

# BFO-FOL: A First-Order Logic Formalization of Basic Formal Ontology 2.0

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## Abstract

This article presents a first-order logic formalization of the revised 2.0 version of Basic Formal Ontology (BFO).

BFO-FOL is a formal system specifying the axioms and definitions for expressing Basic Formal Ontology version 2.0 in classical first-order formal logic.

Basic Formal Ontology (BFO) is an upper level ontology initially developed by Barry Smith and Pierre Grenon. The BFO specification is currently undergoing a major revision to version 2.0, which will be supported by a number of formal implementations, including implementations using OWL and CLIF, among others. The first-order logic formalization in BFO-FOL will serve as a foundation for all such implementations.

The BFO 2.0 specification is currently under development, so the formalization presented here represents the state of the specification at the time of writing. The bracketed references of the form [nnn-*nnn*] are to the correspondingly identified definitions, elucidations, axioms, and theorems in the BFO 2.0 specification document [1].

## 1 Formalization

BFO-FOL is an extension of classical first-order formal logic with identity. It can be represented using any standard axiomatization of the logical calculus. The formalization presented here uses the following symbols for negation, conjunction, disjunction, material implication, biconditional implication, universal and particular quantification, respectively:  $\neg, \wedge, \vee, \supset, \equiv, \forall, \exists$ .

## 2 Predicates

The predicates of BFO-FOL are divided into categorial predicates, which are intended to represent categories or universals, and relational predicates, which are intended to represent relations that hold between individuals within those categories.

According to the meta-theory of BFO, categorial predicates are interpreted as expressing the instantiation of the universal indicated by the categorial predicate name. For example, *Object(a)* signifies the instantiation of the universal *Object* by the particular *a*.

Where feasible, predicates have been defined in terms of more primitive predicates. While it is preferable to minimize the number of primitive predicates, some predicates that would seem to be definable needed to be taken as primitive. One reason is that the likely definitions for these predicates would rely on more primitive predicates that are not asserted as categories or relations in BFO. For example, given the primitive category *SpatialRegion*, it would seem that the category *OneDimensionalSpatialRegion* should be definable in terms of that primitive category. However, such a definition would need to rely on dimensions, and *Dimension* is not asserted as a category of BFO.

### 2.1 Primitive Categorial Predicates

The following categorial predicates are taken as primitive:

**Entity(a)** — Intended interpretation: “a is an entity”. [001-001]

**Continuant(a)** — “a is a continuant”. [008-001]

**MaterialEntity(a)** — “a is a material entity”. [019-001]

**Object(a)** — “a is an object”. [024-001]

**ObjectAggregate(a)** — “a is an object aggregate”. [025-002]

**Site(a)** — “a is a site”. [034-001]

**SpatialRegion(a)** — “a is a spatial region”. [035-001]

**ZeroDimensionalSpatialRegion(a)** — “a is a zero-dimensional spatial region”. [037-001]

**OneDimensionalSpatialRegion(a)** — “a is a one-dimensional spatial region”.  
[038-001]

**TwoDimensionalSpatialRegion(a)** — “a is a two-dimensional spatial region”.  
[039-001]

**ThreeDimensionalSpatialRegion(a)** — “a is a three-dimensional spatial region”. [040-001]

**Quality(a)** — “a is a quality”. [055-001]

**RealizableEntity(a)** — “a is a realizable entity”. [058-001]

**Role(a)** — “a is a role”. [061-001]

**Disposition(a)** — “a is a disposition”. [062-001]

**Function(a)** — “a is a function”. [064-001]

**Occurrent(a)** — “a is an occurrent”. [077-001]

**ProcessProfile(a)** — “a is a process profile”. [093-001]

**SpatioTemporalRegion(a)** — “a is a spatio-temporal region”. [095-001]

**TemporalRegion(a)** — “a is a temporal region”. [100-001]

**ZeroDimensionalTemporalRegion(a)** — “a is a zero-dimensional temporal region”. [102-001]

**OneDimensionalTemporalRegion(a)** — “a is a one-dimensional temporal region”. [103-001]

## 2.2 Defined Categorical Predicates

The following categorical predicates are defined as indicated:

**IndependentContinuant(a)** — “a is an independent continuant”. [017-002]

$$\begin{aligned} \text{IndependentContinuant}(a) &=_{df} \\ (Continuant(a) \wedge \neg \exists(b, t) \text{specificallyDependsOn}(a, b, t)) \end{aligned} \quad (1)$$

**FiatObjectPart(a)** — “a is a fiat object part”. [027-001]

$$\begin{aligned} \text{FiatObjectPart}(a) &=_{df} \\ (MaterialEntity(a) \wedge \neg Object(a) \wedge \\ \exists(b, t)(Object(b) \wedge properContinuantPartOfAt(a, b, t))) \end{aligned} \quad (2)$$

**ImmaterialEntity(a)** — “a is an immaterial entity”. [028-001]

$$\begin{aligned} & \text{ImmaterialEntity}(a) =_{df} \\ & (\text{IndependentContinuant}(a) \wedge \\ & \neg \exists(b, t)(\text{MaterialEntity}(b) \wedge \text{continuantPartOfAt}(b, a, t))) \end{aligned} \quad (3)$$

**ContinuantFiatBoundary(a)** — “a is a continuant fiat boundary”. [029-001]

$$\begin{aligned} & \text{ContinuantFiatBoundary}(a) =_{df} (\text{ImmaterialEntity}(a) \wedge \\ & \exists(b)((\text{ZeroDimensionalSpatialRegion}(b) \vee \\ & \text{OneDimensionalSpatialRegion}(b) \vee \\ & \text{TwoDimensionalSpatialRegion}(b)) \wedge \\ & \forall(t)\text{locatedInAt}(a, b, t)) \wedge \\ & \neg \exists(c, t)(\text{SpatialRegion}(c) \wedge \text{continuantPartOfAt}(c, a, t))) \end{aligned} \quad (4)$$

**ZeroDimensionalContinuantFiatBoundary(a)** — “a is a zero-dimensional continuant fiat boundary”. [031-001]

$$\begin{aligned} & \text{ZeroDimensionalContinuantFiatBoundary}(a) =_{df} \\ & \text{ContinuantFiatBoundary}(a) \wedge \\ & \exists(b)(\text{ZeroDimensionalSpatialRegion}(b) \wedge \\ & \forall(t)\text{locatedInAt}(a, b, t)) \end{aligned} \quad (5)$$

**OneDimensionalContinuantFiatBoundary(a)** — “a is a one-dimensional continuant fiat boundary”. [032-001]

$$\begin{aligned} & \text{OneDimensionalContinuantFiatBoundary}(a) =_{df} \\ & (\text{ContinuantFiatBoundary}(a) \wedge \\ & \exists(b)(\text{OneDimensionalSpatialRegion}(b) \wedge \\ & \forall(t)\text{locatedInAt}(a, b, t))) \end{aligned} \quad (6)$$

**TwoDimensionalContinuantFiatBoundary(a)** — “a is a two-dimensional continuant fiat boundary”. [033-001]

$$\begin{aligned} & \text{TwoDimensionalContinuantFiatBoundary}(a) =_{df} \\ & (\text{ContinuantFiatBoundary}(a) \wedge \\ & \exists(b)(\text{TwoDimensionalSpatialRegion}(b) \wedge \\ & \forall(t)\text{locatedInAt}(a, b, t))) \end{aligned} \quad (7)$$

**SpecificallyDependentContinuant(a)** — “a is a specifically dependent continuant”. [050-002]

$$\begin{aligned} \textit{SpecificallyDependentContinuant}(a) =_{df} \\ & (\textit{Continuant}(a) \wedge \forall(t)(\textit{existsAt}(a, t) \supset \\ & \quad \exists(b)(\textit{IndependentContinuant}(b) \wedge \\ & \quad \textit{specificallyDependsOn}(a, b, t)))) \end{aligned} \quad (8)$$

**RelationalQuality(a)** — “a is a relational quality”. [057-001]

$$\begin{aligned} \textit{RelationalQuality}(a) =_{df} \\ & \exists(b, c, t)(\textit{IndependentContinuant}(b) \wedge \\ & \quad \textit{IndependentContinuant}(c) \wedge \\ & \quad \textit{qualityOfAt}(a, b, t) \wedge \textit{qualityOfAt}(a, c, t)) \end{aligned} \quad (9)$$

**GenericallyDependentContinuant(a)** — “a is a generically dependent continuant”. [074-001]

$$\begin{aligned} \textit{GenericallyDependentContinuant}(a) =_{df} \\ & (\textit{Continuant}(a) \wedge \exists(b, t)\textit{genericallyDependsOnAt}(a, b, t)) \end{aligned} \quad (10)$$

**Process(a)** — “a is a process”. [083-002]

$$\begin{aligned} \textit{Process}(a) =_{df} \\ & (\textit{Occurrent}(a) \wedge \exists(b)\textit{properTemporalPartOf}(b, a) \wedge \\ & \quad \exists(c, t)(\textit{MaterialEntity}(c) \wedge \textit{specificallyDependsOn}(a, c, t))) \end{aligned} \quad (11)$$

**ProcessBoundary(a)** — “a is a process boundary”. [084-001]

$$\begin{aligned} \textit{ProcessBoundary}(a) =_{df} \\ & \exists(p)(\textit{Process}(p) \wedge \textit{temporalPartOf}(a, p) \wedge \\ & \quad \neg \exists(b)\textit{properTemporalPartOf}(b, a)) \end{aligned} \quad (12)$$

## 2.3 Primitive Relational Predicates

The following relational predicates are taken as primitive:

**existsAt(a, t)** — “a exists at time t”. [118-001]

**continuantPartOfAt(a, b, t)** — “a is a part of b at time t”, where a and b are continuants. [002-001]

**occurentPartOf(a, b)** — “a is a part of b”, where a and b are occurents. [003-002]

**specificallyDependsOn(a, b, t)** — “a specifically depends on b at time t”. [012-002]

**memberPartOfAt(a, b, t)** — “a is a member of b at time t”. [026-002]

**locatedAt(a, r, t)** — “a is located at r at time t”. [041-002]

**realizesAt(a, b, t)** — “a realizes b at time t”. [059-002]

**hasMaterialBasisAt(a, b, t)** — “a has the material basis b at time t”. [071-001]

**genericallyDependsOnAt(a, b, t)** — “a generically depends on b at time t”. [072-002]

**concretizesAt(a, b, t)** — “a concretizes b at time t” where a is a specifically dependent continuant and b is a generically dependent continuant. [075-001]

**projectsOnto(a, b)** — “a projects onto b”, where a is a spatiotemporal region, and b is a temporal region. [080-001]

**projectsOntoAt(a, b, t)** — “a projects onto b at time t”, where a is a spatiotemporal region and b is a spatial region. [081-001]

**occupies(a, r)** — “a occupies r”, where a is an occurrent, and r is a temporal or spatiotemporal region. [082-001]

**hasParticipantAt(a, b, t)** — “a has participant b at time t”. [086-002]

**processProfileOf(a, b)** — “a is a process profile of b”. [094-001]

## 2.4 Defined Relational Predicates

The following relational predicates are defined as indicated:

**properContinuantPartOfAt(a, b, t)** — “a is a proper part of b at time t”, where a and b are continuants. [004-001]

$$\begin{aligned} \text{properContinuantPartOfAt}(a, b, t) =_{df} \\ (\text{continuantPartOfAt}(a, b, t) \wedge \neg(a = b)) \end{aligned} \quad (13)$$

**properOccurrentPartOf(a, b)** — “a is a proper part of b”, where a and b are occurrents. [005-001]

$$\begin{aligned} \text{properOccurrentPartOf}(a, b) =_{df} \\ (\text{occurentPartOf}(a, b) \wedge \neg(a = b)) \end{aligned} \quad (14)$$

**hasContinuantPartAt(a, b, t)** — “a has b as a part at time t”, where a and b are continuants. [006-001]

$$\text{hasContinuantPartAt}(a, b, t) =_{df} \text{continuantPartOfAt}(b, a, t) \quad (15)$$

**hasOccurrentPart(a, b)** — “a has b as a part”, where a and b are occurrents. [007-001]

$$\text{hasOccurrentPart}(a, b) =_{df} \text{occurentPartOf}(b, a) \quad (16)$$

**locatedInAt(a, b, t)** — “a is located in b at time t”. [045-001]

$$\begin{aligned} \text{locatedInAt}(a, b, t) =_{df} \\ (\text{IndependentContinuant}(a) \wedge \text{IndependentContinuant}(b) \wedge \\ \exists(r_1, r_2)(\text{locatedAt}(a, r_1, t) \wedge \text{locatedAt}(b, r_2, t) \wedge \\ \text{continuantPartOfAt}(r_1, r_2, t))) \end{aligned} \quad (17)$$

**inheresInAt(a, b, t)** — “a inheres in b at time t”. [051-001]

$$\begin{aligned} \text{inheresInAt}(a, b, t) =_{df} \\ (\text{DependentContinuant}(a) \wedge \text{IndependentContinuant}(b) \wedge \\ \text{specificallyDependsOn}(a, b, t)) \end{aligned} \quad (18)$$

**bearerOfAt(a, b, t)** — “a is the bearer of b at time t”. [053-001]

$$\begin{aligned} \text{bearerOfAt}(a, b, t) =_{df} \\ (\text{specificallyDependsOn}(b, a, t) \wedge \text{IndependentContinuant}(a) \wedge \\ \text{existsAt}(b, t)) \end{aligned} \quad (19)$$

**qualityOfAt(a, b, t)** — “a is a quality of b at time t”. [056-001]

$$\begin{aligned} \text{qualityOfAt}(a, b, t) =_{df} \\ (Quality(a) \wedge IndependentContinuant(b) \wedge \\ specificallyDependsOn(a, b, t)) \end{aligned} \quad (20)$$

**roleOfAt(a, b, t)** — “a is a role of b at time t”. [065-001]

$$\text{roleOfAt}(a, b, t) =_{df} (Role(a) \wedge inheresInAt(a, b, t)) \quad (21)$$

**dispositionOf(a, b, t)** — “a is a disposition of b at time t”. [066-001]

$$\text{dispositionOf}(a, b, t) =_{df} (Disposition(a) \wedge inheresInAt(a, b, t)) \quad (22)$$

**functionOf(a, b, t)** — “a is a function of b at time t”. [067-001]

$$\text{functionOf}(a, b, t) =_{df} (Function(a) \wedge inheresInAt(a, b, t)) \quad (23)$$

**hasRoleAt(a, b, t)** — “a has the role b at time t”. [068-001]

$$\text{hasRoleAt}(a, b, t) =_{df} \text{roleOfAt}(b, a, t) \quad (24)$$

**hasDispositionAt(a, b, t)** — “a has the disposition b at time t”. [069-001]

$$\text{hasDispositionAt}(a, b, t) =_{df} \text{dispositionOf}(b, a, t) \quad (25)$$

**hasFunctionAt(a, b, t)** — “a has the function b at time t”. [070-001]

$$\text{hasFunctionAt}(a, b, t) =_{df} \text{functionOf}(b, a, t) \quad (26)$$

**temporalPartOf(a, b)** — “a is a temporal part of b”, where a and b are occurrents. [078-001]

$$\begin{aligned} \text{temporalPartOf}(a, b) =_{df} & (\text{occurrentPartOf}(a, b) \wedge \\ & \exists(r)(TemporalRegion(r) \wedge occupies(a, r)) \wedge \\ & \forall(c, r_1)((Occurrent(c) \wedge occupies(c, r_1) \wedge \\ & \text{occurrentPartOf}(r_1, r)) \supset \\ & (\text{occurrentPartOf}(c, a) \equiv \text{occurrentPartOf}(c, b)))) \end{aligned} \quad (27)$$



**properTemporalPartOf(a, b)** — “a is a proper temporal part of b”. [116-001]

$$\begin{aligned} \text{properTemporalPartOf}(a, b) =_{df} \\ (\text{temporalPartOf}(a, b) \wedge \neg(a = b)) \end{aligned} \quad (28)$$

### 3 Axioms

The following formulas are asserted as axioms in the system:

$$\begin{aligned} \forall(x, y, t)((\text{continuantPartOfAt}(x, y, t) \wedge \\ \text{continuantPartOfAt}(y, x, t)) \supset \\ (x = y)) \end{aligned} \quad (29)$$

[120-001]

$$\begin{aligned} \forall(x, y, z, t)((\text{continuantPartOfAt}(x, y, t) \wedge \\ \text{continuantPartOfAt}(y, z, t)) \supset \\ \text{continuantPartOfAt}(x, z, t)) \end{aligned} \quad (30)$$

[110-001]

$$\begin{aligned} \forall(x, y, t)((\text{continuantPartOfAt}(x, y, t) \wedge \neg(x = y)) \supset \\ \exists(z)(\text{continuantPartOfAt}(z, y, t) \wedge \\ \neg\exists(w)(\text{continuantPartOfAt}(w, x, t) \wedge \text{continuantPartOfAt}(w, z, t)))) \end{aligned} \quad (31)$$

[121-001]

$$\begin{aligned} \forall(x, y, t)(\exists(v)(\text{continuantPartOfAt}(v, x, t) \wedge \\ \text{continuantPartOfAt}(v, y, t)) \supset \\ \exists(z)\forall(u, w)((\text{continuantPartOfAt}(w, u, t) \equiv \\ (\text{continuantPartOfAt}(w, x, t) \wedge \\ \text{continuantPartOfAt}(w, y, t))) \equiv (z = u))) \end{aligned} \quad (32)$$

[122-001]

$$\begin{aligned} \forall(x, y, t)((\text{occurrentPartOf}(x, y, t) \wedge \text{occurrentPartOf}(y, x, t)) \supset \\ (x = y)) \end{aligned} \quad (33)$$

[123-001]

$$\forall(x, y, z)((\text{occurentPartOf}(x, y) \wedge \text{occurentPartOf}(y, z)) \supset \text{occurentPartOf}(x, z)) \quad (34)$$

[112-001]

$$\begin{aligned} \forall(x, y, t)((\text{occurentPartOf}(x, y, t) \wedge \neg(x = y)) \supset \\ \exists(z)(\text{occurentPartOf}(z, y, t) \wedge \\ \neg \exists(w)(\text{occurentPartOf}(w, x, t) \wedge \text{occurentPartOf}(w, z, t)))) \quad (35) \end{aligned}$$

[124-001]

$$\begin{aligned} \forall(x, y, t)(\exists(v)(\text{occurentPartOf}(v, x, t) \wedge \text{occurentPartOf}(v, y, t)) \supset \\ \exists(z)\forall(u, w)((\text{occurentPartOf}(w, u, t) \equiv \\ (\text{occurentPartOf}(w, x, t) \wedge \text{occurentPartOf}(w, y, t))) \equiv (z = u))) \quad (36) \end{aligned}$$

[125-001]

$$\forall(x)(\text{Continuant}(x) \supset \text{Entity}(x)) \quad (37)$$

[008-001]

$$\begin{aligned} \forall(x, y, t)(\text{specificallyDependsOn}(x, y, t) \supset \\ \neg \exists(z)(\text{continuantPartOfAt}(z, x, t) \wedge \text{continuantPartOfAt}(z, y, t))) \quad (38) \end{aligned}$$

[012-002]

$$\begin{aligned} \forall(x, y)((\text{Continuant}(x) \wedge \exists(t)\text{continuantPartOfAt}(y, x, t)) \supset \\ \text{Continuant}(y)) \quad (39) \end{aligned}$$

[009-002]

$$\begin{aligned} \forall(x, y)((\text{Continuant}(x) \wedge \exists(t)\text{hasContinuantPartOfAt}(y, x, t)) \supset \\ \text{Continuant}(y)) \quad (40) \end{aligned}$$

[126-001]

$$\begin{aligned} \forall(x)(\text{Continuant}(x) \supset \\ \exists(y, t)(\text{TemporalRegion}(y) \wedge \text{existsAt}(y, t) \wedge \text{existsAt}(x, t))) \quad (41) \end{aligned}$$

[011-001]

$$\begin{aligned} \forall(x, y, t)((Occurrent(x) \wedge IndependentContinuant(y) \wedge \\ specificallyDependsOn(x, y, t)) \supset \\ \forall(t_1)(existsAt(x, t_1) \supset \\ \exists(z)(IndependentContinuant(z) \wedge specificallyDependsOn(x, z, t_1)))) \end{aligned} \quad (42)$$

[015-001]

$$\begin{aligned} \forall(x, y, t)((Continuant(x) \wedge specificallyDependsOn(x, y, t)) \supset \\ \forall(t_1)(existsAt(x, t_1) \supset specificallyDependsOn(x, y, t_1))) \end{aligned} \quad (43)$$

[016-001]

$$\begin{aligned} \forall(x, y, t)((Continuant(x) \wedge specificallyDependsOn(x, y, t)) \supset \\ existsAt(x, t)) \end{aligned} \quad (44)$$

[127-001]

$$\begin{aligned} \forall(x, y, t)((Continuant(x) \wedge specificallyDependsOn(x, y, t)) \supset \\ existsAt(y, t)) \end{aligned} \quad (45)$$

[128-001]

$$\begin{aligned} \forall(x, y, t)((Occurrent(x) \wedge Continuant(y) \wedge \\ specificallyDependsOn(x, y, t)) \supset \\ \forall(t_1)(existsAt(y, t_1) \supset existsAt(x, t_1))) \end{aligned} \quad (46)$$

[129-001]

$$\begin{aligned} \forall(x, y, t)((Occurrent(x) \wedge Occurrent(y) \wedge \\ specificallyDependsOn(x, y, t)) \supset \\ existsAt(y, t)) \end{aligned} \quad (47)$$

[130-001]

$$\begin{aligned} \forall(x, t)((IndependentContinuant(x) \wedge existsAt(x, t)) \supset \\ \exists(y)(Entity(y) \wedge specificallyDependsOn(y, x, t))) \end{aligned} \quad (48)$$

[018-002]

$$\forall(x)(MaterialEntity(x) \supset IndependentContinuant(x)) \quad (49)$$

[019-001]

$$\begin{aligned} & \forall(x)((Entity(x) \wedge \\ & \exists(y,t)(MaterialEntity(y) \wedge continuantPartOfAt(y,x,t)) \supset \\ & MaterialEntity(x)) \end{aligned} \quad (50)$$

[020-001]

$$\begin{aligned} & \forall(x)(ObjectAggregate(x) \supset (MaterialEntity(x) \wedge \\ & \forall(t)(existsAt(x,t) \supset \exists(y,z)(Object(y) \wedge Object(z) \wedge \\ & memberPartOfAt(y,x,t) \wedge memberPartOfAt(z,x,t) \wedge \\ & \neg(y=z))) \wedge \neg \exists(w,t_1)(memberPartOfAt(w,x,t_1) \wedge \neg Object(w)))) \end{aligned} \quad (51)$$

[025-002]

$$\forall(x)(SpatialRegion(x) \supset Continuant(x)) \quad (52)$$

[035-001]

$$\begin{aligned} & \forall(x,y,t)((SpatialRegion(x) \wedge continuantPartOfAt(y,x,t)) \supset \\ & SpatialRegion(y)) \end{aligned} \quad (53)$$

[036-001]

$$\forall(x)(ZeroDimensionalSpatialRegion(x) \supset SpatialRegion(x)) \quad (54)$$

[037-001]

$$\forall(x)(OneDimensionalSpatialRegion(x) \supset SpatialRegion(x)) \quad (55)$$

[038-001]

$$\forall(x)(TwoDimensionalSpatialRegion(x) \supset SpatialRegion(x)) \quad (56)$$

[039-001]

$$\forall(x)(ThreeDimensionalSpatialRegion(x) \supset SpatialRegion(x)) \quad (57)$$

[040-001]

$$\begin{aligned} \forall(x, r, t)(locatedAt(x, r, t) \supset \\ (SpatialRegion(r) \wedge IndependentContinuant(x))) \end{aligned} \quad (58)$$

[041-002]

$$\forall(r, t)(Region(r) \supset locatedAt(r, r, t)) \quad (59)$$

[042-001]

$$\begin{aligned} \forall(x, y, r_1, t)((locatedAt(x, r_1, t) \wedge continuantPartOfAt(y, x, t)) \supset \\ \exists(r_2)(continuantPartOfAt(r_2, r_1, t) \wedge locatedAt(y, r_2, t))) \end{aligned} \quad (60)$$

[043-001]

$$\begin{aligned} \forall(x, y, z, t)((locatedInAt(x, y, t) \wedge locatedInAt(y, z, t)) \supset \\ locatedInAt(x, z, t)) \end{aligned} \quad (61)$$

[046-001]

$$\forall(x, y, t)(continuantPartOfAt(x, y, t) \supset locatedInAt(x, y, t)) \quad (62)$$

[047-001]

$$\begin{aligned} \forall(x, y, z, t)((IndependentContinuant(x) \wedge IndependentContinuant(y) \wedge \\ IndependentContinuant(z) \wedge \\ continuantPartOfAt(x, y, t) \wedge locatedInAt(y, z, t)) \supset \\ locatedInAt(x, z, t)) \end{aligned} \quad (63)$$

[048-001]

$$\begin{aligned} \forall(x, y, z, t)((IndependentContinuant(x) \wedge IndependentContinuant(y) \wedge \\ IndependentContinuant(z) \wedge \\ locatedInAt(x, y, t) \wedge continuantPartOfAt(y, z, t)) \supset \\ locatedInAt(x, z, t)) \end{aligned} \quad (64)$$

[049-001]

$$\forall(x)(\exists(y,t)specificallyDependsOn(x,y,t) \supset \neg MaterialEntity(x)) \quad (65)$$

[052-001]

$$\begin{aligned} \forall(x,y,z,t)((specificallyDependsOn(x,y,t) \wedge \\ specificallyDependsOn(y,z,t)) \supset \\ specificallyDependsOn(x,z,t)) \end{aligned} \quad (66)$$

[054-002]

$$\forall(x)(Quality(x) \supset SpecificallyDependentContinuant(x)) \quad (67)$$

[055-001]

$$\begin{aligned} \forall(x)(\exists(t)(existsAt(x,t) \wedge Quality(x)) \supset \forall(t_1)(existsAt(x,t_1) \supset \\ Quality(x))) \end{aligned} \quad (68)$$

[105-001]

$$\begin{aligned} \forall(x)(RealizableEntity(x) \supset \\ (SpecificallyDependentContinuant(x) \wedge \\ \exists(y)(MaterialEntity(y) \wedge inheresIn(x,y)))) \end{aligned} \quad (69)$$

[058-001]

$$\begin{aligned} \forall(x,y,t)(realizesAt(x,y,t) \supset \\ (Process(x) \wedge (Disposition(y) \vee Role(y)) \wedge \\ \exists(z)(MaterialEntity(z) \wedge \\ hasParticipantAt(x,z,t) \wedge bearerOfAt(z,y,t)))) \end{aligned} \quad (70)$$

[059-002]

$$\begin{aligned} \forall(x,y,t)((RealizableEntity(x) \wedge bearerOfAt(y,x,t)) \supset \\ (MaterialEntity(y) \vee Site(y))) \end{aligned} \quad (71)$$

[060-001]

$$\forall(x)(Role(x) \supset RealizableEntity(x)) \quad (72)$$

[061-001]

$$\forall(x)(Disposition(x) \supset RealizableEntity(x)) \quad (73)$$

[062-001]

$$\begin{aligned} &\forall(x, t)((RealizableEntity(x) \wedge existsAt(x, t)) \supset \\ &\exists(y)(MaterialEntity(y) \wedge specificallyDepends(x, y, t))) \end{aligned} \quad (74)$$

[063-002]

$$\forall(x)(Function(x) \supset Disposition(x)) \quad (75)$$

[064-001]

$$\begin{aligned} &\forall(x, y, t)(hasMaterialBasisAt(x, y, t) \supset \\ &\quad (Disposition(x) \wedge MaterialEntity(y) \wedge \\ &\exists(z)(bearerOfAt(z, x, t) \wedge continuantPartOfAt(y, z, t) \wedge \\ &\quad \exists(w)(Disposition(w) \wedge \\ &\quad (hasDisposition(z, w) \supset continuantPartOfAt(y, z, t)))))) \end{aligned} \quad (76)$$

[071-001]

$$\begin{aligned} &\forall(x, y)(\exists(t)genericallyDependsOnAt(x, y, t) \supset \forall(t_1)(existsAt(x, t_1) \supset \\ &\quad \exists(z)genericallyDependsOnAt(x, z, t_1))) \end{aligned} \quad (77)$$

[073-001]

$$\begin{aligned} &\forall(x, y, t)(genericallyDependsOnAt(x, y, t) \supset \\ &\exists(z)(concretizesAt(z, x, t) \wedge specificallyDependsOn(z, y, t))) \end{aligned} \quad (78)$$

[076-001]

$$\begin{aligned} &\forall(x, y)(properTemporalPartOf(x, y) \supset \\ &\quad \exists(z)(properTemporalPartOf(z, y) \wedge \\ &\quad \neg \exists(w)(temporalPartOf(w, x) \wedge temporalPartOf(w, z)))) \end{aligned} \quad (79)$$

[117-001]

$$\forall(x)(Occurrent(x) \equiv (Entity(x) \wedge \exists(y)temporalPartOf(y, x))) \quad (80)$$

[079-001]

$$\forall(x)(ProcessBoundary(x) \supset \exists(y)(ZeroDimensionalTemporalRegion(y) \wedge occupies(x, y))) \quad (81)$$

[085-001]

$$\forall(x, y, t)(hasParticipantAt(x, y, t) \supset Occurrent(x)) \quad (82)$$

[087-001]

$$\forall(x, y, t)(hasParticipantAt(x, y, t) \supset Continuant(y)) \quad (83)$$

[088-001]

$$\forall(x, y, t)(hasParticipantAt(x, y, t) \supset existsAt(y, t)) \quad (84)$$

[089-001]

$$\begin{aligned} &\forall(x, y, t)((hasParticipantAt(x, y, t) \wedge \\ &SpecificallyDependentContinuant(y)) \supset \\ &\exists(z)(MaterialEntity(z) \wedge \\ &specificallyDependsOn(x, z, t) \wedge specificallyDependsOn(y, z, t))) \quad (85) \end{aligned}$$

[090-002]

$$\begin{aligned} &\forall(x, y, t)((hasParticipantAt(x, y, t) \wedge \\ &GenericallyDependentContinuant(y)) \supset \\ &\exists(z)(MaterialEntity(z) \wedge \\ &genericallyDependsOn(y, z, t) \wedge specificallyDependsOn(x, z, t))) \quad (86) \end{aligned}$$

[091-002]

$$\begin{aligned} &\forall(x, y)(processProfileOf(x, y) \supset \\ &(ProcessProfile(x) \wedge properContinuantPartOf(x, y) \wedge \\ &\exists(r)(TemporalRegion(r) \wedge occupies(x, r) \wedge occupies(y, r)))) \quad (87) \end{aligned}$$

[094-002]

$$\forall(x)(SpatioTemporalRegion(x) \supset Occurrent(x)) \quad (88)$$



[095-001]

$$\forall(x, y)((SpatioTemporalRegion(x) \wedge occurrentPartOf(y, x)) \supset SpatioTemporalRegion(y)) \quad (89)$$

[096-001]

$$\forall(x)(SpatioTemporalRegion(x) \supset \exists(y)(TemporalRegion(y) \wedge projectsOnto(x, y))) \quad (90)$$

[098-001]

$$\forall(x, t)(SpatioTemporalRegion(x) \supset \exists(y)(SpatialRegion(y) \wedge projectsOntoAt(x, y, t))) \quad (91)$$

[099-001]

$$\forall(r)(TemporalRegion(r) \supset occupies(r, r)) \quad (92)$$

[107-001]

$$\forall(r)(SpatioTemporalRegion(r) \supset occupies(r, r)) \quad (93)$$

[119-001]

$$\forall(x)(Occurrent(x) \supset \exists(r)(SpatioTemporalRegion(r) \wedge occupies(x, r))) \quad (94)$$

[108-001]

$$\forall(x)(TemporalRegion(x) \supset Occurrent(x)) \quad (95)$$

[100-001]

$$\forall(x, y)((TemporalRegion(x) \wedge occurrentPartOf(y, x)) \supset TemporalRegion(y)) \quad (96)$$

[101-001]

$$\forall(x)(ZeroDimensionalTemporalRegion(x) \supset TemporalRegion(x)) \quad (97)$$

[102-001]

$$\forall(x)(OneDimensionalTemporalRegion(x) \supset TemporalRegion(x)) \quad (98)$$

[103-001]

## 4 Theorems

The following formulas are noted as theorems in the *BFO 2.0 Draft Specification and User's Guide* and are derivable from the definitions and axioms of the system. Of course, these explicitly noted theorems are only a small subset of what is derivable within BFO-FOL.

$$\forall(x, t)(\textit{Continuant}(x) \supset \textit{continuantPartOfAt}(x, x, t)) \quad (99)$$

[111-002]

$$\forall(x)(\textit{Occurrent}(x) \supset \textit{occurrentPartOf}(x, x)) \quad (100)$$

[113-002]

$$\begin{aligned} \forall(x, y, t)((\textit{Entity}(x) \wedge \\ (\textit{continuantPartOfAt}(y, x, t) \vee \textit{continuantPartOfAt}(x, y, t) \vee \\ \textit{occurrentPartOf}(x, y) \vee \textit{occurrentPartOf}(y, x))) \supset \\ \neg \textit{specificallyDependsOn}(x, y, t)) \end{aligned} \quad (101)$$

[013-001]

$$\begin{aligned} \forall(x)((\textit{Entity}(x) \wedge \\ \exists(y, t)(\textit{MaterialEntity}(y) \wedge \textit{continuantPartOfAt}(x, y, t))) \supset \\ \textit{MaterialEntity}(x)) \end{aligned} \quad (102)$$

[021-001]

$$\forall(x, y, t)(\textit{memberPartOfAt}(x, y, t) \supset \textit{continuantPartOfAt}(x, y, t)) \quad (103)$$

[104-001]

$$\begin{aligned} \forall(x, y, z, t)((\textit{RealizableEntity}(x) \wedge \textit{Process}(y) \wedge \\ \textit{realizesAt}(y, x, t) \wedge \textit{bearerOfAt}(z, x, t)) \supset \\ \textit{hasParticipantAt}(y, z, t)) \end{aligned} \quad (104)$$

[106-002]

$$\forall(x)(\textit{Occurrent}(x) \supset \exists(r)(\textit{TemporalRegion}(r) \wedge \textit{occupies}(x, r))) \quad (105)$$

[109-001]

## 5 Conclusion

As noted above, the BFO 2.0 specification is currently under development, and thus the axiomatization of the specification in BFO-FOL is accordingly subject to modification and refinement. Of particular interest is the question of what consequences can be derived from these definitions and axioms, both with regard to the formal consistency of BFO-FOL and with regard to whether these consequences would run counter to the basic principles and intentions of BFO. Since BFO-FOL contains a large number of definitions and axioms, the working group is investigating formal tools capable of automating the investigation into these consequences.

## References

- [1] Barry Smith, et al. *Basic Formal Ontology 2.0: Draft Specification and User's Guide*. Manuscript.