An Axiomatisation of Basic Formal Ontology with Projection Functions

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Abstract

This paper proposes a reformulation of the treatment of boundaries, fiat parts and aggregates of entities in Basic Formal Ontology. These are currently treated as mutually exclusive, which is inadequate for biological representation since some entities may simultaneously be fiat parts, boundaries and/or aggregates. We introduce functions which map entities to their boundaries, fiat parts or aggregations. We make use of time, space and spacetime projection functions which, along the way, allow us to develop a simple temporal theory.

Keywords: ontology, mereology, axiomatisation

1 Introduction

Developed at the Institute for Formal Ontology and Medical Information Science, Basic Formal Ontology (BFO) is a theory of the basic structures of reality. BFO endorses the view that the world contains occurrents and continuants. Occurrents are entities which unfold, or develop in time. Continuants are entities which have a continuous existence and a capacity to endure through time. Both types of entities exist in time in different ways. By heeding a notion of (Zemach 1970), namely that distinct modes of being generate distinct ontologies, BFO distinguishes between two kinds of ontologies: one for continuants, the other for occurrents. The Open Biomedical Ontologies consortium's Relation Ontology describes inter and intra relations between the two ontologies in order to support automated reasoning about the spatiotemporal, temporal and spatial dimensions of biological and medical phenomena. We refer to BFO merged with the Relation Ontology simply as 'BFO'.

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In BFO there are three main categories of occurrents: processes, spatiotemporal regions and temporal regions. Examples of processes include the process of respiration, a human life, the development of an embryo, the flight of a bird, and the functioning of the heart. Examples of spatiotemporal regions include the spatiotemporal location of an individual organism's life and the spatiotemporal location of a replicating strand of DNA, whereas examples of temporal regions include the time taken by a cell undergoing meiosis, and the moment a finger is detached in an industrial accident. In BFO there are three main categories of continuants: dependent continuants, independent continuants and spatial regions. Dependent continuants are entities such as qualities, roles and dispositions that inhere in independent continuants. Independent continuants are entities in which dependent continuants, such as qualities and dispositions can inhere in. Examples of independent continuants include a human individual and a heart, whereas examples of dependent continuants include the mass of a cloud, the role of being a doctor, the disposition of a vase to break when dropped, the function of the heart to pump blood and the spectrum of the sun. Examples of spatial regions include a cubed-shape part of space, and a point in space.

The paper is structured as follows. Section 2 provides an overview of the BFO type hierarchy and is based on work found in (Spear 2006). Sections 3 and 4 describe mereological relations which are required in later sections. Section 3 is influenced by theory described by (Simons 1987), whereas Section 4 is influenced by (Smith 1996). Section 5.2 is based on work in (Smith et al. 2005), however the rest of Section 5 is new material. Here we introduce our time, space and spacetime projection functions and outline a simple temporal theory. Section 6 is entirely new and introduces functions which handle boundaries, fiat parts and aggregates. Section 7 draws conclusions. Throughout this paper we rely on the typography described in Figure 1. Relations between types are depicted in *italics*, whereas all other relations are depicted in **bold**. The logical connectors $\neg$, $=$, $\wedge$, $\vee$, $\Rightarrow$ and $\Leftrightarrow$ have their usual interpretation. The symbol $=_{def}$ is used for definitions, $\forall$ for universal quantification, $\exists$ for existential quantification, and ι for the definite descriptor. We omit leading universal quantifiers in our formulae. Names of axioms begin with 'A', names of definitions begin with 'D', and names of theorems begin with 'T'.

Occurrent	O	o
Process	P	p
Spatiotemporal_Region	U	u
Scattered_Spatiotemporal_Region		$u^{s(k)}$
Connected_Spatiotemporal_Region		u^c
Temporal_Region	T	t
Scattered_Temporal_Region		$t^{s(k)}$
Connected_Temporal_Region		t^c
Temporal_Instant		i
Temporal_Interval		v
Continuant	C	c
Spatial_Region	S	s
Independent_Continuant	A	a
Material_Continuant	M	m
Site		
Dependent_Continuant		
Generic_Dependent_Continuant		
Specific_Dependent_Continuant		
Quality		
Realisable_Entity		
Role		
Disposition		
Function		
time projection	τ	
space projection	ψ, ψ^i	
spacetime projection	μ	
boundary function	β, β^i	
fiat part function	φ, φ^i	
aggregation function	α, α^i	

Figure 1: Candidate BFO 2.0 type hierarchy and typography used throughout this paper. Upper-case roman letters denote occurrent and continuant types and lower-case roman letters denote instances.

2 The type hierarchy

BFO distinguishes between types and instances. Types are what all members of a natural kind, grouping or species have in common. For example: cat, cell and photosynthesis are types. Instances can be thought of as the individual occupants of reality. For example: my neighbour's cat, the red blood cell on this microscope slide, and the process of photosynthesis the gum tree in my backyard performs throughout its lifetime are all instances. Types can be instantiated by more than one entity at more than one time, whereas instances are one-off, they can exist only in one place at one time. Types exist when and where their instantiations exist. Instances exist in space and time, and come into and pass out of existence.

2.1 Instances and subtypes

The primitive binary relational assertion x **instance_of** X has the meaning: instance x is an instantiation of type X. We say 'x is an occurrent instance' (or, more simply, 'x is an occurrent') if and only if x **instance_of** *Occurrent*. A type X is a subtype of *Occurrent* if and only if all instances of X are occurents. In that case we call X an 'occurrent type'. In the following we use $O, O_1, \ldots$ and $o, o_1, \ldots$ to range over occurrent types and occurrents, respectively. In BFO, an example of an occurrent type is *Temporal_Instant*. We say "x is a temporal instant instance' (or, more simply, x is a 'temporal instant') if and only if x **instance_of** *Temporal_Instant*. We use $i, i_1, \ldots$ to range over temporal instants.

The primitive ternary relational assertion x **instance_of** X **at** i has the meaning: instance x is an instantiation of type X at the temporal instant i. We say 'x is a continuant instance at temporal instant i' (or, more simply 'x is a continuant at i') if and only if x **instance_of** *Continuant* **at** i. A type X is a subtype of *Continuant* if and only if all instances of X at any temporal instant are continuants at that instant. In that case we call X a 'continuant type'. We use C, $C_1, \ldots$ and c, $c_1, \ldots$ to range over continuant types and continuants. We furthermore write $o{:}O$ as an abbreviation for o **instance_of** O and $c{:}C$ **at** i as an abbreviation for c **instance_of** C **at** i.

Any two occurrent types are such that the instances of one are not the instances of the other. Any two continuants types are such that the instances of one at any given temporal instant are not the instances of the other at that same temporal instant.

$$O_1 = O_2 \Rightarrow \forall o.\ (o{:}O_1 \Leftrightarrow o{:}O_2) \quad \text{(A2.1)}$$
$$C_1 = C_2 \Rightarrow \forall c, i.\ (c{:}C_1 \textbf{ at } i \Leftrightarrow c{:}C_2 \textbf{ at } i) \quad \text{(A2.2)}$$

An occurrent type O_1 *is_a* (subtype of) occurrent type O_2 if and only if all instances of O_1 are also instances of O_2. A continuant type C_1 *is_a* continuant type C_2 if and only if all instances of C_1 at any temporal instant i are also instances of C_2 at i.

$$O_1 \textit{ is\_a } O_2 =_{def} \forall o.\ o{:}O_1 \Rightarrow o{:}O_2 \quad \text{(D2.1)}$$
$$C_1 \textit{ is\_a } C_2 =_{def} \forall c, i.\ c{:}C_1 \textbf{ at } i \Rightarrow c{:}C_2 \textbf{ at } i \quad \text{(D2.2)}$$

For example: *DNA is_a nucleic acid*; *photosynthesis is_a physiological_process*.

Although we do not show them here, using defintions D2.1 and D2.2, and axioms A2.1 and A2.2, we can prove theorems which state that the subtype relation is reflexive, antisymmetric and transitive. Moreover we can trivially prove two theorems (that appear in later proofs) which tell us that occurrent and continuant types inherit their subtype instances.

$$o{:}O_1 \wedge O_1 \textit{ is\_a } O_2 \Rightarrow o{:}O_2 \quad \text{(T2.1)}$$
$$c{:}C_1 \textbf{ at } i \wedge C_1 \textit{ is\_a } C_2 \Rightarrow c{:}C_2 \textbf{ at } i \quad \text{(T2.2)}$$

2.2 Occurrent types

Occurrents are entities that happen, unfold, or develop in time. They are sometimes referred to as 'perdurant' entities. The type *Occurrent* has three mutually exclusive subclasses: *Process*, *Spatiotemporal_Region* and *Temporal_Region*. Processes always depend on one or more independent continuants. For example the flight of a bird, the life of an organism, the process of cell division, or the course of a disease.

$$\textit{Process is\_a Occurrent} \quad \text{(A2.3)}$$
$$\textit{Spatiotemporal\_Region is\_a Occurrent} \quad \text{(A2.4)}$$
$$\textit{Temporal\_Region is\_a Occurrent} \quad \text{(A2.5)}$$

We differentiate between connected and scattered spatiotemporal regions. A scattered spatiotemporal region is the mereological sum of multiple connected spatiotemporal regions which are separated in spacetime. A connected spatiotemporal region is any spatiotemporal region that is not scattered.

$$\textit{Scattered\_Spatiotemporal\_Region} \\ \textit{is\_a Spatiotemporal\_Region} \tag{A2.6}$$

$$\textit{Connected\_Spatiotemporal\_Region} \\ \textit{is\_a Spatiotemporal\_Region} \tag{A2.7}$$

A spatiotemporal interval is a connected spatiotemporal region that endures for more than a single instant of time. A spatiotemporal instant is a connected spatiotemporal region at a specific instant in time.

$$\textit{Spatiotemporal\_Interval} \\ \textit{is\_a Connected\_Spatiotemporal\_Region} \tag{A2.8}$$

$$\textit{Spatiotemporal\_Instant} \\ \textit{is\_a Connected\_Spatiotemporal\_Region} \tag{A2.9}$$

We also differentiate between connected and scattered temporal regions. A scattered temporal region is the mereological sum of multiple connected temporal regions which are separate in time. A connected temporal region is any temporal region that is not scattered.

$$\textit{Connected\_Temporal\_Region} \\ \textit{is\_a Temporal\_Region} \tag{A2.10}$$

$$\textit{Scattered\_Temporal\_Region} \\ \textit{is\_a Temporal\_Region} \tag{A2.11}$$

A temporal interval is a connected temporal region that lasts for more than a single instant of time. A temporal instant is a connected temporal region comprising a single instant in time.

$$\textit{Temporal\_Interval} \\ \textit{is\_a Connected\_Temporal\_Region} \tag{A2.12}$$

$$\textit{Temporal\_Instant} \\ \textit{is\_a Connected\_Temporal\_Region} \tag{A2.13}$$

2.3 Continuant types

Continuants are entities that exists in full at any time at which they exist at all, persist through time while maintaining their identity, and have no temporal parts. They are sometimes referred to as 'endurant' entities. The type *Continuant* has three mutually exclusive subclasses: *Spatial_Region*, *Independent_Continuant* and *Dependent_Continuant*.

$$\textit{Spatial\_Region is\_a Continuant} \tag{A2.14}$$

$$\textit{Independent\_Continuant is\_a Continuant} \tag{A2.15}$$

$$\textit{Dependent\_Continuant is\_a Continuant} \tag{A2.16}$$

Any point, line, surface or volume is an instance of *Spatial_Region*.

Material continuants are entities which are the bearers of dependent continuants. They are entities in which dependent continuants inhere. Material continuants themselves cannot inhere in anything. Sites (such as hollows, cavities and tunnels) are entities which can move through space and also can be occupied by material continuants.

$$\textit{Material\_Continuant} \\ \textit{is\_a Independent\_Continuant} \tag{A2.17}$$

$$\textit{Site is\_a Independent\_Continuant} \tag{A2.18}$$

Dependent continuants are entities which inhere in independent continuants. Thus in order to exist, some independent continuant must also exist. Dependent continuants can be either specific or generic. An existing specific dependent continuant inheres in a single, specific bearer, whereas an existing generic dependent continuant can inhere in multiple bearers. For example the redness of this apple is not identical to the redness of that apple, but the pdf file in my inbox and on my desktop are identical. For each entity in which a generic dependent continuant inheres there exists a 'concretization' of the generic dependent continuant which is itself specific.

$$\textit{Specific\_Dependent\_Continuant} \\ \textit{is\_a Dependent\_Continuant} \tag{A2.19}$$

$$\textit{Generic\_Dependent\_Continuant} \\ \textit{is\_a Dependent\_Continuant} \tag{A2.20}$$

Qualities (such as temperature, shape and mass) are entities which inhere in a specific bearer and are such that they are exhibited in full whenever they are borne. Realisable entities are entities which inhere in a specific bearer and are sometimes (not always) realised as processes. For example the role of being a surgeon may inhere in a person, but that role is not realised when that person is away from work. Likewise the disposition of a match to ignite is realised when the match is struck and starts to burn.

$$\textit{Quality} \\ \textit{is\_a Specific\_Dependent\_Continuant} \tag{A2.21}$$

$$\textit{Realisable\_Entity} \\ \textit{is\_a Specific\_Dependent\_Continuant} \tag{A2.22}$$

We do not further address dependent continuants in this paper. We instead refer the reader to (Arp & Smith 2008) for more details.

3 Basic mereological relations

3.1 Parthood

In BFO, occurrent parthood is specified using the primitive binary relational assertion $o_1\, \mathbf{part\_of}\, o_2$. A time-indexed version $c_1\, \mathbf{part\_of}\, c_2\, \mathbf{at}\, i$ is used for continuants where i is a temporal instant. The instance level parthood relation is reflexive (A3.1 and A3.2), antisymmetric (A3.3 and A3.4) and transitive (A3.5 and A3.6).

$$o\, \mathbf{part\_of}\, o \tag{A3.1}$$

$$c\, \mathbf{part\_of}\, c\, \mathbf{at}\, i \tag{A3.2}$$

$$o_1\, \mathbf{part\_of}\, o_2 \wedge o_2\, \mathbf{part\_of}\, o_1 \Leftrightarrow o_1 = o_2 \tag{A3.3}$$

$$c_1\, \mathbf{part\_of}\, c_2\, \mathbf{at}\, i \wedge c_2\, \mathbf{part\_of}\, c_1\, \mathbf{at}\, i \\ \Leftrightarrow c_1 = c_2 \tag{A3.4}$$

$$o_1\, \mathbf{part\_of}\, o_2 \wedge o_2\, \mathbf{part\_of}\, o_3 \\ \Rightarrow o_1\, \mathbf{part\_of}\, o_3 \tag{A3.5}$$

$$c_1\, \mathbf{part\_of}\, c_2\, \mathbf{at}\, i \wedge c_2\, \mathbf{part\_of}\, c_3\, \mathbf{at}\, i \\ \Rightarrow c_1\, \mathbf{part\_of}\, c_3\, \mathbf{at}\, i \tag{A3.6}$$

If a spatial region s_1 is part of a spatial region s_2 at a given temporal instant, then s_1 is part of s_2 at all times. For example, at this instant in time the spatial region occupied by Tokyo is part of the spatial region occupied by Japan, but that same spatial configuration held before Tokyo was even built (and will hold after the city is demolished by Godzilla). Instead of the ternary relational assertion s_1 **part_of** s_2 **at** i we write the binary assertion s_1 **part_of** s_2 without the time-index.

An occurrent type O_1 is *part_of* occurrent type O_2 if and only if for all instances o_1 of O_1, there exists an instance o_2 of O_2 such that o_1 is part of o_2. A continuant type C_1 is *part_of* continuant type C_2 if and only if for all instances c_1 of C_1 at any temporal instant i, there exists an instance c_2 of C_2 at i such that c_1 is part of c_2 at i.

$$O_1\ \mathit{part\_of}\ O_2 =_{def} \forall o_1.\ o_1 : O_1 \Rightarrow \exists o_2.\ o_2 : O_2 \wedge o_1\ \mathbf{part\_of}\ o_2 \tag{D3.1}$$

$$C_1\ \mathit{part\_of}\ C_2 =_{def} \forall c_1, i.\ c_1 : C_1\ \mathbf{at}\ i \Rightarrow \exists c_2.\ c_2 : C_2\ \mathbf{at}\ i \wedge c_1\ \mathbf{part\_of}\ c_2\ \mathbf{at}\ i \tag{D3.2}$$

For example: *nucleoplasm part_of nucleus*; *gastrulation part_of embryonic development*. Note the definitions make use of an 'all-some' structure. O_1s in every case exist as parts of O_2s, however O_2s may exist without having O_1s as parts. For example: *menopause part_of ageing*

Although we do not show them here, using definitions D3.1 and D3.2, and the reflexivity and transitivity of the instance-level parthood relation, we can prove theorems stating that the type-level parthood relation is reflexive and transitive. We specify axioms which tell us that the relation is antisymmetric.

3.2 Overlaps

Another relation we use frequently in this paper is **overlaps**. A spatiotemporal region u_1 **overlaps** a spatiotemporal region u_2 if and only if there exists a spatiotemporal region u which is both a part of u_1 and u_2. We define similar relations for temporal regions and spatial regions (not shown here). An independent continuant a_1 **overlaps** an independent continuant a_2 at a temporal instant i if and only if there exists an independent continuant a which is both a part of a_1 and a_2 at i.

$$u_1\ \mathbf{overlaps}\ u_2 =_{def} \exists u.\ u\ \mathbf{part\_of}\ u_1 \wedge u\ \mathbf{part\_of}\ u_2 \tag{D3.3}$$

$$a_1\ \mathbf{overlaps}\ a_2\ \mathbf{at}\ i =_{def} \exists a.\ a\ \mathbf{part\_of}\ a_1\ \mathbf{at}\ i \wedge\ a\ \mathbf{part\_of}\ a_2\ \mathbf{at}\ i \tag{D3.4}$$

The instance-level overlap relation is reflexive, symmetric and intransitive.

$$u\ \mathbf{overlaps}\ u \tag{T3.1}$$

$$a\ \mathbf{overlaps}\ a\ \mathbf{at}\ i \tag{T3.2}$$

$$u_1\ \mathbf{overlaps}\ u_2 \Rightarrow u_2\ \mathbf{overlaps}\ u_1 \tag{T3.3}$$

$$a_1\ \mathbf{overlaps}\ a_2\ \mathbf{at}\ i \Rightarrow a_2\ \mathbf{overlaps}\ a_1\ \mathbf{at}\ i \tag{T3.4}$$

$$\exists u_1, u_2, u_3.\ \neg(u_1\ \mathbf{overlaps}\ u_2 \wedge u_2\ \mathbf{overlaps}\ u_3 \Rightarrow u_1\ \mathbf{overlaps}\ u_3) \tag{T3.5}$$

$$\exists a_1, a_2, a_3.\ \neg(a_1\ \mathbf{overlaps}\ a_2\ \mathbf{at}\ i \wedge a_2\ \mathbf{overlaps}\ a_3\ \mathbf{at}\ i \Rightarrow a_1\ \mathbf{overlaps}\ a_3\ \mathbf{at}\ i) \tag{T3.6}$$

Proof. Since u is an occurrent by A2.4 and T2.1, we can prove T3.1 by A3.1 and D3.3. Similarly since a is a continaunt by A2.15 and T2.2, we can prove T3.2 by A3.2 and D3.4. T3.3 and T3.4 follow from D3.3 and D3.4, respectively. In order to prove T3.5 by contradiction we choose spatiotemporal regions u_1, u_2 and u_2 such that u_1 **overlaps** u_2, u_2 **overlaps** u_3 and $\neg(u_1\ \mathbf{overlaps}\ u_2)$. We prove T3.6 in a similar fashion. □

A spatiotemporal region type U_1 *overlaps* a spatiotemporal region type U_2 if and only if for all instances u_1 of U_1, there exists an instance u_2 of U_2 such that u_1 **overlaps** u_2. We define similar relations for temporal region types and spatial regions types (not shown here). An independent continuant type A_1 *overlaps* an independent continuant type A_2 if and only if for all instances a_1 of A_1 at any temporal instant i, there exists an instance a_2 of A_2 at i such that a_1 overlaps a_2 at i.

$$U_1\ \mathit{overlaps}\ U_2 =_{def} \forall u_1.\ u_1 : U_1 \Rightarrow \exists u_2.\ u_2 : U_2 \wedge u_1\ \mathbf{overlaps}\ u_2 \tag{D3.5}$$

$$A_1\ \mathit{overlaps}\ A_2 =_{def} \forall a_1, i.\ a_1 : A_1\ \mathbf{at}\ i \Rightarrow \exists a_2.\ a_2 : A_2\ \mathbf{at}\ i \wedge a_1\ \mathbf{overlaps}\ a_2\ \mathbf{at}\ i \tag{D3.6}$$

For example: *cube overlaps cube face*; *nucleus overlaps cell*.

The type-level overlap relation is reflexive (T3.7 and T3.8), symmetric between spatiotemporal region types (A3.7), antisymmetric between independent continuant types (A3.8), and intransitive (A3.9 and A3.10). Note that T3.7, A3.7 and A3.9 also hold for temporal region types and spatial region types. It is possible that A_1 in general overlaps A_2 while no analogous statement holds for A_2 in relation to A_1. For example, although *uterine tract overlaps urogenital system*, it is not the case in general that *urogenital system overlaps uterine tract*.

$$U\ \mathit{overlaps}\ U \tag{T3.7}$$

$$A\ \mathit{overlaps}\ A \tag{T3.8}$$

$$U_1\ \mathit{overlaps}\ U_2 \Rightarrow U_2\ \mathit{overlaps}\ U_1 \tag{A3.7}$$

$$A_1\ \mathit{overlaps}\ A_2 \wedge A_2\ \mathit{overlaps}\ A_1 \Rightarrow A_1 = A_2 \tag{A3.8}$$

$$\exists U_1, U_2, U_3.\ \neg(U_1\ \mathit{overlaps}\ U_2 \wedge U_2\ \mathit{overlaps}\ U_3 \Rightarrow U_1\ \mathit{overlaps}\ U_3) \tag{A3.9}$$

$$\exists A_1, A_2, A_3.\ \neg(A_1\ \mathit{overlaps}\ A_2 \wedge A_2\ \mathit{overlaps}\ A_3 \Rightarrow A_1\ \mathit{overlaps}\ A_3) \tag{A3.10}$$

Proof. T3.7 can be proved by D3.5 and T3.1. T3.8 can be proved by D3.6 and T3.2. □

4 Connected and scattered regions

This section describes standard mereological relations, for example as outlined in (Casati & Varzi 1999) and (Smith 1996), which allow us to define connected and scattered spatiotemporal and temporal regions.

For every property or condition φ that is true of at least one spatiotemporal region, there is a spatiotemporal region consisting precisely of all the φers. This spatiotemporal region is called the spatiotemporal fusion of the φers and is denoted $\sigma u(\varphi u)$. We define the temporal fusion and spatial fusion of the φers in a similar fashion (not shown here).

$$\sigma u(\varphi u) =_{def} \iota u_1 \forall u_2.\ (u_1\ \mathbf{overlaps}\ u_2 \Leftrightarrow \exists u.\ (\varphi u \wedge u\ \mathbf{overlaps}\ u_2)) \tag{D4.1}$$

We call $u_1 + u_2$ the sum of spatiotemporal regions u_1 and u_2, and define it as the spatiotemporal fusion of parts of u_1 or u_2. We define the sum of temporal regions and the sum of spatial regions in a similar fashion (not shown here).

$$u_1 + u_2 =_{def} \sigma u(u\, \textbf{part\_of}\, u_1 \vee u\, \textbf{part\_of}\, u_2) \quad \text{(D4.2)}$$

We call $u_1 - u_2$ the difference of spatiotemporal region u_1 from spatiotemporal region u_2, and define it as the spatiotemporal fusion of parts of u_1 which don't overlap u_2. We call $\bar{u}$ the complement of spatiotemporal region u, and define it as the spatiotemporal fusion of spatiotemporal regions which don't overlap u. We define the difference of temporal regions and the difference of spatial regions, and the complement of both temporal and spatial regions in a similar fashion (not shown here).

$$u_1 - u_2 =_{def} \sigma u(u\, \textbf{part\_of}\, u_1 \wedge \neg(u\, \textbf{overlaps}\, u_2)) \quad \text{(D4.3)}$$

$$\bar{u} =_{def} \sigma u'(\neg(u'\, \textbf{overlaps}\, u)) \quad \text{(D4.4)}$$

The spatiotemporal region u_1 is an **interior_part_of** the spatiotemporal region u_2 if and only if u_1 is a non-equivalent (*i.e.* proper) part of u_2 and any spatiotemporal region which partially overlaps u_1 also overlaps the difference of u_2 from u_1. We define similar relations for temporal regions and spatial regions (not shown here).

$$\begin{aligned} u_1\, \textbf{interior\_part\_of}\, u_2 =_{def}\ & u_1\, \textbf{part\_of}\, u_2 \wedge u_1 \neq u_2 \\ \wedge (\forall u'.\ & u'\, \textbf{overlaps}\, u_1 \wedge \neg(u'\, \textbf{part\_of}\, u_1) \\ & \wedge \neg(u_1\, \textbf{part\_of}\, u') \\ & \Rightarrow u'\, \textbf{overlaps}\, (u_2 - u_1)) \end{aligned} \quad \text{(D4.5)}$$

A spatiotemporal region u_1 **crosses** a spatiotemporal region u_2 if and only if u_1 overlaps both u_2 and its complement. A spatiotemporal region u_1 **straddles** a spatiotemporal region u_2 if and only if any spatiotemporal region for which u_1 is an interior part also crosses u_2. We define similar relations for temporal regions and spatial regions (not shown here).

$$u_1\, \textbf{crosses}\, u_2 =_{def} u_1\, \textbf{overlaps}\, u_2 \wedge u_1\, \textbf{overlaps}\, \bar{u_2} \quad \text{(D4.6)}$$

$$u_1\, \textbf{straddles}\, u_2 =_{def} \forall u.\ u_1\, \textbf{interior\_part\_of}\, u \Rightarrow u\, \textbf{crosses}\, u_2 \quad \text{(D4.7)}$$

A spatiotemporal region u' is the **boundary_of** a spatiotemporal region u if and only if any part of u' also straddles u. We call $\hat{u}$ the closure of a spatiotemporal region u and define it as the sum of u and its boundaries. A spatiotemporal region u_1 is **separate_from** a spatiotemporal region u_2 if and only if the closure of u_1 does not overlap u_2 and u_1 does not overlap the closure of u_2. We define similar relations for temporal regions and spatial regions (not shown here).

$$u'\, \textbf{boundary\_of}\, u =_{def} \forall u''.\ u''\, \textbf{part\_of}\, u' \Rightarrow u''\, \textbf{straddles}\, u \quad \text{(D4.8)}$$

$$\hat{u} =_{def} u + \sigma u'(u'\, \textbf{boundary\_of}\, u) \quad \text{(D4.9)}$$

$$u_1\, \textbf{separate\_from}\, u_2 =_{def} \neg(\hat{u_1}\, \textbf{overlaps}\, u_2) \wedge \neg(u_1\, \textbf{overlaps}\, \hat{u_2}) \quad \text{(D4.10)}$$

A connected spatiotemporal region u^c is not the sum of separate spatiotemporal regions. Nor is a connected temporal region t^c the sum of separate temporal regions.

$$\neg(\exists u_1, u_2.\ u^c = u_1 + u_2 \wedge u_1\, \textbf{separate\_from}\, u_2) \quad \text{(A4.1)}$$

$$\neg(\exists t_1, t_2.\ t^c = t_1 + t_2 \wedge t_1\, \textbf{separate\_from}\, t_2) \quad \text{(A4.2)}$$

A scattered spatiotemporal region $u^{s(k)}$ is the sum of k separate connected spatiotemporal regions. Likewise a scattered temporal region $t^{s(k)}$ is the sum of k separate connected temporal regions. We use the notation $\bigwedge_{j=1}^{k-1} x_j\, \textbf{rel}\, x_{j+1}$ to mean $x_1\, \textbf{rel}\, x_2 \wedge x_2\, \textbf{rel}\, x_3 \wedge \ldots \wedge x_{k-1}\, \textbf{rel}\, x_k$ for relation **rel**.

$$\exists u_1^c, \ldots, u_k^c.\ u^{s(k)} = u_1^c + \cdots + u_k^c \wedge \bigwedge_{j=1}^{k-1} u_j^c\, \textbf{separate\_from}\, u_{j+1}^c \quad \text{(A4.3)}$$

$$\exists t_1^c, \ldots, t_k^c.\ t^{s(k)} = t_1^c + \cdots + t_k^c \wedge \bigwedge_{j=1}^{k-1} t_j^c\, \textbf{separate\_from}\, t_{j+1}^c \quad \text{(A4.4)}$$

We represent a scattered temporal region comprised of k separate temporal intervals by $v^{s(k)}$.

5 Spatial, temporal and spatiotemporal projections

The time projection function τ maps a process to its 'spell', *i.e.* the temporal region corresponding to the time during which the process endures. In BFO, we make the assumption that there is no such thing as an instantaneous process, hence any process endures through either a temporal interval v, or through a scattered temporal region $v^{s(k)}$ comprised of k temporal intervals separated in time (A5.1). The spacetime projection function μ maps a process to its 'span', *i.e.* the spatiotemporal region corresponding to the area of spacetime in which the process unfolds. For any process there exists a spatiotemporal region in which that process unfolds (A5.2). The space projection function ψ maps a process to its 'spread', *i.e.* the spatial region corresponding to the area of space over which the process covers. For any process there exists a spatial region over which that process covers (A5.3). The time-indexed space projection function ψ^i maps an independent continuant at the temporal instant i to the spatial region corresponding to the area of space which the independent continuant occupies at i. If an independent continuant exists at a given temporal instant, then there is a unique spatial region occupied by that continuant (A5.4 and A5.5).

$$\exists v.\ (\tau(p) = v) \vee \exists v^{s(k)}.\ (\tau(p) = v^{s(k)}) \quad \text{(A5.1)}$$

$$\exists u.\ \mu(p) = u \quad \text{(A5.2)}$$

$$\exists s.\ \psi(p) = s \quad \text{(A5.3)}$$

$$a : \mathit{Independent\_Continuant}\ \textbf{at}\ i \Rightarrow \exists s.\ \psi^i(a) = s \quad \text{(A5.4)}$$

$$\psi^i(a) = s_1 \wedge \psi^i(a) = s_2 \Rightarrow s_1 = s_2 \quad \text{(A5.5)}$$

If a process p_1 is part of a process p_2, then p_1's spell is part of p_2's spell. If a process p_1 is part of a process p_2, then p_1's span is part of p_2's span; moreover if p_1's span is part of p_2's span then p_1 is part of p_2. If a process p_1 is part of a process p_2, then p_1's spread is part of p_2's spread.

$$p_1 \,\textbf{part\_of}\, p_2 \Rightarrow \tau(p_1) \,\textbf{part\_of}\, \tau(p_2) \tag{A5.6}$$

$$p_1 \,\textbf{part\_of}\, p_2 \Leftrightarrow \mu(p_1) \,\textbf{part\_of}\, \mu(p_2) \tag{A5.7}$$

$$p_1 \,\textbf{part\_of}\, p_2 \Rightarrow \psi(p_1) \,\textbf{part\_of}\, \psi(p_2) \tag{A5.8}$$

Note that BFO already features an expression a **located_in** s **at** i which is semantically equivalent to $\psi^i(a) = s$. Moreover in (Smith et al. 2005) an independent continuant a_1 is **located_in** an independent continuant a_2 at temporal instant i if and only if there are spatial regions s_1 and s_2 such that a_1 **located_in** s_1 **at** i and a_2 **located_in** s_2 **at** i and s_1 **part_of** s_2.

5.1 Temporal ordering

In BFO, all times are with respect to a single inertial frame of reference (making the ontology inadequate for describing special relativity). The primitive binary relational assertion i_1 **earlier_than** i_2 is used to order temporal instants along the time line. Although we do not show them here, we specify axioms which tell us that the temporal ordering relation is irreflexive, asymmetric and transitive.

Two non-equivalent temporal instants are seperate and one is earlier than the other.

$$\begin{aligned} i_1 \neq i_2 \Leftrightarrow i_1 \,\textbf{separate\_from}\, i_2 \\ \Leftrightarrow (i_1 \,\textbf{earlier\_than}\, i_2 \vee i_2 \,\textbf{earlier\_than}\, i_1) \end{aligned} \tag{A5.9}$$

According to the ontology, temporal instants only exist at the boundary of temporal intervals. Hence the boundary of any temporal interval v is the sum of two temporal instants which are separated.

$$\begin{aligned} \exists i_1, i_2.\ \ \sigma t(t \,\textbf{boundary\_of}\, v) = i_1 + i_2 \\ \wedge\, i_1 \,\textbf{separate\_from}\, i_2 \end{aligned} \tag{A5.10}$$

A temporal instant i_1 **starts** a temporal interval v if and only if i_1 is the earlier instant lying at v's boundary. Likewise, a temporal instant i_2 **ends** a temporal interval v if and only if i is the later instant lying at v's boundary.

$$\begin{aligned} i_1 \,\textbf{starts}\, v =_{def} \\ \exists i_2.\ \ \sigma t(t \,\textbf{boundary\_of}\, v) = i_1 + i_2 \\ \wedge\, i_1 \,\textbf{earlier\_than}\, i_2 \end{aligned} \tag{D5.1}$$

$$\begin{aligned} i_2 \,\textbf{ends}\, v =_{def} \\ \exists i_1.\ \ \sigma t(t \,\textbf{boundary\_of}\, v) = i_1 + i_2 \\ \wedge\, i_1 \,\textbf{earlier\_than}\, i_2 \end{aligned} \tag{D5.2}$$

Every temporal interval is started and ended by a temporal instant.

$$\begin{aligned} \exists i_1, i_2.\ \ i_1 \,\textbf{starts}\, v \wedge i_2 \,\textbf{ends}\, v \\ \wedge\, i_1 \,\textbf{earlier\_than}\, i_2 \end{aligned} \tag{T5.1}$$

Proof. T5.1 follows from A5.10 with A5.9 and using D5.1 and D5.2. □

5.2 Participation

The primitive ternary relational assertion p **has_participant** a **at** i is used to specify that independent continuant a at temporal instant i participates in some way in process p. A process type P *has_participant* independent continuant type A if and only if for all instances p of P there exists some a of A at some temporal instant i such that p **has_participant** a **at** i.

$$\begin{aligned} P \; \textit{has\_participant}\; A =_{def} \forall p.\ \ p\!:\!P \\ \Rightarrow \exists a, i.\ \ a\!:\!A \,\textbf{at}\, i \\ \wedge\, p \,\textbf{has\_participant}\, a \,\textbf{at}\, i \end{aligned} \tag{D5.3}$$

For example: *cell division has_participant chromosome*; *photosynthesis has_participant chlorophyll*.

An independent continuant a **exists_at** a temporal instant i if and only if there is some process in which a is a participant at i. (An independent continuant will at least participate in its own life.) A process p **occurs_at** at i if and only if there is some independent continuant a which is a participant of p at i.

$$\begin{aligned} a \,\textbf{exists\_at}\, i =_{def} \\ \exists p.\ \ p \,\textbf{has\_participant}\, a \,\textbf{at}\, i \end{aligned} \tag{D5.4}$$

$$\begin{aligned} p \,\textbf{occurs\_at}\, i =_{def} \\ \exists a.\ \ p \,\textbf{has\_participant}\, a \,\textbf{at}\, i \end{aligned} \tag{D5.6}$$

If an independent continuant is instantiated at a temporal instant, then it exists at that temporal instant and *vice versa.* There are at least two temporal instants at which any process occurs.

$$\begin{aligned} a\!:\!\textit{Independent\_Continuant} \,\textbf{at}\, i \\ \Leftrightarrow a \,\textbf{exists\_at}\, i \end{aligned} \tag{A5.11}$$

$$\begin{aligned} \exists i_1, i_2.\ \ p \,\textbf{occurs\_at}\, i_1 \wedge p \,\textbf{occurs\_at}\, i_2 \\ \wedge\, i_1 \neq i_2 \end{aligned} \tag{T5.2}$$

Proof. From A5.1 we know the spell of p is either a temporal interval or a scattered temporal region $v^{s(k)}$ comprised of k temporal intervals. If we choose the former, T5.2 follows from A5.12 and T5.1. If we choose the latter, T5.2 follows from A5.13, A4.4 and T5.1. We can use A4.4 since we know $v^{s(k)}$ is a temporal region by A2.12, A2.10, the transitivity of the subtype relation, and T2.1. □

5.3 First and last instants

A temporal instant i is the **first_instant_of** a process p if and only if p occurs at i and does not occur at any temporal instant before i. A temporal instant i is the **last_instant_of** a process p if and only if p occurs at i and does not occur at any temporal instant after i.

$$\begin{aligned} i \,\textbf{first\_instant\_of}\, p =_{def} p \,\textbf{occurs\_at}\, i \\ \wedge\, \forall i'.\ \ i' \,\textbf{earlier\_than}\, i \\ \Rightarrow \neg(p \,\textbf{occurs\_at}\, i') \end{aligned} \tag{D5.7}$$

$$\begin{aligned} i \,\textbf{last\_instant\_of}\, p =_{def} p \,\textbf{occurs\_at}\, i \\ \wedge\, \forall i'.\ \ i \,\textbf{earlier\_than}\, i' \\ \Rightarrow \neg(p \,\textbf{occurs\_at}\, i') \end{aligned} \tag{D5.8}$$

A process has unique first and last temporal instants.

$$i_1 \textbf{ first\_instant\_of } p \wedge i_2 \textbf{ first\_instant\_of } p \Rightarrow i_1 = i_2 \quad \text{(T5.3)}$$

$$i_1 \textbf{ last\_instant\_of } p \wedge i_2 \textbf{ last\_instant\_of } p \Rightarrow i_1 = i_2 \quad \text{(T5.4)}$$

Proof. T5.3 can be proved by contradiction using D5.7 and A5.9. T5.4 can be proved by contradiction using D5.8 and A5.9. □

If the spell of a process p is the temporal interval v and temporal instants i_1 and i_2 start and end v, respectively, then p occurs at both i_1 and i_2 and does not occur at any instant before i_1 or after i_2. If the spell of a process p is the scattered temporal region $v^{s(k)}$ comprised of temporal intervals $v_1, \ldots, v_k$ and temporal instant i_1 starts v_1 and temporal instant i_2 ends v_k, then p occurs at both i_1 and i_2 and does not occur at any instant before i_1 or after i_2.

$$\begin{aligned} &\tau(p) = v \wedge i_1 \textbf{ starts } v \wedge i_2 \textbf{ ends } v \quad \text{(A5.12)} \\ &\Rightarrow p \textbf{ occurs\_at } i_1 \wedge p \textbf{ occurs\_at } i_2 \\ &\wedge \forall i', i''.\ (i' \textbf{ earlier\_than } i_1 \\ &\wedge i_2 \textbf{ earlier\_than } i'' \\ &\Rightarrow \neg(p \textbf{ occurs\_at } i') \wedge \neg(p \textbf{ occurs\_at } i'')) \end{aligned}$$

$$\begin{aligned} &\tau(p) = v^{s(k)} \wedge i_1 \textbf{ starts } v_1 \wedge i_2 \textbf{ ends } v_k \quad \text{(A5.13)} \\ &\Rightarrow p \textbf{ occurs\_at } i_1 \wedge p \textbf{ occurs\_at } i_2 \\ &\wedge \forall i', i''.\ (i' \textbf{ earlier\_than } i_1 \\ &\wedge i_2 \textbf{ earlier\_than } i'' \\ &\Rightarrow \neg(p \textbf{ occurs\_at } i') \wedge \neg(p \textbf{ occurs\_at } i'')) \end{aligned}$$

Any process has a first and last temporal instant such that the former is earlier than the latter. If the spell of a process p is the temporal interval v and temporal instant i starts (ends) v, then i is the first (last) instant of p. If the spell of a process p is the scattered temporal region $v^{s(k)}$ comprised of temporal intervals $v_1, \ldots, v_k$ and temporal instant i starts (ends) v_1 (v_k), then i is the first (last) instant of p.

$$\exists i_1, i_2.\ i_1 \textbf{ first\_instant\_of } p \wedge i_2 \textbf{ last\_instant\_of } p \wedge i_1 \textbf{ earlier\_than } i_2 \quad \text{(T5.6)}$$

$$\tau(p) = v \wedge i \textbf{ starts } v \Rightarrow i \textbf{ first\_instant\_of } p \quad \text{(T5.7)}$$

$$\tau(p) = v^{s(k)} \wedge i \textbf{ starts } v_1 \Rightarrow i \textbf{ first\_instant\_of } p \quad \text{(T5.8)}$$

$$\tau(p) = v \wedge i \textbf{ ends } v \Rightarrow i \textbf{ last\_instant\_of } p \quad \text{(T5.9)}$$

$$\tau(p) = v^{s(k)} \wedge i \textbf{ ends } v_k \Rightarrow i \textbf{ last\_instant\_of } p \quad \text{(T5.10)}$$

Proof. Using A5.1, if the spell of p is a temporal interval, then T5.6 can be proved by T5.1, A5.12, D5.7 and D5.8. If the spell of p is a scattered temporal region comprised of temporal intervals, then T5.6 can be proved by T5.1, the transitivity of the temporal ordering relation **earlier_than**, A5.13, D5.7 and D5.8. T5.7 can be proved by A5.12 and D5.7, whereas T5.8 can be proved by A5.13 and D5.7. T5.9 can be proved by A5.12 and D5.8, whereas T5.10 can be proved by A5.13 and D5.8. □

Using this theory we can define relations such as **preceded_by** and **immediately_preceded_by**, whereby a process p' is **preceded_by** a process p if and only if the last temporal instant of p is earlier than the first temporal instant of p', and a process p' is **immediately_preceded** by a process p if and only if there exists a temporal instant which is both the first instant of p' and the last instant of p.

6 Boundaries, fiat parts and aggregates of independent continuants and processes

As shown in Figure 2, *Boundary_Of_Object*, *Fiat_Part_Of_Object* and *Object_Aggregate* were featured in the original BFO (1.0 version) continuant type hierarchy, along with *Object* and *Site*, as subclasses of *Independent_Continuant*. All five types were considered mutually exclusive. *Boundary_Of_Process*, *Fiat_Part_Of_Process* and *Process_Aggregate*, along with *Process*, were featured in the BFO 1.0 occurrent type hierarchy as subclasses of *Processual_Entity*. These four types were also deemed mutually exclusive. In the 1.0 version, *Processual_Entity* has the same interpretation we (in this paper) have provided for *Process*, *i.e.* an entity which unfolds or develops in time, and which depends on one or more independent continuants. The type *Process* is interpreted as an entity that is a maximally connected spatiotemporal whole which has *bona fide* beginnings and endings. The candidate BFO (2.0 version) reflects the type hierarchy given in Figure 1. The type *Processual_Entity* has been renamed *Process*, and the old sense of *Process* (as being an entity that is a maximally connected spatiotemporal whole) has been removed, since these kinds of processes occur rarely in reality. The type *Object* has been renamed *Material_Continuant*. The types *Boundary_Of_Object*, *Fiat_Part_Of_Object* and *Object_Aggregate* along with *Boundary_Of_Process*, *Fiat_Part_Of_Process* and *Process_Aggregate* have been entirely removed. This is because we may only talk of aggregations of material continuants, we cannot talk of, say, aggregations of fiat parts and/or boundaries. Moreover, it rules out entities which are simultaneously fiat parts and aggregates, or say, boundaries and fiat parts.

Processual_Entity
- *Process*
- *Boundary_Of_Process*
- *Fiat_Part_Of_Process*
- *Process_Aggregate*

Independent_Continuant
- *Object*
- *Site*
- *Boundary_Of_Object*
- *Fiat_Part_Of_Object*
- *Object_Aggregate*

Figure 2: Original BFO 1.0 *Processual_Entity* and *Independent_Continuant* type hierarchy.

In this section we introduce a number of functions which still allow us to talk of boundaries, fiat parts and aggregates, as well as aggregations which include boundaries and fiat parts.

6.1 Boundaries

The function β^i maps a material continuant at the temporal instant i to its boundary at i. We refer to its boundary as an 'object boundary'. The function β maps a process to its boundary. Here we refer to its boundary as a 'process boundary'. An object (or process) boundary can be thought of as that part of a material continuant (or process) that exists exactly at the limitation of that material continuant (or process). If a material continuant m exists at a temporal instant i, then it has a boundary which is part of m at i. Furthermore, every process p has a boundary which is part of p.

$$m \,\textbf{exists\_at}\, i \Rightarrow \beta^i(m) \,\textbf{part\_of}\, m \,\textbf{at}\, i \tag{A6.1}$$

$$\beta(p) \,\textbf{part\_of}\, p \tag{A6.2}$$

The spatial region occupied by the object boundary of a material continuant m at the temporal instant i is the boundary of the spatial region occupied by m at i. The spread of a process boundary is the boundary of that process' spread. Similar axioms hold for the span and spell of a process and its boundary.

$$\psi^i(\beta^i(m)) \,\textbf{boundary\_of}\, \psi^i(m) \tag{A6.3}$$

$$\psi(\beta(p)) \,\textbf{boundary\_of}\, \psi(p) \tag{A6.4}$$

$$\mu(\beta(p)) \,\textbf{boundary\_of}\, \mu(p) \tag{A6.5}$$

$$\tau(\beta(p)) \,\textbf{boundary\_of}\, \tau(p) \tag{A6.6}$$

A material continuant type M *has_object_boundary* material continuant type M' if and only if for all instances m of M at any temporal instant i, $\beta^i(m)$ is an instance of M' at i. A process type P *has_process_boundary* process type P' if and only if for all instances p of P, $\beta(p)$ is an instance of P'.

$$M \textit{ has\_object\_boundary } M' =_{def} \tag{D6.1}$$

$$\forall m, i.\;\; m\!:\!M \,\textbf{at}\, i \Rightarrow \beta^i(m)\!:\!M' \,\textbf{at}\, i$$

$$P \textit{ has\_process\_boundary } P' =_{def} \tag{D6.2}$$

$$\forall p.\;\; p\!:\!P \Rightarrow \beta(p)\!:\!P'$$

The boundary of a cavity is the internal boundary of the material continuant which fully surrounds it. Other sites (such as hollows and tunnels) have a boundary which is the boundary of the containing walls of the hollow or tunnel. For example a boundary of the interior of my coffee mug is the boundary of the solid, containing part of the mug. The external, infinitely thin surface of an apple is a boundary of the apple, but it is also a boundary of the surrounding air. A boundary of a tunnel bored into the apple is the boundary of the tunnel walls. Note that the boundary of the apple is part of the apple, but the boundary of the surrounding air (tunnel) it is not part of the surrounding air (tunnel).

6.2 Fiat parts

The function φ^i maps an independent continuant at the temporal instant i to its fiat part. We refer to its fiat part as a 'fiat object part'. The function φ maps a process to its fiat part. Here we refer to its fiat part as a 'fiat process part'. A fiat object (or process) part can be thought of as a part of an independent continuant (or process) which is demarcated by human partitioning. In contrast, a *bona fide* object (or process) part of an independent continuant (or process) is demarcated by a discontinuity present at physical gradients which is independent of human partitioning. We refer the reader to (Smith 2001) for further discussion regarding fiat parts. A fiat part of an independent continuant a at the temporal instant i is a part of a at i. A fiat part of a process p is a part of p.

$$\varphi^i(a) \,\textbf{part\_of}\, a \,\textbf{at}\, i \tag{A6.9}$$

$$\varphi(p) \,\textbf{part\_of}\, p \tag{A6.10}$$

An independent continuant type A *has_fiat_object_part* independent continuant type A' if and only if for all instances a of A at any temporal instant i, there exists a fiat part which is an instance of A' at i. A process type P *has_fiat_process_part* process type P' if and only if for all instances p of P, there exists a fiat part which is an instance of P'.

$$A \textit{ has\_fiat\_object\_part } A' =_{def} \tag{D6.3}$$

$$\forall a, i.\;\; a\!:\!A \,\textbf{at}\, i \Rightarrow \varphi^i(a)\!:\!A' \,\textbf{at}\, i$$

$$P \textit{ has\_fiat\_process\_part } P' =_{def} \tag{D6.4}$$

$$\forall p.\;\; p\!:\!P \Rightarrow \varphi(p)\!:\!P'$$

For example: *lung has_fiat_object_part upper lobe*; *body has_fiat_object_part ventral surface*.

If an independent continuant type A has a fiat object part type A' then A' is a part of A. Likewise if a process type P has a fiat process part P' then P' is a part of P.

$$A \textit{ has\_fiat\_object\_part } A' \Rightarrow A' \textit{ part\_of } A \tag{T6.1}$$

$$P \textit{ has\_fiat\_process\_part } P' \Rightarrow P' \textit{ part\_of } P \tag{T6.2}$$

Proof. T6.1 can be proved by D6.3, A6.9 and D3.2, since any independent continuant is a continuant by A2.15 and T2.1. Since any process is an occurrent by A2.3 and T2.1, T6.2 can be proved by D6.4, A6.10 and D3.1. □

6.3 Aggregates

The function α^i maps k independent continuants $a_1, \ldots, a_k$ to their aggregation at the temporal instant i. We refer to their aggregation as an 'object aggregate'. The function α maps k processes $p_1, \ldots, p_k$ to their aggregation. Here we refer to their aggregation as a 'process aggregate'. If k independent continuants occupy separate spatial regions we can form their object aggregate. If k processes span separate spatiotemporal regions we can form their process aggregate.

$$\bigwedge_{j=1}^{k} a_j \,\textbf{exists\_at}\, i \tag{A6.11}$$

$$\wedge \bigwedge_{j=1}^{k-1} \psi^i(a_j) \,\textbf{separate\_from}\, \psi^i(a_{j+1})$$

$$\Rightarrow \exists a.\;\; \alpha^i(a_1, \ldots, a_k) = a$$

$$\bigwedge_{j=1}^{k} p_j \,\textbf{occurs\_at}\, i \tag{A6.12}$$

$$\wedge \bigwedge_{j=1}^{k-1} \mu(p_j) \,\textbf{separate\_from}\, \mu(p_{j+1})$$

$$\Rightarrow \exists p.\;\; \alpha(p_1, \ldots, p_k) = p$$

If a is the object aggregate of independent continuants $a_1, \ldots, a_k$ at temporal instant i, then a_j is part of a at i for $1 \leq j \leq k$. If p is the process aggregate of processes $p_1, \ldots, p_k$, then p_j is part of p for $1 \leq j \leq k$.

$$\alpha^i(a_1, \ldots, a_k) = a \Rightarrow \bigwedge_{j=1}^{k} a_j \ \mathbf{part\_of}\ a \ \mathbf{at}\ i \tag{A6.13}$$

$$\alpha(p_1, \ldots, p_k) = p \Rightarrow \bigwedge_{j=1}^{k} p_j \ \mathbf{part\_of}\ p \tag{A6.14}$$

If a is the object aggregate of independent continuants $a_1, \ldots, a_k$ at temporal instant i, then the spatial region occupied by a at i is the sum of the spatial regions occupied by each a_j at i for $1 \leq j \leq k$. If p is the process aggregate of processes $p_1, \ldots, p_k$, then the spread of p is the sum of the spreads of each p_j for $1 \leq j \leq k$. Similar axioms hold for the span and spell of a process aggregate.

$$\alpha^i(a_1, \ldots, a_k) = a \Rightarrow \psi^i(a) = \sum_{j=1}^{k} \psi^i(a_j) \tag{A6.15}$$

$$\alpha(p_1, \ldots, p_k) = p \Rightarrow \psi(p) = \sum_{j=1}^{k} \psi(p_j) \tag{A6.16}$$

$$\alpha(p_1, \ldots, p_k) = p \Rightarrow \mu(p) = \sum_{j=1}^{k} \mu(p_j) \tag{A6.17}$$

$$\alpha(p_1, \ldots, p_k) = p \Rightarrow \tau(p) = \sum_{j=1}^{k} \tau(p_j) \tag{A6.18}$$

An independent continuant type A is an *object_aggregate_of* independent continuant types $A_1, \ldots, A_k$ if and only if for all instances a of A at any temporal instant i, there exists instances $a_1, \ldots, a_k$ such that $\alpha^i(a_1, \ldots, a_k) = a$ and a_j is an instance of A_j at i for $1 \leq j \leq k$. A process type P is a *process_aggregate_of* process types $P_1, \ldots, P_k$ if and only if for all instances p of P, there exists instances $p_1, \ldots, p_k$ such that $\alpha(p_1, \ldots, p_k) = p$ and p_j is an instance of P_j for $1 \leq j \leq k$.

$$\begin{aligned} &A \ \textit{object\_aggregate\_of}\ (A_1, \ldots, A_n) =_{def} \\ &\quad \forall a, i.\ \ a\!:\!A \ \mathbf{at}\ i \\ &\quad \Rightarrow \exists a_1, \ldots, a_k.\ \ \alpha^i(a_1, \ldots, a_k) = a \\ &\quad \wedge \bigwedge_{j=1}^{k} a_j\!:\!A_j \ \mathbf{at}\ i \end{aligned} \tag{D6.5}$$

$$\begin{aligned} &P \ \textit{process\_aggregate\_of}\ (P_1, \ldots, P_n) =_{def} \\ &\quad \forall p.\ \ p\!:\!P \\ &\quad \Rightarrow \exists p_1, \ldots, p_k.\ \ \alpha(p_1, \ldots, p_k) = p \\ &\quad \wedge \bigwedge_{j=1}^{k} p_j\!:\!P_j \end{aligned} \tag{D6.6}$$

For example: *string trio object_aggregate_of* (*violinist*, *violist*, *cellist*); *playing of a string trio process_aggregate_of* (*playing of violinist*, *playing of violist*, *playing of cellist*).

If independent continuant type A is an object aggregate of independent continuant types $A_1, \ldots, A_k$ then each A_j is a part of A for $1 \leq j \leq k$. Likewise if process type P is a process aggregate of process types $P_1, \ldots, P_k$ then each P_j is a part of P for $1 \leq j \leq k$.

$$A \ \textit{object\_aggregate\_of}\ (A_1, \ldots, A_n) \Rightarrow \bigwedge_{j=1}^{k} A_j \ \textit{part\_of}\ A \tag{T6.3}$$

$$P \ \textit{process\_aggregate\_of}\ (P_1, \ldots, P_n) \Rightarrow \bigwedge_{j=1}^{k} P_j \ \textit{part\_of}\ P \tag{T6.4}$$

Proof. T6.3 can be proved by D6.5, A6.13 and D3.2, since any independent continuant is a continuant by A2.15 and T2.1. Since any process is an occurrent by A2.3 and T2.1, T6.4 can be proved by D6.6, A6.14 and D3.1. □

6.4 DOLCE comparison

As a summary, it may be instructive for the reader to compare the approach of BFO by contrast to that of DOLCE. The Descriptive Ontology for Linguistic and Cognitive Engineering has been developed at the Laboratory for Applied Ontology as a reference module for a library of ontologies which aims to provide ontology infrastructure for the Semantic Web. DOLCE has a cognitive bias since it aims at capturing the ontological categories underlying natural language (Gangemi et al. 2002).

The hierarchy for DOLCE's *Endurant* and *Perdurant* universals are shown in Figure 3. DOLCE treats boundaries as instances of the universal *Feature*. Other features include holes, bumps, surfaces and stains. Features are specifically dependent on physical objects which act as their hosts. DOLCE does not consider boundaries of perdurants, nor does it consider fiat parts. Fiat parts are necessary for biological representation, but have little relevance linguistically.

- *Endurant*
  - *Physical_Endurant*
    - *Amount_Of_Matter*
    - *Feature*
    - *Physical_Object*
  - *Non_Physical_Endurant*
  - *Arbitrary_Sum*

- *Perdurant*
  - *Event*
    - *Achievement*
    - *Accomplishment*
  - *Stative*
    - *State*
    - *Process*

Figure 3: DOLCE *Endurant* and *Perdurant* universal hierarchy.

In DOLCE, an amount of matter refers to mass nouns, such as some air, some gold, some coffee. A physical object is an endurant with unity, and is allowed to change parts while keeping its identity. Examples of non-physical endurants include poems, or ideas. An arbitrary sum is a collection of endurants which has no overall unity and cannot be considered an essential whole. Arbitrary sums change identity when they change parts. For example both my left foot and my car is an arbitrary sum. Thus DOLCE allows aggregations of boundaries and features along with other physical objects.

Perdurants (also called occurrences) in DOLCE are classified according to their cumulativity and homeomericity. Hence the way aggregations of perdurants are formed in DOLCE is built into the universal hierarchy itself. Events are non-cumulative. For example the aggregation of two consecutive events of finishing a book does not form a new event which is the finishing of a book. Events are differentiated as achievements or accomplishments. Achievements are instantaneous, for example: reaching the top of a mountain, departing somewhere, or dying. Accomplishments are non-instantaneous, for example: a performance, or climbing a mountain. Statives are cumulative and are differentiated as states and processes. States, such as sitting, or being red, are homeomeric. Each stage of a sitting occurrence is still a sitting occurrence. Processes, such as running or writing are non-homeomeric, since there are very short stages of these occurrences which do not involve running or writing.

7 Conclusion

We have proposed the introduction of a number of new functions in order to deal with boundaries, fiat parts and aggregates in Basic Formal Ontology. These functions are flexible enough to handle aggregations of processes and independent continuants along with their fiat parts and boundaries, and we can also use these functions to express the fiat parts of boundaries. We have introduced time, space and spacetime projection functions to improve upon the ontology's expressibility. We have formalised a simple temporal theory using these functions.

References

Arp, R. & Smith, B. (2008), 'Function, role and disposition in Basic Formal Ontology', in *Proceedings of the 11th Annual Bio-Ontologies Meeting*.

Basic Formal Ontology website (2010), `http://www.ifomis.org/bfo/home`.

Casati, R. & Varzi, A. (1999), *Parts and Places: The Structures of Spatial Representation*, The MIT Press.

DOLCE website (2010), `http://www.loa-cnr.it/DOLCE.html`.

Gangemi, A., Guarino, N., Masolo C., Oltramari A. & Schneider, L. (2002), 'Sweetening Ontologies with DOLCE', in *Proceedings of the 13th International Conference on Knowledge Engineering and Knowledge Management* LNCS 2473, 223–233.

Grenon, P. & Smith, B. (2004), 'SNAP and SPAN: towards dynamic spatial ontology', *Spatial Cognition and Computation: An Interdisciplinary Journal* **4**(1), 69–104.

Relation Ontology website (2010), `http://www.obofoundry.org/ro/`.

Simons, P. (1987), *Parts: A Study In Ontology*, Oxford University Press.

Smith, B., Ceusters, W., Klagges B., Köhler, J., Kumar, A., Lomax, J., Mungall, C., Neuhaus, F., Rector, A. & Rosse, C. (2005), 'Relations in biomedical ontologies', *Genome Biology* **6**(R46).

Smith, B. (1996), 'Mereotopology: a theory of parts and boundaries', *Data and Knowledge Engineering* 20, 287–303.

Smith, B. (2001), 'Fiat objects', *Topoi* **20**(2), 131–148.

Spear, A.D. (2006), Ontology for the twenty first century: an introduction with recommendations, `http://www.ifomis.org/bfo/manual`.

Zemach, E.M. (1970), 'Four ontologies', *The Journal of Philosophy* **67**(8), 231–247.