = 5 \cdot (-w_0) \implies \sum_1^n w_n \cdot x_n = -w_0$
So this scalar cancels. No change to update. For the firing rule however we have
$\sum_1^n (-5) \cdot w_n \cdot x_n > (-5) \cdot (-w_0) \implies \sum_1^n w_n \cdot x_n < -w_0$
So the firing rule inverted.

H

2/2

For H we have $\sum_0^n 5 \cdot w_n \cdot x_n = 0 \implies \sum_0^n 5 \cdot w_n \cdot x_n = 0$.
This does also does not change our plane. it does not change our firing rule.
But for our firing rule we have $\sum_0^n -5 \cdot w_n \cdot x_n > 0 \implies \sum_0^n 5 \cdot w_n \cdot x_n < 0$.
So our firing rule inverted.

e,f

2/2

e, f : since w is a vector in our vector space then $\frac{1}{w_0}$ must be a scalar in our field.
So from the previous logic, we know that $\underline{H}, H$ do not have its plane changed.
The firing rule depends on the sign of w_n. For $w_n > 0$ there is no change to the firing rule. For $w_n < 0$ it inverts.

3

3/3

3a,b

1 dimension is a line. A plane is just a point
1 point can split the line into 2 segments.
2 lines split it into 3 segments.
3 lines split it into 4 segments
So we get $m + 1$ divisions.
Geometrically, this makes sense because we cant draw divisions through the same point and divide the region differently.

3/3

c

next time add more explanation

The largest term in the series is $\frac{m!}{n!(m-n)!} = \Pi_0^{n-1}(m-k) \approx m^n$