$\frac{13}{18}$

Math Assignment 1

Brad Tait

04/19/2023

• forgot additive inverse $-v+v=0$

$\frac{1.5}{2}$

1a) Compatibility of scalar multiplication with field multiplication.

$$a(bc) = (ab)c$$

where 1 denotes the multiplicative identity in F. Identity element of scalar multiplication.

$$1a = a$$

Distributivity of scalar multiplication with respect to vector addition.

$$a(b + c) = ab + ac$$

Distributivity of scalar multiplication with respect to field addition.

$$(a + b)c = ac + bc$$

$1/1$

1b) The scalars of a vector space must form a field. The real numbers are a field. The reason why the scalars must form a field is because the operations of vector addition and scalar multiplication are defined in terms of the field operations.

2a) The two hyperplanes are equivalent because they are both defined by the same set of linear equations.

The n-dimensional hyperplane is defined by the equations.

$1.5/2$

• right idea wrong math

$$\sum_{i=0}^{n} w_i x_i = -w_0$$

The n+1-dimensional hyperplane is defined by the equations

$$\sum_{i=0}^{n} w_i x_i = 0$$

We can rewrite the n+1-dimensional hyperplane as follows:

$$\sum_{i=0}^{n} w_i x_i = w_0 x_0 + \sum_{i=1}^{n} w_i x_i \cancel{=} w_0(x_0 \cancel{-1}) + \sum_{i=1}^{n} w_i x_i = -w_0 + \sum_{i=1}^{n} w_i x_i$$

The n+1-dimensional hyperplane is simply the n-dimensional hyperplane with an additional dimension that is always equal to 1. This additional dimension does not affect the set of points that lie on the hyperplane, so the two hyperplanes are equivalent.

$1/1$

2b) If $w_0 \neq 0$, then the n+1-dimensional hyperplane H passes through the origin. However, the n-dimensional hyperplane $\underline{H}$ does not pass through the origin. This is because the n+1-dimensional hyperplane is defined by the equations $\sum_{i=0}^{n} w_i x_i = 0$, and if we set $x_0 = 1$, then these equations become $\sum_{i=1}^{n} w_i x_i = -w_0$, which is the equation for the n-dimensional hyperplane $\underline{H}$.

Therefore, the n+1-dimensional hyperplane H has a special property that the n-dimensional hyperplane $\underline{H}$ lacks: it passes through the origin.

The n+1-dimensional hyperplane H is a translation of the n-dimensional hyperplane $\underline{H}$ by the vector $(1, 0, 0, \ldots, 0)^T$.

• hyperplanes don't change

2c) If $w_0, w_1, \dots, w_n$ are all multiplied by 5, then both hyperplanes will be scaled by a factor of 5. This is because the equations for the hyperplanes are linear, and multiplying all the coefficients by a constant will scale the hyperplane by the same constant.

2/2

The firing rules will not change, because they are defined in terms of the dot product of the input vector and the weight vector. The dot product is also linear, so multiplying all the weights by a constant will not change the dot product.

$w_0, w_1, \dots, w_n$ are all multiplied by -5, then both hyperplanes will be reflected through the origin. This is because the equations for the hyperplanes are linear, and multiplying all of the coefficients by -1 will reflect the hyperplane through the origin.

The firing rules will not change, because they are defined in terms of the dot product of the input vector and the weight vector. The dot product is also linear, so multiplying all the weights by -1 will not change the dot product.

1/2

2d) No, the firing rule does not change. The firing rule is defined as follows:

The neuron fires on inputs x where $\sum_{i=0}^{n} w_i x_i > 0$. • changes for -5

Multiplying all the weights by a constant does not change the dot product of the input vector and the weight vector, so the firing rule will not change.

$w_0 = 1$, $w_1 = 2$, and $w_2 = 3$, then the firing rule will fire on inputs x where $x_0 + 2x_1 + 3x_2 > 0$. If we multiply all the weights by 5, then the firing rule will still fire on inputs x where $5x_0 + 10x_1 + 15x_2 > 0$. The dot product of the input vector and the weight vector is still positive, so the neuron will still fire.

1/1

2e) No, the hyperplane does not change under this reduced parameter representation. This is because the equations for the hyperplane are linear, and dividing all the coefficients by a constant does not change the equation of the hyperplane.

If $w_0 = 1$, $w_1 = 2$, and $w_2 = 3$, then the equation for the hyperplane is $\sum_{i=0}^{2} w_i x_i = 0$. If we divide all the weights by $w_n = 3$, then the equation for the hyperplane becomes $\sum_{i=0}^{2} \frac{w_i}{w_n} x_i = 0$. This is the same equation as the original equation, so the hyperplane does not change.

0/1

2f) No, the firing rule does not change. The firing rule is defined as follows:

The neuron fires on inputs x where $\sum_{i=0}^{n} w_i x_i > 0$.

Dividing all the weights by a constant does not change the dot product of the input vector and the weight vector, so the firing rule will not change.

For example, if $w_0 = 1$, $w_1 = 2$, and $w_2 = 3$, then the firing rule will fire on inputs x where $x_0 + 2x_1 + 3x_2 > 0$. If we divide all the weights by $w_n = 3$, then the firing rule will still fire on inputs x where $\frac{x_0}{3} + \frac{2x_1}{3} + \frac{x_2}{3} > 0$. The dot product of the input vector and the weight vector is still positive, so the neuron will still fire.

The reduced parameter representation is simply the original parameter representation with one of the weights set to 1. This does not change the dot product of the input vector and the weight vector, so the firing rule does not change.

2/2

3a) $\Xi(1, m) = (m + 1)$

1/1

3b) $\Xi(1, m) = (m + 1)$ each line divides the line into two regions, and the regions on either side of the lines are distinct.

1/3

3c) $\Xi(3, m) = \binom{m}{0} + \binom{m}{1} + \binom{m}{2} + \binom{m}{3}$. If $m \gg n$, then

$$\frac{m!}{0!(m-0)!} + \frac{m!}{1!(m-1)!} + \frac{m!}{2!(m-2)!} + \frac{m!}{3!(m-3)!} \approx m^3.$$

• do a proof

• study Θ notation