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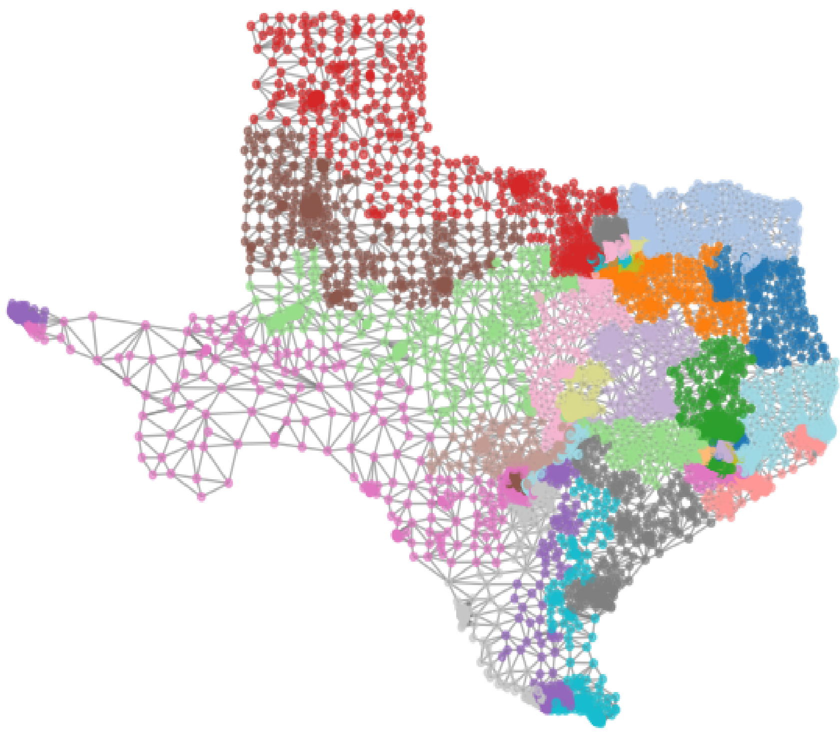
Applying GerryChain:
A User's Guide for Redistricting Problems



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Preface

This guide provides a general overview of how to apply GerryChain to redistricting problems in the United States. The introduction briefly summarizes the context relevant for understanding the redistricting process and the potential for computational redistricting tools like GerryChain to help address some of the many challenges in this space. The Getting Started with GerryChain section provides an overview of the general workflow for applying GerryChain to redistricting problems. This section is a great place to start for anyone wanting to understand how to begin using GerryChain to assist with redistricting in their state or local context. Finally, the Applying GerryChain in State Contexts section offers brief summaries of three state level case studies and discusses the approach we used in each state based on different analytic goals. The text summaries of these case studies are supplemented by the full code and annotation used for each state case study on our Github repository.

This guide was produced by four graduate student fellows under the guidance of Dr. Daryl Deford at Washington State University and with the support of two data scientists from the University of Washington's e-Science Institute as the culminating project of the 2021 Data Science for Social Good Fellowship program. More details about the team can be found on the project website. For questions or comments about the guide or its contents, please contact Dr. Daryl Deford at daryl.deford@wsu.edu.

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Introduction

Background

The Problem of Redistricting

Every ten years the Constitution requires the 435 seats in the U.S. House of Representatives be divided among the states based on population counts determined by the newly taken census. This redistribution of congressional seats is called **reapportionment**. Each state has a unique process and set of laws that govern drawing new federal and state legislative districts to reflect shifts in population. This drawing of new legislative boundaries is called **redistricting**; and it is a legally and technically challenging process with serious consequences. The demographic and geographic makeup of the resulting districts profoundly shapes legislative electoral outcomes for at least a decade. Because currently elected state legislatures are typically responsible for determining the process for drawing new district maps, representatives of both major political parties have a long history of exploiting this power to secure future electoral advantage for themselves. Such manipulation of district boundaries for personal or partisan gain is commonly known as **gerrymandering**.

Gerrymandering is fundamentally undemocratic. Drawing district lines to favor particular political candidates or parties undermines competition in elections. It also violates the principles of equal representation enshrined in the US constitution by giving outsized influence to some groups, while marginalizing the voices of others. Gerrymandering has gotten worse over the last several decades. Partisan redistricting bodies have become increasingly adept at using computational geospatial mapping tools and voter databases to create safe districts for political parties and individual politicians with high accuracy. In an intensely polarized political climate, neither major party is likely to unilaterally disarm by refusing to gerrymander when they have the power to do so. Thus, we are unlikely to arrive at a political solution to this problem. Instead, we need to democratize the process of redistricting and empower citizen groups, activists, and non-partisan map drawing commissions to become more involved.

Though gerrymandering has been a persistent problem in the US since at least 1812, new developments in mathematics offer promising new computational tools to confront this centuries old political issue. The 2021 redistricting cycle is the first where these new tools are available with the potential to help reshape the process. This guide provides an overview of one powerful new tool called GerryChain, and offers advice on applying it to the 2021 redistricting cycle. Our hope is to increase access and participation in the redistricting process by empowering people to use GerryChain to inform and assist with their specific local redistricting problems.

The Promise of GerryChain

GerryChain uses Markov Chain Monte Carlo (MCMC) methods to help address some of the most challenging problems posed by redistricting. There are an enormous number of ways to partition the geography of a state into electoral districts that satisfy the basic legal requirements. As a point of comparison, there are at least as many valid congressional district maps as there are atoms in the universe. It is therefore impossible for human map drawers to consider the entire range of possible plans. The MCMC approach implemented by GerryChain is computationally efficient enough to generate hundreds of thousands of viable districting plans on a home laptop in a few hours. These large collections of plans are called **ensembles**.

GerryChain also allows users to track multiple properties of interest for each individual plan in an ensemble, such as: population, demographics, and historical voting patterns by district. This enables users to conduct a variety of statistical tests and analyses to evaluate how proposed maps perform across several metrics of interest. For example, GerryChain has built-in functions to help users compute proposed metrics, like partisan bias, wasted votes, and the efficiency gap based on past election data. Ensemble distributions can then be used as a representative baseline with which to compare the properties of proposed or enacted maps. Plans that are outliers on distributions of thoughtfully applied metrics can then be identified as potential cases of gerrymandering.

GerryChain is flexible enough to be applied at all phases of the redistricting process for both state and federal legislatures. Users with a modest level of experience working with the Python programming language and some familiarity with statistical techniques can configure GerryChain to assist with redistricting problems by producing valid comparison maps to assist redistricting commissions in assessing the representativeness of proposed or enacted maps across many different properties or metrics. GerryChain and other computational redistricting approaches using MCMC methods can, and have, even been used as a basis for supporting legal challenges against enacted maps suspected of being gerrymandered, such as in the 2018 Supreme Court case *Rucho v. Common Cause*.

The Need for Thoughtful Human Modeling

However, GerryChain is not a silver bullet. The effective and ethical application of GerryChain requires thoughtful and transparent modeling choices and critical analysis of its outputs. This guide is an overview of both the significant strengths and important limitations of GerryChain. It outlines the major data, modeling, and analysis decisions that must be made and illustrates the downstream implications of each choice. Computational redistricting is not a solved problem. Therefore, no single approach to applying these tools to the complex problem of redistricting exists. Our goal is to help map drawing commissions, community groups, concerned citizens, and activists to use GerryChain to participate in the redistricting process while being informed of its limitations. In addition to generalizable guidelines for applying

GerryChain to common redistricting problems, we provide state-level case studies for Georgia, Colorado, and Texas, which illustrate the benefits and challenges of using GerryChain.

Overview

The Purpose of this Guide

We provide a set of general guidelines and example case studies for users of the GerryChain library. Keeping in mind that no claims of “best” practices can yet be made given the nascency of this field, we outline a number of good practices for applying GerryChain and underscore some common questions that are likely to arise for users. We illustrate good practices via a road map (see Figure 3 on page 13), sketching multiple ways to reach a given destination, and discussing possible routes while highlighting forks in the road where choices must be made to satisfy analysis goals. We provide a series of traffic lights at each turn emphasizing the importance of making thoughtful and informed decisions at each of these points, and exploring the implications of these divergent choices.

To help ground and contextualize this guide, we provide links to our [Github repository] throughout, which includes runnable code for the general workflow of GerryChain and the included case studies. This code is provided primarily for reproducibility and context, so caution is warranted in adapting it for use in other contexts. Although reusing code is often helpful as a starting point for new analyses, it will always be important to carefully and thoughtfully customize the code based on applicable state law and regulatory guidance, data availability and quality, project goals, and other factors.

Audience

Effective use of GerryChain requires some knowledge of Python. We therefore advise users lacking this experience to complete one of the many excellent Python tutorials available online. For readers with some Python experience, we recommend starting with this guide introducing the use of Gerrychain. Though we anticipate most readers will already be familiar with redistricting problems and statistical methods broadly speaking, we provide additional context to minimize the amount of detailed background knowledge required. There are also beginning guides to the mathematics of redistricting and MCMC methods. These and many other resources are linked throughout this guide for additional context.

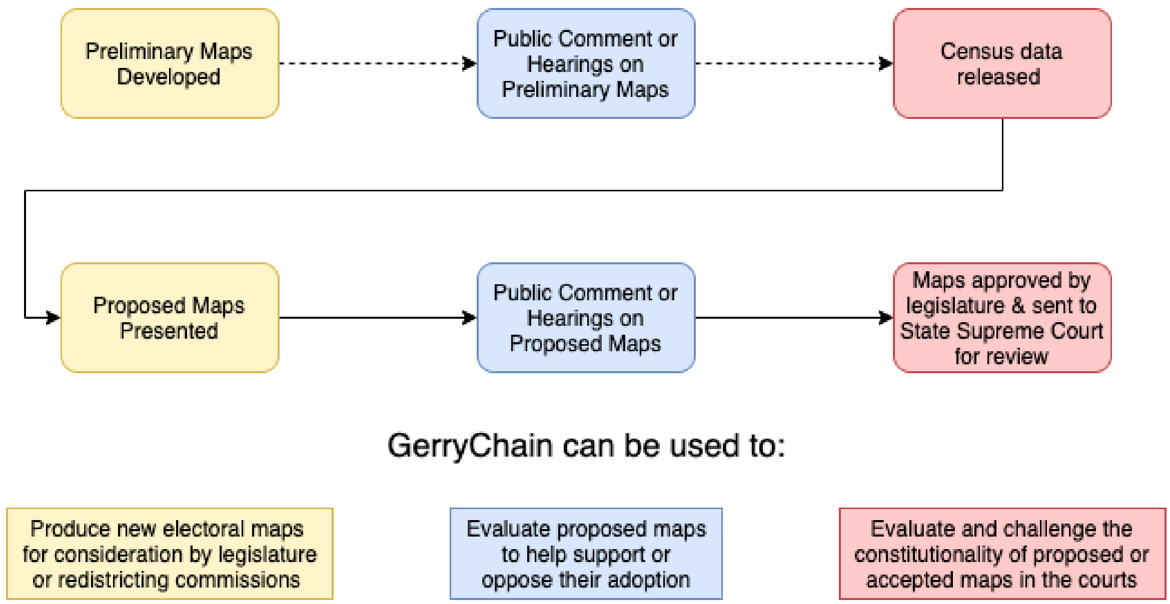
Image source: freepik

Readers without the above technical prerequisites may nevertheless benefit from surveying this guide to help understand the utility and limitations of applying GerryChain to redistricting problems. This guide is primarily intended for citizen groups, activists, and non-partisan, community-led map drawing commissions who wish to get involved in the redistricting process and would benefit from the powerful computational tools that GerryChain offers. Our goal is to help empower anyone with modest technical experience to use GerryChain to contribute to their community or state's redistricting process.

Use Cases

GerryChain can be fruitfully applied to assist with the redistricting process in a number of ways. The particular application of GerryChain will depend upon which phase of the redistricting process your state is in and the goals of your analysis. Though each state's timeline and process for redistricting vary, below (Figure 1) is an abstracted example of the typical sequence of events in most states and highlights where users of GerryChain might most effectively engage.

Figure 1: Typical Sequence of Events in Redistricting Process



Our three state level case studies are intended to further illustrate some common use cases for applying GerryChain to the redistricting process (some other examples are discussed here). For instance, our case study of Georgia uses one of GerryChain's built in metrics of partisan symmetry to analyze the wasted votes for each party under the state's current electoral district map.

Our case study of Colorado focuses on the map drawing phase and demonstrates some approaches to using GerryChain during this time period. As we show, applying GerryChain at this stage requires translating the often vague language of constitutional or legislative texts that dictate the redistricting process into a series of quantitative specifications to ensure that the district maps produced by GerryChain conform to state guidelines and respect the priorities of the legislature. In particular, we demonstrate one approach to operationalizing the political competitiveness requirement outlined in Colorado's state constitution and associated statutes. Our goal is not to provide a universal test for political competitiveness, but rather to underscore the fact that there are multiple ways to interpret and operationalize abstract concepts like competitiveness and that decisions made in this regard will have notable consequences for how GerryChain is applied and how the lines are eventually drawn.

Similarly, in our Texas case study, we examine ways in which GerryChain can be used to evaluate a proposed or enacted map of electoral districts along a number of metrics of interest, with particular attention paid to compliance with the 1965 Voting Rights Act (VRA). As there is a precedent of maps that violate the VRA being challenged in court, and a history of racial gerrymandering in Texas, we use this example to demonstrate how GerryChain might be useful in establishing whether or not a map potentially violates the VRA. Additionally, because Texas will get two new Congressional districts to reflect increases in the state's population, we assess the most suitable geographic areas for those two new districts based both on population growth and demographic shifts to comply with the VRA and maximize minority representation.

Outline of the Guide

The remainder of this guide proceeds as follows. In the next section we provide an overview of the general workflow for applying GerryChain to redistricting problems. This section is a great place to start for anyone wanting to understand how to begin using GerryChain to assist with redistricting in their state or local context. Next we provide brief summaries of our three state level case studies and discuss the approach we used in each state based on different analytic goals. The text summaries of these case studies are supplemented by the full code and annotation used for each state case study on our Github repository. Finally, we review some of the major lessons learned through these case studies and our work with GerryChain more broadly, before outlining some recommendations for future work in this area that could build off of our protocol.

Getting Started with GerryChain

GerryChain is a publicly available library with extensive documentation. First time users can consult these resources for setup instructions and to familiarize themselves with its basic functions. Though each use case of GerryChain will vary considerably depending on the specific state context and goals of analysis, we provide here a high-level generalizable workflow for GerryChain. Most applications of GerryChain will follow this workflow: (1) Data cleaning and wrangling, (2) Exploratory data analysis and modeling decisions, and (3) Applying GerryChain. The case study code in our Github repo offers in-depth examples of how to navigate each step. We provide here a general overview of each step.

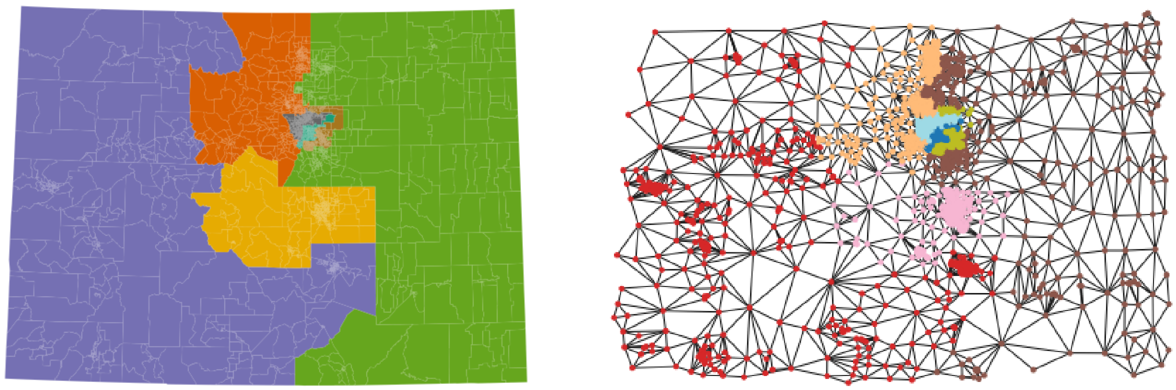
Stage 1. Data Cleaning/Wrangling

The first step of any data driven project is pre-processing and data cleaning. The specific steps required at this stage will vary based on the availability and format of data for your state. The primary form of data for analysis in GerryChain is a **dual graph**, which lends itself to a discrete graph partitioning problem.

Dual graphs are built using shapefiles containing geographic data that tracks the boundaries of voting precincts or census geographies and tabular data containing population, demographic, election, and voting data within those precincts. Additional steps are necessary to convert a shapefile to a dual graph. Figure 2 below shows an example of the 2012 Colorado

Congressional map (left) and its resulting dual graph (right) after processing. Sample code converting Colorado from a shapefile to a dual graph can be viewed [on our Github](#).

Figure 2: Colorado 2012 Congressional Map and Dual Graph



Although the data needs of each redistricting analysis will depend on the specific questions of interest, the most common components of a GerryChain ready shapefile include six attributes: voting precinct ID, district IDs, population totals, racial demographics, election results, and geometry. Additional technical reading about shapefiles can be found in original documentation from ESRI and through the Library of Congress.

- *Voting Precinct ID:* Official identifier that indicates the unique voting or electoral precincts that voters are assigned to and where they cast their votes for each election
- *District ID*: Current and past congressional district or state legislative district the voting precinct is located in
- *Population Totals*: Count of the total population assigned to the voting precinct
- *Racial Demographics*: Count of racial demographic group voting age population assigned to the voting precinct.
- *Election Results:* Vote counts for each election for each precinct
- *Geometry*: Geometries, typically polygon or multipolygon, indicating the shape and area of the precinct

Common Data Sources

Users may elect to create their own shapefiles, or find pre-existing cleaned shapefiles. Many publicly available shapefiles are available through the Redistricting Data Hub, the Princeton Gerrymandering Project, and the Metrics Geometry and Gerrymandering (MGGG) group at Tufts University. All three organizations host open collections of precinct-level processed shapefiles covering all U.S. states and Puerto Rico. These shapefiles are pre-constructed, publicly available for use, and can be readily loaded into GerryChain for analysis.

However, pre-processed data files have some limitations and may not be suitable for all user needs or redistricting questions. In particular, the MGGG shapefiles are not inclusive of all elections held in a given state in a given year. The working groups producing shapefiles often need to make pre-processing decisions when preparing their data. Many groups, such as MGGG detail these data processing decisions in their metadata documentation, but it is important to be aware of and understand the steps taken in this regard.

For instance, MGGG employs an open-source geospatial toolkit called maup that assists with fixing common raw shapefile problems such as topological issues, overlaps, and gaps in districts. This toolkit also allows users to align data sources that use different geographical units (e.g. voting precincts and census blocks). This alignment process is often arduous, and MGGG has created a template to assist in aggregating data from different sources. The GerryChain documentation also includes some useful suggestions for dealing with map-related problems such as islands or districts that otherwise do not fully connect. In general, though using third party cleaned data is convenient, it requires careful examination of what pre-processing decisions were made and their implications.

Alternatively, users may choose to process their own shapefiles using publicly available or requested data from government entities such as the state demographer's office, the secretary of state, and others. Our Github contains some examples of code used to download, process, and clean data from Colorado's Secretary of State website, which may be helpful for users wanting to import their own elections data. However, in many cases users will have to compile data from multiple sources as a single government entity rarely collects all of the types of data necessary for most analyses related to redistricting.

Those who want to participate in the 2021 redistricting cycle will likely be interested in utilizing the 2021 census data for analyses in GerryChain. As the census data release date was delayed until August 12, 2021, many states will be operating on a shortened timeline for redistricting in some cases it may not be feasible to wait for third party organizations to process the data for public access. Ambitious users with sufficient technical abilities may find the GerryChain guidelines for constructing their own dual graphs using publicly available data sources helpful. Our Github also contains an example of combining American Community Survey (ACS) demographic data with an existing shapefile. A similar process could be used to combine more recent data with existing shapefiles if users wish to evaluate plans as demographic shifts occur in between redistricting cycles. Otherwise, the data resource pages of MGGG, Princeton Gerrymandering Project, and the Redistricting Data Hub, will be updated as these groups continue to prepare shapefiles with the updated census data.

Data Cleaning Questions to Ask Yourself:

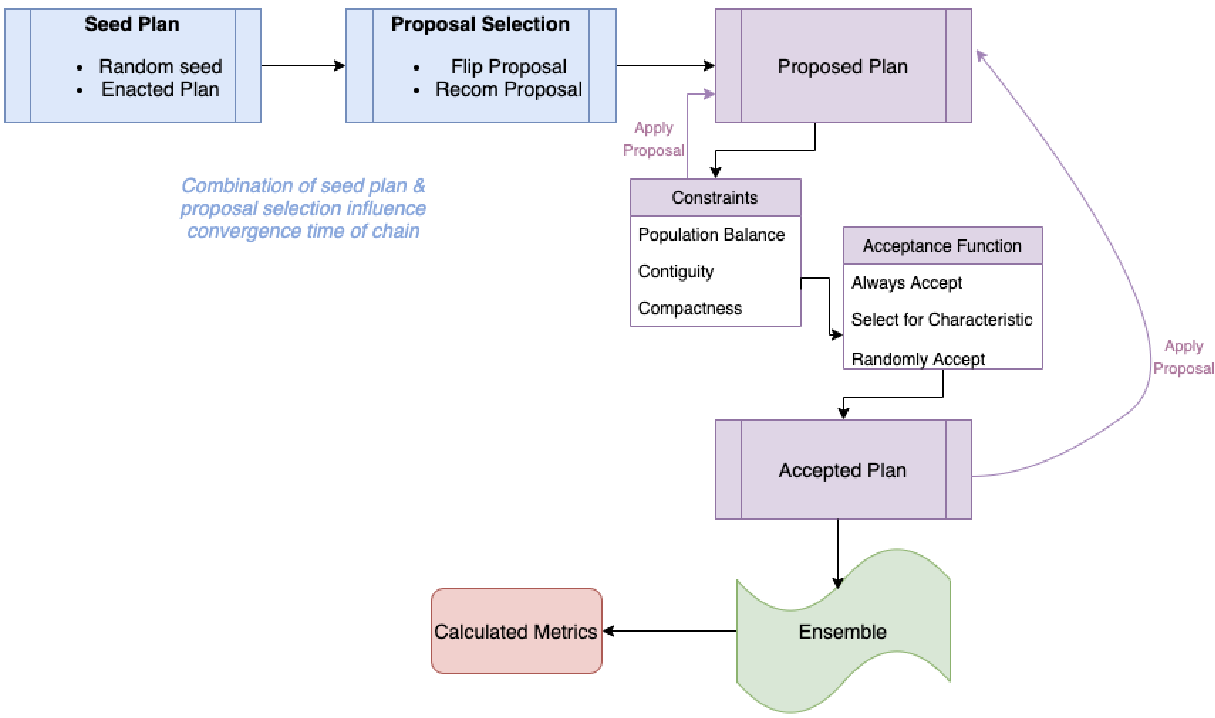
- *What offices and agencies can I source additional data from if needed?*
  - The U.S. Census Bureau, state-level Secretary of State, and state demographer's office are all great places to start.
- *What are the potential limitations of this additional data?*
  - Make sure the data includes some indexing files that can be used to merge the data sources. Data with precinct IDs or census block IDs are usually the easiest to merge with existing shapefiles, but be sure to check that the identification system is consistent as state and federal agency IDs do not always match.
- *If the data sources are available at different geographical units (e.g. voting precinct vs census block level), how can I reconcile the datasets?*
  - Best case scenario the different geographical units nest within each other. For example if you have one set of data at the voting precinct level, these might map into congressional districts without too much trouble. In other cases, you may need to apply a prorating method to align the two geographies.
- *If I'm using preprocessed data from another organization, like MGGG, what decisions were made to build the final shapefile? What were the data inputs? What limitations are associated with their decisions?*
  - Shapefiles prepared by reputable organizations like MGGG will almost always have metadata files associated with them, which you can consult to understand the data processing decisions they made. Be sure to read and understand them.
- *What changes occurred in my state of interest over the ten years since the last census that might impact my data?*
  - For example, some states allow precinct boundaries to be redrawn to adjust for population growth and decline in between censuses. This may mean that precinct boundaries are not consistent across your data set if you're using data collected in multiple years. Be sure to check for inconsistencies like this.
- *If merging multiple data sources, how can I double check that the datasets merged cleanly?*
  - Verifying that geographical units such as census blocks or precincts have consistent identifiers is important before merging. But you might also need to perform some other checks after merging to ensure that it did so correctly. This is also an area where some exploratory data analysis might be helpful.
- *If my data is insufficient to conduct my initial analysis of interest, are there alternative analyses that I could run with the data you have that get at the same question(s)?*
  - Data limitations are a reality of any social science inquiry. While we can often imagine a perfect dataset that would allow us to run an analysis of interest, it is rare to have such ideal data. Given the complexity of redistricting problems and the limitations of existing data, it is often necessary to be creative and come up with analyses that will work with the data limitations.

Stage 2. Exploratory Data Analysis & Modeling Decisions

This section is the most critical for understanding how to apply GerryChain effectively. In addition to reviewing the major steps required to implement the code, we also underscore the importance of making thoughtful modeling decisions to ensure your application of GerryChain is consistent with your analysis goals.

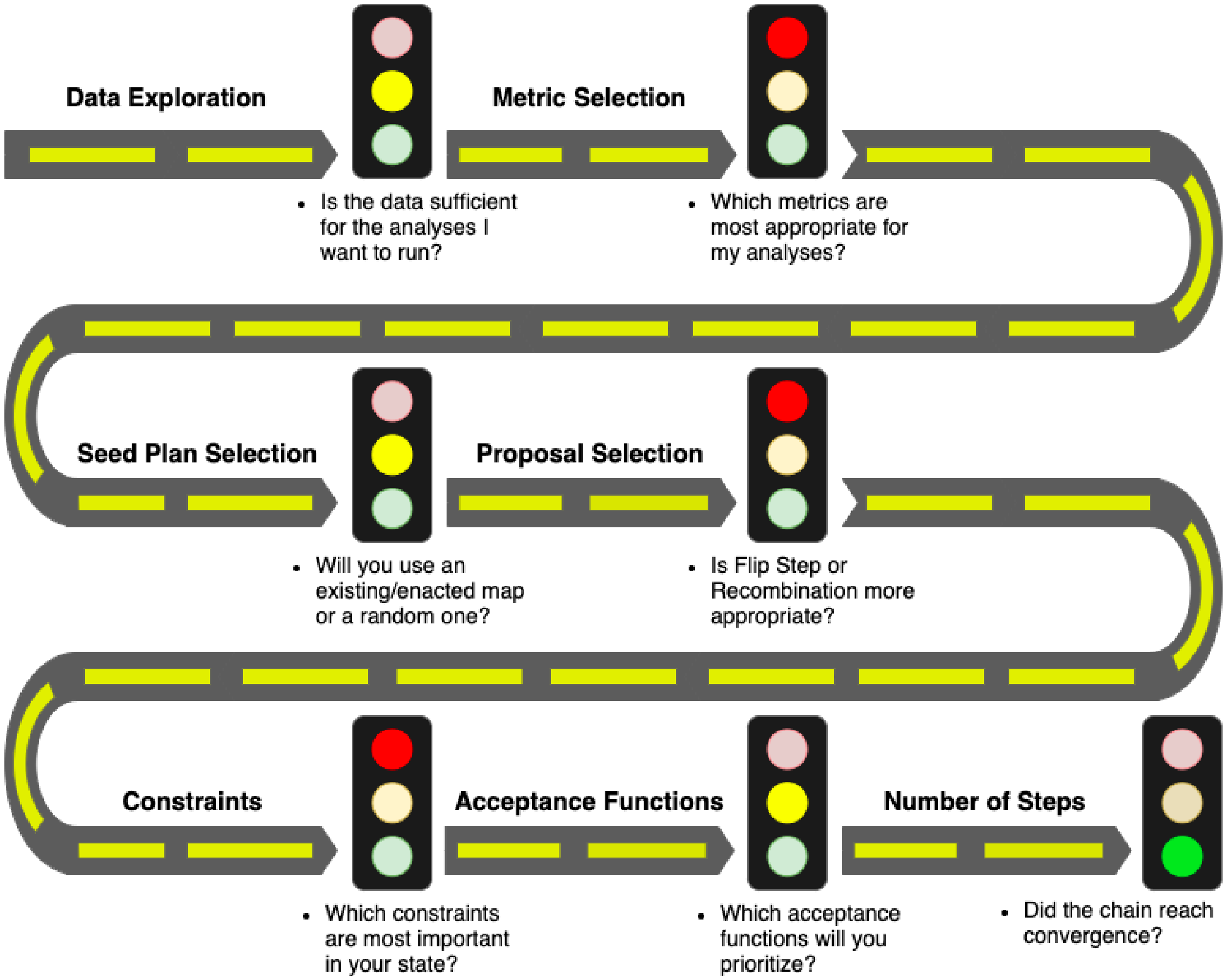
Before we delve into specific modeling decisions, it is helpful to understand the general flow of how MCMC methods work in the redistricting space. As illustrated in the flowchart below (Figure 3), the first step in the Markov chain is an initial seed plan. A proposal function is used to alter the seed plan in some random manner, and this proposed plan is then validated to ensure that any applicable constraints are met so the plan can be considered for inclusion in the ensemble. If the plan does not meet the constraints, then the proposal function is applied iteratively until a plan is created that does satisfy all of the constraints. Once a valid plan is created, it can either be accepted into the ensemble 100% of the time or with an additional random probability based on a selection criteria. As we progress through each step in the chain, each accepted plan becomes part of the ensemble, and this set of sampled plans is then used to calculate relevant metrics from.

Figure 3: Flow Chart of MCMC Methods Applied to Redistricting



The diagram below (Figure 4) traces the major decision points in the GerryChain workflow and highlights some key questions that you will need to ask yourself at each stage to ensure that you are making appropriate modeling choices. The rest of this section details each major step in more detail and explains what goes into coding them appropriately.

Figure 4: Roadmap of the GerryChain Workflow



Data Exploration

Is the data sufficient for the analyses I want to run?

Before delving into the application of GerryChain, it is important to spend some time familiarizing yourself with state level data you are working with and make careful, intentional modeling decisions based on the redistricting criteria unique to that context. It may also be helpful to read over the work that has already been done in your particular state or local context when it comes to computational redistricting. MGGG has published a variety of [state- and local-level reports]() and [research on relevant metrics](). Initial steps of the exploratory data analysis (EDA) should include identifying the analyses you are interested in running and determining whether or not the data you have is sufficient for that purpose.

For example, some state's legislatures like Virginia have entertained adopting specific criteria for using computational tools to test whether or not district maps have been potentially gerrymandered. The language of this proposed legislation specifies that the expected electoral outcomes of a districting plan should be determined by using data from a representative sample of statewide elections. The proposed legislative text in Virginia denoted a representative sample of statewide elections as "a collection of no fewer than four contests from the preceding four general election cycles which featured candidates from the two political parties whose candidates for president garnered the most votes in the most recent presidential election in the state."

Taking this example, a basic but very important EDA step would be to ensure that you have data on at least four elections meeting this criteria before beginning your analysis. If not, you will either need to find and merge sufficient election data from other sources, or conduct a different set of analyses. Once you are confident that your data is complete enough for your analyses of interest, you will need to make a series of modeling decisions in order to apply GerryChain effectively.

Metric Selection

Which metrics are most appropriate for my analyses?

The next step is **metric selection.** GerryChain has a number of built-in metrics functions available, which we cover in more detail in the Georgia case study below. Depending on your analysis of interest and your state context you may need to develop your own metrics. A small but growing number of states have begun adopting specific language about how plans should be evaluated for potential gerrymandering or require that features like the competitiveness of districts be assessed in the redistricting process. The language of these tests differ, but our case study of Colorado demonstrates one approach to applying a test for competitiveness and provides an example of how a user might translate state redistricting rules into code. Metric selection is a key component of running analyses in GerryChain, and as such your choice of metrics will depend upon your goals. Our case studies of Georgia and Texas provide additional examples of selecting appropriate metrics for analyzing general partisan trends of existing maps or making decisions regarding VRA compliance when designing new maps, respectively.

Seed Plan Selection

Will you use an existing/enacted map or a random one?

The selection of a **seed plan** is a bit more straightforward, but still depends on the goals of your analysis. A seed plan is the starting point for the Markov Chain algorithm. GerryChain is capable of producing a randomly generated districting plan through a recursive tree partition function that conforms to your state requirements, which can then be used as the starting point for your chain. This approach is most appropriate if you are interested in generating a neutral ensemble of plans with which to compare proposed or existing maps to the distribution of metrics. Alternatively, you can use an existing districting plan as the starting point for your Markov Chain. This approach may be best if you are interested in exploring the space of similar plans that could produce different outcomes without dramatically altering the existing map. While the seed plan selection is consequential for Markov Chains with fewer steps, in most cases you will want to run the chain for enough steps that it converges on a stable distribution. If the chain reaches convergence, the seed plan starting point will not ultimately matter because the resulting distribution will cover enough plans in the ensemble that a similar distribution could have been reached even if a different seed plan was used. For more details about seed independence, MGGG's analysis on Virginia's House of Delegate provides further elaboration.

Proposal Function Selection

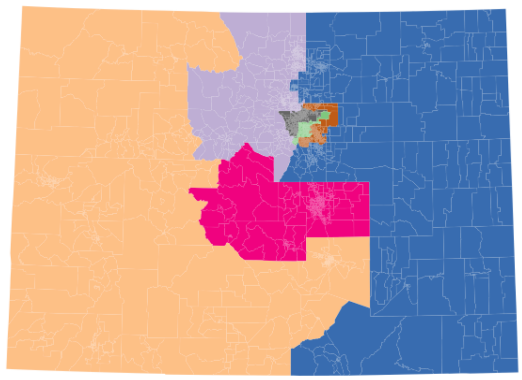
Is Flip Step or Recombination more appropriate?

The **proposal function** determines the type of steps that the Markov Chain takes to explore the state space of plans. There are two primary proposal functions used in GerryChain: **Flip** and **Recombination (ReCom)**. The Flip proposal "flips" a single randomly selected precinct at the boundaries of one district with each step of the chain. Because it makes only incremental changes by altering districts only at boundary points, each step requires negligible computing time but it takes a very large number of steps (often in the millions) to explore the state space sufficiently to reach a stable distribution. The Flip proposal is best suited to analyses interested in exploring state spaces similar to existing districting plans. For instance, if you wanted to assess the extent to which it would be possible to increase the political competitiveness of districts without completely altering the current electoral map, Flip would be the logical choice.

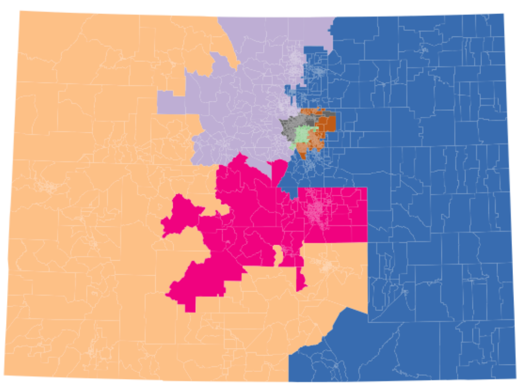
The ReCom proposal, on the other hand, randomly selects two adjacent districts and briefly merges them together before using a spanning tree algorithm to re-divide the joined districts into two new contiguous and roughly equal population balanced districts. Each ReCom step, therefore, makes much larger changes to the map. This means ReCom steps are more computationally intensive, but fewer total steps are required to explore a much more diverse space of plans and can often converge on a stable distribution after tens of thousands of steps (as opposed to millions). If the goal of your analysis is to produce a distributional baseline with which to compare existing maps (e.g. for outlier tests), then ReCom will usually be the right choice. If the Markov Chain is run for enough steps, it will eventually converge on a similar stable distribution, so the choice of proposal function becomes less important if you have the time and computing power to run a very large number of steps. For more detail about the development of the ReCom proposal and how it differs from the Flip proposal, see DeFord et al. (2021).

Below (Figure 5) is an example of the map generation process using the Flip proposal (lower left) and the ReCom (lower right) using Colorado’s 2012 Congressional district map

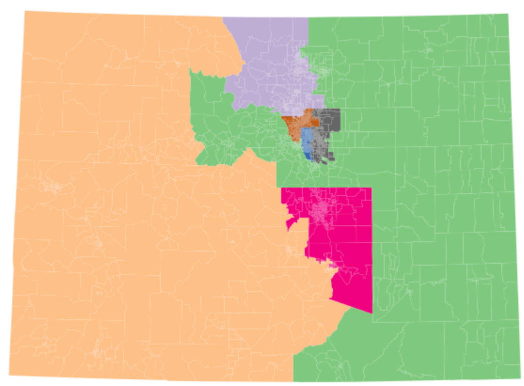
Figure 5: Markov Chain Examples using Flip versus ReCom Proposal



2012 Congressional District Map



100 Flip Steps



10 ReCom Steps

Constraints

Which constraints are most important in your state?

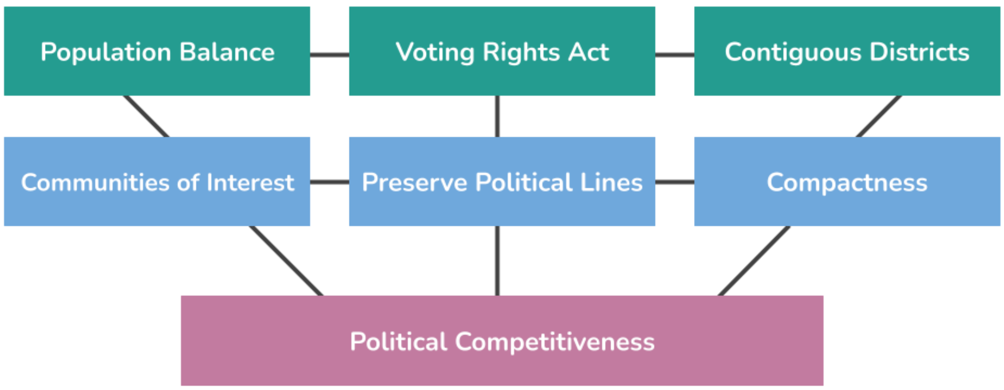
Constraints provide important limits on the types of districting plans that will be produced by the GerryChain algorithm. For instance, to ensure that districts are reasonably compact, a requirement in all states, you will need to include a compactness constraint. GerryChain has many common constraint functions built-in, but depending on your state context you may need to either produce some constraint functions from scratch or tune them specifically to meet your state’s guidelines. Similarly, contiguity of districts is generally required in most states. Though the ReCom proposals automatically maintain district contiguity by design, if using a Flip Step proposal you will need to include a contiguity constraint.

While contiguity and compactness are almost always required and tend to be similar regardless of state context, the inclusion and specification of other constraints will depend on the state context in which you are working. A necessary step at this stage, therefore, is to familiarize yourself with your state's redistricting guidelines to ensure that your approach in GerryChain conforms to those basic requirements. Otherwise you risk producing maps that are not viable under your state's laws. In addition, you will need to make modeling decisions based on how your state prioritizes these requirements in the districting plan generation process. Many of these decisions are left up to interpretation, so it is important to be transparent about how and why you make the modeling decisions that you do.

Each state has its own legislation to guide the redistricting process, and each varies considerably in its level of detail, and number of requirements. In addition to contiguity and compactness, all states require some version of population balance between districts (i.e. roughly the same number of people living in each district) and all states must comply with the 1965 Voting Rights Act. Vague language in these guidelines is common. Few specify how compact (or by what measure) districts must be and phrases like "to the extent practicable" frequently raise more questions than they answer when it comes to implementing this legislation. There is also a trend in some states toward adopting requirements for political competitiveness of districts, but unfortunately there is no agreed upon definition or measure of competitiveness (see our Colorado case study for more on political competitiveness).

Some states also specify hierarchical preferences regarding which requirements are given weight above others. For example, in 2018 Colorado's State Legislature adopted a new framework for redistricting, which requires that electoral maps 1) keep district populations as equal as possible; 2) preserve communities of interest and political subdivisions; 3) minimize the number of divisions when a city, county, or town is divided; 4) have districts as compact as reasonably possible; and 5) maximize the number of politically competitive districts. The State Legislative Council then developed the following hierarchy to guide the 2021 redistricting cycle:

Figure 6: Hierarchy of Redistricting in Colorado



From these legislative requirements, a user of GerryChain in the context of Colorado will therefore have to decide how to translate this text into their code. Though not all states have such explicit hierarchy guidelines, most states do specify some level of prioritization among the different requirements. Understanding these guidelines and their hierarchy is important for determining the constraints that you use. For example, you will have to decide the level of compactness required for each district (and what measure of compactness to use) and the acceptable level of deviation from perfect population balance across districts. Constraints will therefore prevent GerryChain from producing districting plans that exceed the levels of compactness or population imbalance that you specify. If there are lower priority guidelines outlined by a state, then these desired (but not required) rules might be operationalized through the use of acceptance functions described below instead of constraints, which are more stringent and must be met for a plan to be considered valid for inclusion in the ensemble.

Acceptance Functions

Which acceptance functions will you prioritize?

Acceptance functions determine which districting plans are (or are not) ultimately accepted into your ensemble. Compared to constraints, acceptance functions offer more flexibility and can be designed to gradually move the districting plans proposed by GerryChain toward a desired outcome while still allowing the algorithm to explore a sufficiently diverse set of plans to reach a stable distribution. The use of acceptance functions can therefore serve the role of an objective function in this context.

The logic behind an acceptance function is to set conditions that dictate whether the random walk should take a step to the next state. As detailed in the GerryChain API, the acceptance function either take GerryChain's always accept function, or a custom function. The custom function itself creates an acceptance rule and returns either True or False to determine whether the random walk should indeed proceed to the proposed plan. Custom acceptance functions can be found in our case study EDA notebooks, such as those that minimize county splits, explore different measures to maximize political competitiveness, and those that combine multiple conditions such as one that minimizes county splits as well as maximizing political competitiveness. For acceptance functions that include tight or multiple acceptance criteria, a degree of randomness can be added to assist the walk to "move along". This can help run time and to ensure the Markov chain explores different configurations of the state space. The specific random value a user selects is a decision suited for the level of randomness they want in their exploration of the state space and the degree to which they want to preferentially select for specific characteristics (e.g. minimal county splits). A user should consider testing the impact of varying this random probability, and tuning the value to build an ensemble with desired characteristics.

Of note is the need to consider a combination of constraints and acceptance functions. The distinction is that constraints define whether districting plans are valid, while acceptance functions dictate whether the random walk should move to the next state or not. Acceptance functions serve as alternatives to hard constraints since they provide a secondary criteria that are preferred, but not a hard requirement. This is especially relevant on how to interpret legal redistricting rules along a hierarchy, or proposed rules to investigate what's possible. Thoughtful combinations of constraints and acceptance functions will be crucial for enabling GerryChain to run in a way that explores a sufficient sample of districting plans, while also limiting the types of plans that are ultimately accepted into the ensemble to ensure that they still conform to state guidelines and to optimize user conditions. Each of our case studies illustrate some approaches to thoughtfully applying constraints and acceptance functions, but this is an area of GerryChain that is highly customizable with plenty of possibilities depending on your analytic goals.

Number of Steps

Did the chain reach convergence?

Finally, you will need to set the number of steps your Markov Chain will take before finishing. The ultimate goal in most cases will be to reach convergence. This ensures that your ensemble contains a steady distribution of plans and is therefore likely representative of the distribution of plans that are possible in your state given its constitutional guidelines. While this suggests that more steps are always better, the tradeoff is the amount of computing time required to complete them.

A statistically motivated way to select an appropriate number of steps is given in Clelland, et al. (2020) but often heuristic checks are used in practice to evaluate mixing. A good starting point for ReCom proposals is 20,000 steps. Though this number will need to be adjusted depending on the number of districts in your state, computing power, and time available. The test for whether or not the number of steps you've selected are satisfactory will be whether or not convergence is reached. If using Flip proposals, a good starting point is one million steps. Remember, Flip is much faster so running this many steps is feasible.

To test for convergence, there are two classes of approaches often used: (1) multi-start heuristics to compare chains, and (2) within chain tests. A user can check for convergence by comparing distributions of metric values from chains that started with different seed plans, and if the resulting distributions are independent of the seed plan, then it can be assumed convergence has occurred. Additionally, once a Markov chain has converged, the distribution of the measured metrics should not continue to shift as the length of the walk increases. Thus, to test for convergence, we can also examine whether the metric distribution for the first 50% of the steps resembles the full distribution, and if so, then convergence is achieved at 50% of the full chain length. Other within chain tests include checking for autocorrelation between steps, although less rigorous methods of comparing distributions of target metrics is often a sufficient indication of convergence. Both across chain and within chain tests are illustrated in the MGGG technical report examining districting plans in Virginia.

Modeling Questions to Ask Yourself:

- *What is the ultimate goal of my analysis?*
  - Answers will vary widely here since GerryChain is applicable to a diverse array of analytic questions. But common analyses might include to assess the partisan balance of a proposed or existing map; to evaluate the political competitiveness of proposed districts; to perform an outlier analysis to determine whether or not a map has potentially been gerrymandered; to assess how many districts are considered minority opportunity districts, etc.
- *What computational redistricting work has already been done in my state?*
  - MGGG and other groups have already done a robust number of state level analyses which may be pertinent to your interests. Rather than starting from scratch, it may be helpful to look into other efforts already made in your state.
  - State-level computational redistricting research has been published in peer-reviewed journals. These can be publicly searchable through Google Scholar, using keywords such as: "Markov chain Monte Carlo methods", "computational redistricting", "GerryChain". While these articles often contain more methodological detail than may be desired by the user, their findings often highlight nuanced modeling decisions within a state that users might find informative.

- *How do you justify your choices of seed plan, proposal type, constraint, acceptance function, and number of steps?*
    - The above sections give some general guidance on when and why to select different seed plans, proposals, constraints, etc., but the ultimate decision will depend on the goals of your analysis. There is rarely a “right” answer to any of these questions. Rather, the more important thing is to understand the reasons for making these decisions and be able to explain the appropriateness of those choices.
- *What are the implications of my decisions and the trade-offs associated with each decision point?*
    - We outline decision points and questions users should ask themselves at each step of the EDA and modeling process. Being transparent about your decisions, their reasoning throughout your analysis is key to using GerryChain ethically and effectively. Furthermore, it’s important to document and highlight the trade-offs and implications of such decisions -- the ways your unique analysis does not consider or apply specific modeling considerations and the effect that decision has on the findings.

Stage 3. Applying GerryChain:

The final step is actually running your analyses of interest using the GerryChain library. If you have already thought through the modeling decisions above, this step can be quite straightforward. GerryChain’s API documentation and Getting Started Guide outlines exactly how to execute a GerryChain file from start to finish. Each of our case studies contain example code for running the entire GerryChain sequence for our state specific analyses and may be helpful for those wishing to apply similar approaches.

We offer two considerations at this stage: memory management and data visualization. Users can have GerryChain write statistics of interest to a list, but longer chains may tax memory requirements. We have updated and refactored existing code from the GerryChain Getting Started Guide into a utility function that exports all metrics of interest to a csv file, which the user can then read back to conduct analysis and create visualizations. Creating visualizations of the data with Matplotlib or any of its higher level interfaces such as Seaborn or interactive libraries such as Altair will rely on user goals. The type of visualization will rely on the type of metric profiled. We encourage users to consider the intended audience of their analysis, and to focus on readability to communicate their intention. GerryChain has several template resources that illustrate the use of histograms, box plots, plots, and map, while our case studies work in Colorado and Texas through the decisions that went into our final visualizations.

GerryChain Application Questions to Ask Yourself:

- *Have you documented your modeling choices and made them explicit in this file?*
  - The purpose of the EDA is for the user to consider the different decision points that emerge as part of the modeling process. Clearly documenting user modeling choices, and the strengths and trade-offs of those choices, will ensure transparency behind your analysis.
- *Have you considered basic data visualization principles?*
  - There are many resources available online that detail basic data visualization principles, not just for GerryChain and Python, but data science and statistics in general. Edward Tufte's *The Visual Display of Quantitative Information* is considered a classic text of data visualization that users might find enlightening.
- *Who is the intended audience for this analysis?*
  - Visualizations used as part of GerryChain's ensemble analysis have generally been histograms, box plots, plots, and maps. However, GerryChain's Markov chain outputs allow for flexibility and creativity to integrate with any type of data visualization library and visualization type.
  - However, data visualizations are meant to be easily consumable, powerful artifacts of data modeling and statistical analysis. The best data visualizations are those that communicate the author's intention simply and directly. We encourage users to think about the intended audience for their analysis, and how the choice of data visualization options can ease the audience's comprehension and information needs.

Applying GerryChain in State Contexts

The following section outlines three different state-level applications of GerryChain. These brief overviews of the state case studies are supplemented by companion code on our Github and summaries of findings on our project website. The purpose of these case studies is to illustrate more concretely the decision process and modeling choices that must be made for different analytic goals. These are by no means the only approach to conducting similar analyses, but rather one example of how to apply GerryChain effectively with a given analytic objective in mind. We strongly encourage readers to consume these case studies in tandem with our annotated code, which provides crucial details explaining each step and will give users a more tangible understanding of the GerryChain process.

Georgia: Using GerryChain's Built in Metrics

We use the case of Georgia to demonstrate the application of some of GerryChain's built-in metrics, and highlight some of the limitations of this approach alone. Political scientists have proposed a number of metrics to quantitatively measure concepts like partisan symmetry and democratic efficiency. GerryChain includes built-in functions to apply many of these metrics. However, all such metrics have significant limitations and the outputs of these functions should not be taken as truth. When applied appropriately and thoughtfully, though, they can give users a better sense of the balance of partisan power within a given state or the degree to which voters are fairly represented within and across districts.

We chose the state of Georgia for this case study due to its history of single-party control. Both the State Legislature and Governorship have long been controlled by one party, giving state officials ample opportunity to engage in partisan and racial gerrymandering. Though several legal challenges have been brought against the State Legislature alleging partisan and racial gerrymandering, most have been dismissed by the courts. GerryChain could potentially be used to assess whether or not district maps have been gerrymandered by comparing the electoral outcomes of current maps with a neutral ensemble of plans. The built-in metrics of GerryChain could be usefully applied in this context to test for partisan gerrymandering. Details of the findings and the analysis on the metric chosen to evaluate electoral outcomes of the state is provided in our Georgia case study report.

Overview of Metrics

Mean-Median

The mean-median score is a test of partisan symmetry that considers the difference between the median and the mean vote share by district. The logic underpinning this metric is that 50% of votes for a given political party "should" translate to 50% of the seats for that party. The mean-median score is vote-denominated and produces a signed number that, in principle, measures how far short of 50% of the vote share a party can fall and still secure 50% of the seats. The ideal value of the score is zero, which would reflect a map that is perfectly balanced between the two parties. A positive or negative score is therefore supposed to identify an advantage or disadvantage of a given party, respectively, and to what degree. The equation for the mean-median score is as follows:

$$\text{Mean-Median (MM)} = X_{median} - X_{mean}$$

In practice, however, the mean-median score is often quite problematic and should not be taken at face value. The metric is prone to sign errors. It has been shown in some cases to flag one party as being the beneficiary or partisan gerrymandering, when in reality the electoral map has been designed to ensure that party receives the fewest seats possible based on the political geography. The metric is also highly gameable and can be manipulated in some contexts to produce a score close to zero suggesting that the district map is fair, when in reality it has been heavily gerrymandered in favor of one party. Thus, this metric should be applied with caution and cannot be taken alone as evidence of gerrymandering or the lack thereof.

Partisan Bias

Partisan bias is another measure of partisan symmetry and like the mean-median score is not a reliable test of gerrymandering in and of itself. This metric is seat denominated, and is supposed to measure how many seats over 50% a given party would secure if they received 50% of the votes. Like the mean-median score, the ideal partisan bias value is zero and it is also a signed score that should, in theory, establish which party has an advantage or disadvantage and to what extent. The following equation is used to determine partisan bias:

$$\text{Partisan Bias (BB)} = \boldsymbol{\gamma}(½) - ½$$

Here, $\boldsymbol{\gamma}$, represents the seat-votes curve, a plot which establishes the connection of the vote to seat shares of the point-of-view party. Like the mean-median score, partisan bias is prone to sign errors and susceptible to manipulation. Thus, it too is not sufficient to establish evidence of gerrymandering or its absence without additional tests or analyses to corroborate.

Wasted Votes

Wasted votes, by definition, are any votes cast for a party in excess of those required for that party to win a seat. The premise of this metric is that votes cast for a party in excess of 50% plus one are effectively wasted within that district. Measuring wasted votes can theoretically provide circumstantial evidence of gerrymandering. For instance, a high number of wasted votes for one party in a district may suggest that voters have been “packed” into a district to produce a safe seat for that party. If all wasted votes belong to the losing party of that district, it is considered competitive. An example of the equation used to calculate the wasted votes for party X (W_x) in district A is defined as follows:

$$W_x = T_x - S \times (T_A/2)$$

In this case T_x represents the total number of votes in district A for party X. S is a binary variable that is 1 if party X won district A and 0 if not. T_A is the voting turnout in district A. Though wasted votes may be used to build circumstantial evidence of packing voters of one party into a given district, this metric alone is insufficient to prove gerrymandering has occurred.

Efficiency Gap

Wasted votes are also the basis for another built-in metric, the efficiency gap (EG) After wasted votes are calculated, the efficiency gap can be computed as follows:

$$\text{EG} = (\text{Party A wasted votes} - \text{Party B wasted votes}) / \text{total votes}$$

When there are two major political parties that are roughly equal in popularity, it is argued that the efficiency gap should be zero. Advocates of the efficiency gap therefore argue that a score greater than 0.08 (i.e., EG ≥ |0.08|) can be interpreted as evidence of potentially gerrymandered districts. Like the other proposed metrics, however, it is possible for EG to produce false positive or negative results, and it is susceptible to gamification. Though, EG is still a potentially useful component for broader analyses that consist of multiple other metrics, alone it is not sufficient evidence for determining whether or not a district map has been gerrymandered.

Colorado: Using GerryChain to Support Map Drawing

New state-specific requirements have emerged for the 2020 redistricting cycle that require additional examination before real-world application. In particular, in 2018 Colorado voters approved Amendment Y to add new language to the state constitution that requires the consideration of political competitiveness in the redrawing of Congressional district maps. Specifically, Article V of the Colorado Constitution now states in Section 44.3: "Thereafter, the commission shall, to the extent possible, maximize the number of politically competitive districts." This constitutional language is exceedingly broad, and is open to broad interpretation of how to qualify and quantify what politically competitive districts mean.

The goal of the following case study is to define and implement a measure of political competitiveness relevant for Colorado's redistricting context. The following section will outline the GerryChain process and methods engaged as part of this case study. A complete narrative of findings can be found on our Colorado case study report.

Stage 1. Data Wrangling and Cleaning

The Colorado case study utilizes a processed shapefile of 2018 precinct-level election results. More details on MGGG's Colorado data source and process can be found here. The shapefile contains one federal election (U.S. House race) and four state-level and, as well as racial demographic data for the population and voting age population of the precinct.

However, there are limitations. Due to raw data quality initially supplied to MGGG from the Colorado State Demographer's Office and the need for extended processing, 81% (n= 2555) of the precincts in the final shapefile do have a state-wide 10-digit unique precinct ID. This made joining MGGG's shapefile with other precinct-level data tricky since only 19% of the IDs matched upon a simple merge. Higher-level unique IDs, such as Voting District, are available, but would aggregate up and lose the precinct distinctions.

Additionally, Colorado's precinct boundaries change annually due to shifting population totals, so while Congressional district boundaries remained the same from 2012-2022, there have been slight shifts of precinct lines over the years. There is currently a lack of publicly available, clean data that keeps track of precinct changes over time and election results tracked across the years.

Therefore only 2018 election data is available and attached to the preprocessed MGGG shapefile used in this case study.

Note: Our team cleaned and merged archival election data from the Colorado Secretary of State, and built a precinct-level panel data of all federal election results from 2004 - 2020. This data is available in our Colorado/Data folder and may be of use for users if they choose to interpret Colorado's language of competitiveness as "measured by factors such as a proposed district's past election results". There are non-trivial challenges associated with merging this panel data to cleaned, preprocessed shapefiles appropriate for GerryChain.

Stage 2. Exploratory Data Analysis (EDA) and Modeling

Before conducting the technical Colorado EDA and modeling work, we first had to engage with the research literature on political competitiveness. We found that political competitiveness can be examined through two distinct bodies of work. Since the inclusion of the political competitiveness language will be relevant for the 2020 redistricting cycle, several papers from mathematicians have recently been published exploring this question in the multiple state contexts. Additionally, political scientists have theorized and tested their own measures of political competitiveness, broadly in relation to voter behavior and partisan politics in single member districts. Yet, despite this rich research literature, there is no prevailing agreed-upon standard on how to best quantify measures of political competitiveness.

After reviewing the literature and examining field-specific consensus, we decided to focus our definition of political competitiveness through the vote band method. Clelland et al. (2021) and DeFord et al. (2020) both investigate political competitiveness using GerryChain, and conclude that a vote band of 45-55% is the simplest starting point. The vote band method has also been adopted by other organizations such as the 538's Redistricting Tracker and Princeton Gerrymandering Project's Report Card (Note: the Princeton Gerrymandering Project uses a narrower band of 46.5-53.5%)

Clelland et al. state that “a district is considered competitive if Democrats and Republicans both receive 45% to 55% of the combined Democratic and Republican votes (votes for candidates of other parties are not considered)... and [the vote band method] requires the fewest assumptions and will be most satisfactory for [their] analysis” (p. 13). We too adopted a vote band method of 45-55%, although we acknowledge that Colorado’s language to “maximize politically competitive districts” is highly interpretable and can take many additional angles. We chose to start with a more generous 45-55% band to maximize the number of competitive districts in our proposed maps.

We walk through how we used GerryChain to explore political competitiveness in our Colorado EDA and modeling notebook available here. We organized this process using the steps outlined in Stage 2 outlined above, with ample documentation to describe our modeling decisions and analytical approach.

- *Metric Selection* - Our Colorado analysis specifically focuses on political competitiveness, particularly in relationship to the hierarchy of redistricting rules criteria. We also include county splits to our analysis to add additional nuance to our analysis of political competitiveness.
- *Seed Plan Selection* - We explore two different seed plans: the Colorado's 2012 Enacted Plan and a neutral random seed plan. The 2012 enacted plan will provide us with a real-world actual plan that survived the political process and has been in use for the past decade. However, it's important to note the limitations with enacted maps. In particular, the 2011 proposed map was challenged in court by both the Democratic and Republican parties, and users should be familiar with the arguments and performance of previous enacted plans. Our neutral random seed plan was generated from GerryChain using its various spanning tree methods.
- *Proposal Selection* - We explore the different performance of four combinations seed starting plans and different Markov chain types. For starting plans, we consider (1.) a random "neutral" seed plan generated from a recursive tree partition and (2.) the 2012 Colorado plan. For chain types we considered (a.) a flip boundary walk and (b.) a ReCom walk. While we only ran the Markov chain for 1,000 ReCom steps and 10,000 Flip steps -- which is not enough steps to reach a steady state distribution – we note that choice of starting plan doesn't matter as much as the proposal type. The ReCom proposals explore a far greater range of potential plans for both county splits and competitive districts. For the rest of our analysis, we use the ReCom proposal with a 2012 starting plan.
- *Constraints* - Based on the Colorado hierarchy of criteria, our constraints are focused on population balance and compactness. We relegate the preservation of county lines and political competitiveness to acceptance functions since we seek more flexibility in those rules. We use a basic population bound constraint available in GerryChain as well as a generic compactness bound constraint. We discuss our exclusion of a more in-depth analysis of compactness score in the context section of the notebook.
- *Acceptance Functions* - We offer four different custom acceptance functions seeking to maximize political competitive districts and to minimize county splits. We present these options as illustrative examples on how to write custom acceptance functions and the decisions necessary for modeling. Each of the four acceptance functions include a 10% degree of randomness in the function to ensure the Markov chain explores different configurations of the state space. This value is intentionally kept low since our ambitions for the chain is to minimize and maximize specific conditions.

  - Competitive Accept: This acceptance function only accepts the next step of a chain if it has the same or more count of districts that fall within the 45-55% vote band.
  - Competitive Nudge Accept: This acceptance function attempts to "nudge" districts that are close to the 45-55% vote band into the band. It does this by coercing the chain to increase the number of districts that fall within a 40-60% vote share band, and then accepts next steps of a chain that has the same or more count of districts that fall within the 45-55% vote band.
  - Competitive County Accept: This acceptance function uses the Competitive Accept function and adds a condition that to minimize county splits. Specifically it will only accept the next step of a chain if it has fewer county splits than the previous step.
  - Competitive Nudge County Accept: This acceptance function uses the Competitive Nudge Accept function and adds a condition that to minimize county splits. Specifically it will only accept the next step of a chain if it has fewer county splits than the previous step.
- *Number of Steps (Chain Length)* - We walk through two classes of approaches to check for chain convergence: multi-start and within chain. Our results demonstrate the independence of seed and that our chain using the ReCom proposal converged within 20,000 steps.

Additionally, a curated reading list about political competitiveness is provided below.

- Abramowitz, A. I., Alexander, B., & Gunning, M. (2006). Incumbency, redistricting, and the decline of competition in US House elections. *The Journal of politics, 68*(1), 75-88.
- Cain, B. E., MacDonald, K., & McDonald, M. (2005). From equality to fairness: The path of political reform since Baker v. Carr. *Party lines: Competition, partisanship, and congressional redistricting, 6*(8).
- Carson, J. L., & Crespin, M. H. (2004). The effect of state redistricting methods on electoral competition in United States House of Representatives races. *State Politics & Policy Quarterly, 4*(4), 455-469.
- Clelland, J., Colgate, H., DeFord, D., Malmskog, B., & Sancier-Barbosa, F. (2021). Colorado in context: Congressional redistricting and competing fairness criteria in Colorado. *Journal of Computational Social Science*, 1-38.
- Cottrell, D. (2019). Using Computer Simulations to Measure the Effect of Gerrymandering on Electoral Competition in the US Congress. *Legislative Studies Quarterly, 44*(3), 487-514.
- Cox, G. W., Fiva, J. H., & Smith, D. M. (2020). Measuring the competitiveness of elections. *Political Analysis, 28*(2), 168-185.
- DeFord, D., Duchin, M., & Solomon, J. (2020). A computational approach to measuring vote elasticity and competitiveness. *Statistics and Public Policy, 7*(1), 69-86.

- Forgette, R., Garner, A., & Winkle, J. (2009). Do Redistricting Principles and Practices Affect US State Legislative Electoral Competition?. *State Politics & Policy Quarterly, 9*(2), 151-175.
- Liller, D. E. (2020). The impact of competitive congressional districts on compactness and political subdivision splits in Colorado (Doctoral dissertation, University of Colorado Colorado Springs).

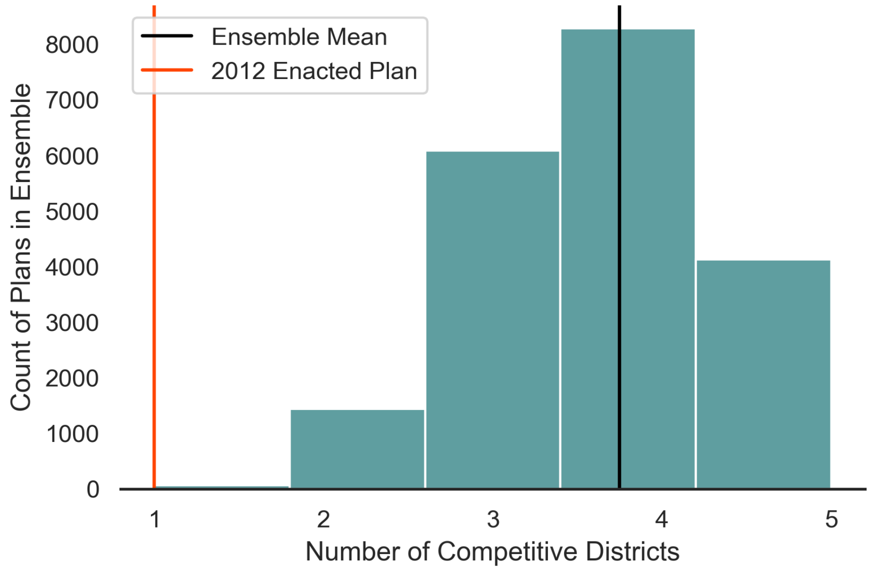
Step 3. Applying GerryChain and Ensemble Analysis

To review, our final decisions that emerged from Stage 2 are as follows:

- *Data*: 2018 U.S. House race
- *Metric*: Count of county splits and count of politically competitive (45-55%) districts
- *Seed Plan*: 2012 enacted plan
- *Proposal*: ReCom
- *Constraints*: Population bound, compactness
- *Acceptance Function*: Minimize county splits while maximizing political competitive districts using a nudge condition
- *Number of Steps*: 20,000

Our GerryChain Colorado script sets up the decisions above and runs a neutral ensemble that maximizes the number of competitive districts while minimizing county splits. The only significant change to our Stage 2 established model is an adjustment to the ReCom proposal to make it county aware (see the specific updated ReCom proposal under the "Proposal" section of the script). After running the model, we conduct an outlier analysis using a histogram to examine how the 2012 Congressional plan compares to the ensemble distribution.

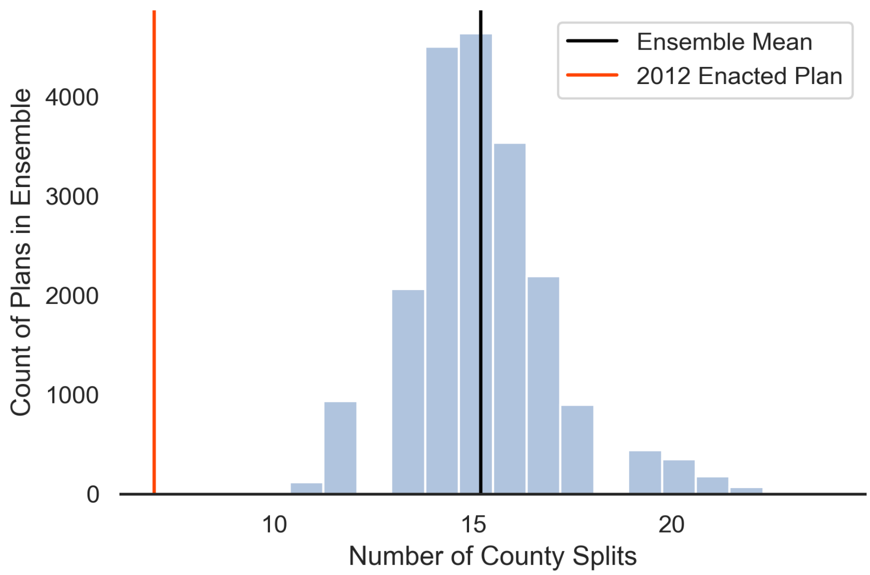
Figure 7: Proposed Plans that Maximize Competitive Districts



In the past 5 US House elections in Colorado using the 2012 map, only 1 of the 7 districts each year was considered competitive. Using GerryChain and 2018 U.S. House election data, we specifically designed the Markov chain to only accept next steps in the chain that contain plans with both a greater or same number of competitive districts and fewer or the same number of county splits. We ran the chain for 20,000 steps and then graphed the number of competitive districts contained in each plan explored. Our ensemble mean is 3-4 districts (specifically, 3.75), which is 2-3 districts more than the 2012 human generated plan.

However, our neutral ensemble did not generate proposed plans that met the number of county splits in the 2012 enacted plan. Even though we applied a county-aware ReCom proposal and an acceptance function with a county split condition, our ensemble mean is 15 county splits while the 2012 human generated map only split 7 counties. This illustrates the complexity that emerges when using ensemble analysis and computational tools to translate legal rules for redistricting. While the 2012 enacted plan was able to minimize county splits to a degree that our ensemble did not, the same plan did not maximize politically competitive districts to the degree our ensemble did.

Figure 8: Proposed Plans that Minimize County Splits



Our takeaway from this analysis is that Colorado Congressional district proposed plans that only contain 1 competitive district can be considered an outlier within the universe of potential plans, and based on the 2018 U.S. House election results, Colorado map drawers can aim to design maps that have up to 4 competitive U.S. congressional districts out of a total of 7 districts. However, balancing this condition with other redistricting rules requires trade-offs – that is, it may not be possible to maintain minimal county splits as seen in previous maps while maximizing politically competitive districts. Modeling requires human judgement and in-depth understanding of state-specific priorities, and our Colorado case study outlines a decision-making process that GerryChain users should consider as part of the process. Additionally, we wish to highlight the important nuanced discussion and debate on the value of political competitiveness (see, Deford et al., 2020) as a metric to enshrine into redistricting legal rules. Our analysis here provides one approach on how to apply this analysis in GerryChain, but the trade-offs that come with optimizing political competitive measures come at the expense of other important rules and measures of interest.

As Colorado is due to receive an additional U.S. House seat as part of the 2020 redistricting cycle – bringing their congressional districts from 7 to 8 – map drawers can also use GerryChain to decide where to selectively place new districts. Our Texas case study outlines how to evaluate the placement of new districts, specifically in compliance with the Voting Rights Act of 1965.

Texas: Using GerryChain to Evaluate Proposed/Enacted Maps

Texas has experienced an estimated 15.9% population growth over the last decade, driven largely by Hispanic groups. This has resulted in two new congressional seats being allocated to Texas in the upcoming 2021 redistricting cycle. As Texas considers where to place these two new districts, it will be important to not only account for changing population patterns within the state, but also to ensure that minority groups have equal representation. In the new redistricting cycle, it is crucial to consider what a fair map looks like for Black and Latino voters, and GerryChain can be leveraged to evaluate whether proposed plans by state legislatures are serving minority groups.

Section II of the Voting Rights Acts passed in 1965 prohibits redistricting plans that abridge any citizen's right to vote "on account of race or color (or membership in a language-minority group)." Advocacy groups can assess whether a map violates the VRA by evaluating whether members of the minority group have "less opportunity than other members of the electorate to nominate and elect representatives of their choice." Section II of the VRA does stipulate that districts should be designed to allow minority groups to elect their preferred candidate, not just the candidate preferred by the (usually White) majority. Thus, the VRA does provision states with the right to replace districts where minority-preferred candidates are not successful with districts where candidates are given the opportunity to win if they "pull, haul, and trade to find common ground" (*Johnson v. De Grandy*, 1994).

Moon et al. considers a district to be 'effective' for minority groups if the districts nominate (primary elections) and elect (general elections) minority-preferred candidates, potentially through coalition building. These districts can be specified for a target minority group (e.g. Hispanic, Black) or can be considered effective if it meets the criteria for at least one minority group. This method differs from traditional VRA assessment approaches where only demographic targets are considered. Researchers at MGGG build a strong case for the alternative approach of measuring minority effective districts based on performance of minority preferred candidates, and this metric is employed in the Texas case study where we examine the interplay of population shifts and minority representation considerations when analyzing the placement of the two new districts.

Stage 1. Data Wrangling

To determine the impact of creating two new districts, we compare ensembles of plans with 36 districts built using 2010 U.S Census population information to ensembles of plans with 38 districts and current population data. Throughout our analysis we leverage publicly available shapefiles and data files provided by MGGG, which include racial demographic information from the 2010 census, state and federal election data from 2012, 2014, 2016, and 2018, as well as geographical information. The data is available at the Voting Tabulation District (VTD) level, which is equivalent to election precincts in Texas. For our desired comparison, we must also augment the original shapefile with more recent population data from the 2019 American Community Survey hosted by Redistricting Hub. The challenges and considerations during this data merging process are outlined in the corresponding section of the Texas Jupyter Notebook (02_EDA_Modeling_Decisions.ipynb), and the data wrangling process is executed in the 01_TX_data_wrangling.py script.

Stage 2. Exploratory Data Analysis (EDA) and Modeling

After the updated shapefile is constructed, we consider the following modeling decisions, which are outlined in more detail in the EDA & Modeling Decisions notebook:

- *Metric Selection* - Our primary objective is to assess the impact of different considerations during the design process of adding two new districts to the Texas map. For example, how does the decision making process change if only population patterns are accounted for versus intentionally accounting for minority effective districts during the design process? GerryChain updaters can track metric values across each step in the MCMC chain, and so at each step, we tally population per district, number of county splits, and number of minority effective districts, among other metrics. Additionally, we keep track of the number of districts being allocated to each county in Texas in order to compare how the counts differ between redistricting cycles.
- *Seed Plan Selection* - because we will rely on a ReCom based proposal, we determine that the choice of seed plan is not crucial in our modeling decision since after the chains converge, the ensembles will be similar regardless of the choice of seed plan. We will employ the enacted 2013 congressional district plan for the previous redistricting cycle chains (36 districts), and we also need to create a constrained random seed plan using the 2019 population data and 38 districts for the new redistricting cycle chains.
- *Constraints* - Texas's legislative requirements for redistricting should be accounted for while building ensembles of plans to ensure that only plans that satisfy the rules are included in the sampled distribution. This requires close analysis of the legislative text for the state, and then translating the text to quantifiable constraints. Please refer to the EDA & Modeling Decision notebook for additional details.
- *Acceptance Functions and Proposal Selection* - To abide by Texas's redistricting rules, which require a minimal number of county splits, we compare the efficacy of both an acceptance function designed to minimize county splits and a "county aware" ReCom proposal. We determine that the county aware ReCom proposal method results in an ensemble with a lower number of county splits, and opt for this method in our final models.
- *Number of Steps (Chain Length)* - initial distributions after 1,000 steps show chains begin to converge for population related metrics. In concordance with best practices for

ReCom proposals, we run our chains for 10,000 steps to ensure full convergence of the distributions.

Stage 3a. Applying GerryChain: Running the Model

For our comparisons, we constructed the following four chains using the 03_TX_model.py script:

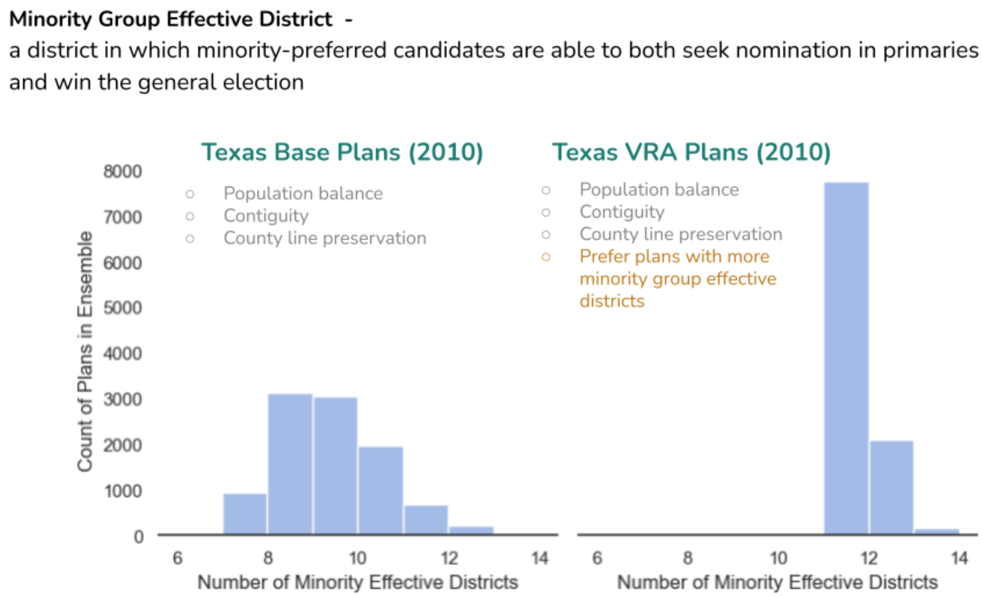
- Previous Redistricting Cycle: Base Plan - 2010 population data, 36 districts, 1% population balance constraint only
- Previous Redistricting Cycle: VRA Conscious Plan - 2010 population data, 36 districts, 1% population balance and inclusion constraint applied to preferentially select plans with more minority effective districts
- Upcoming Redistricting Cycle: Base Plan - 2019 population data, 38 districts, 1% population balance constraint only
- Upcoming Redistricting Cycle: VRA Conscious Plan - 2019 population data, 38 districts, 1% population balance and inclusion constraint applied to preferentially select plans with more minority effective districts

Note, all chains accounted for contiguity and county line preservation by utilizing a county aware ReCom proposal, and compactness was maintained by using VTDs as the building blocks for districts as opposed to smaller geographical units, such as census blocks.

Stage 3b: Applying GerryChain: Ensemble Analysis

Using GerryChain, we are able to examine how a VRA conscious ensemble compares to a base ensemble of plans that only considers population balance, contiguity, and county line preservation goals. This code for this analysis can be found in the Jupyter Notebook 04_TX_Ensemble_Analysis.ipynb. As you can see in the two graphs below, if we add an additional constraint when designing plans that preferentially selects plans with more minority effective districts, we would be able to design plans that have 2 to 3 more minority effective districts on average based on the 2010 population data.

Figure 9. Number of Minority Group Effective Districts

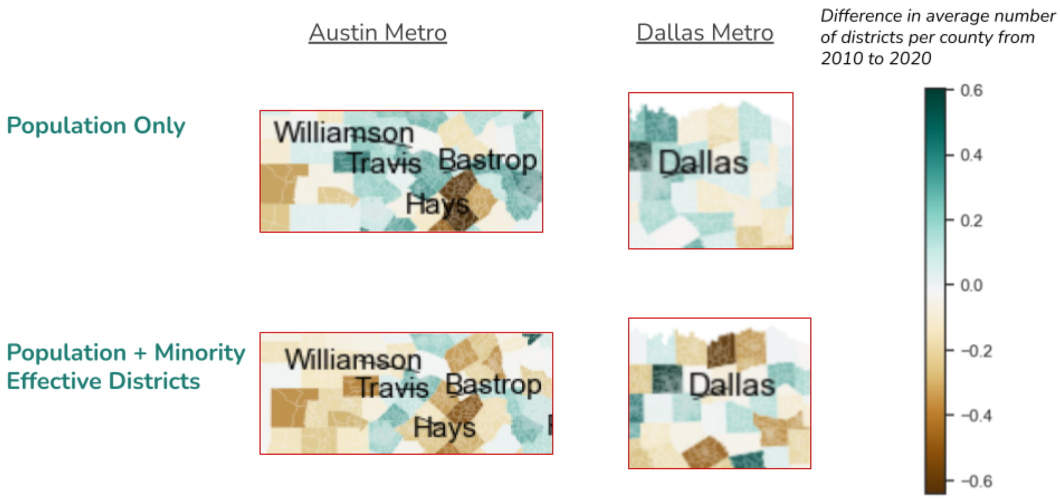


With this in mind, if we now think ahead to the upcoming cycle and consider options for where the two new districts might go, it's important for us to remember that the placement of these districts is crucial to fair representation.

We can visualize the difference in the average number of districts per county in our ensembles when we compare plans from the previous redistricting cycle to proposed plans for the upcoming cycle. The green regions on the maps are areas where we'd expect to see a higher number of districts in 2021 than 2013, and the brown areas are counties where we'd expect to see a decrease in the number of districts. To take a deeper dive, we focused on the Dallas metro area and the Austin metro area, which are visualized below. If we only consider the population differences over the last decade, then on the top row, we can see green regions corresponding to areas that our ensemble analysis indicates could be likely candidates for adding a new district. However, if we build our 2021 ensemble of plans by prioritizing adding more minority effective districts, then the landscape of options changes. For example, when accounting only for population, Hays, Travis, and Bastrop counties are potential candidates for a new district. But if we want to maximize minority opportunity districts, then on the bottom row, we can see that Hays county is a much better option than Bastrop county. Similarly, it would be better to place a new district in select Dallas suburbs shaded in green in the second row if we want to better comply with the Voting Rights Act.

As you can see, the GerryChain library allows for nuanced analysis on possible tradeoffs when considering where the new districts could be placed. A summary of our final results can also be found in our [Texas Case Study Report.]

Figure 10. Where Should the Two New Districts Go?



Conclusion

As readers of this guide are surely aware at this point, the answer to most questions about how to best apply GerryChain is “it depends.” While this can be a frustrating answer to users who are just getting started with the library and anxious to apply it to redistricting problems, it is the only honest one. Hopefully, after reading this document, you have surmised that political redistricting is a complex problem. And, to reiterate, computational redistricting is not a solved problem. This field is new and quickly evolving and some of the most pressing questions have yet to be satisfactorily answered. The translation of often arcane constitutional language into quantitative tests is as much art as it is science. Therefore, few standards have yet to be developed in this regard. Though we hope that this guide helps to make the basics of GerryChain more approachable for new users, there is no way to fast track the thoughtful, ethical, and effective application of this powerful tool. Again, we encourage users to consult our project website and Github repository for more information about our state level analyses and examples of the code used to produce them. For readers wanting to apply GerryChain to redistricting problems in their own state and needing additional support, please consult the extensive GerryChain documentation.