

Explicit Method

Start State at Step n: $\mathbf{x}_p^{n-\frac{1}{2}}, \mathbf{x}_p^n, \mathbf{x}_p^{n+\frac{1}{2}}, \mathbf{v}_p^n, \mathbf{E}^n, \mathbf{B}^n$

Interpolate Fields to Particle Positions

$$\mathbf{E}^n, \mathbf{B}^n \rightarrow \mathbf{E}_p^n, \mathbf{B}_p^n$$

Push Particles with Boris Algorithm

$$\begin{aligned} \mathbf{v}_p^n &\rightarrow \mathbf{v}_p^{n+1} \text{ then } \mathbf{x}_p^{n+\frac{1}{2}} \rightarrow \mathbf{x}_p^{n+\frac{3}{2}} \\ \mathbf{x}_p^{n+1} &= \mathbf{x}_p^{n+\frac{3}{2}} - \frac{\Delta t}{2} \mathbf{v}_p^{n+1} \end{aligned}$$

Deposit Current using $\nabla \cdot \mathbf{J} = \partial \rho / \partial t$

$$\mathbf{J}^{n+1} \text{ from } \mathbf{v}_p^{n+1} \text{ and } \mathbf{x}_p^{n+\frac{1}{2}}, \mathbf{x}_p^{n+1}, \mathbf{x}_p^{n+\frac{3}{2}}$$

Update Fields with Maxwell's Eqs.
FDTD: $\mathbf{E}^n, \mathbf{B}^n, \mathbf{J}^{n+1} \rightarrow \mathbf{E}^{n+1}, \mathbf{B}^{n+1}$

Update Time and State

$$t_{n+1}: \mathbf{x}_p^{n+\frac{1}{2}}, \mathbf{x}_p^{n+1}, \mathbf{x}_p^{n+\frac{3}{2}}, \mathbf{v}_p^{n+1}, \mathbf{E}^{n+1}, \mathbf{B}^{n+1}$$

Implicit Method

Start State at Step n: $\mathbf{x}_p^n, \mathbf{v}_p^n, \mathbf{E}^n, \mathbf{B}^n$

Guess $(k) = (0)$ for state at $t + \Delta t$:

$$\mathbf{x}_p^{(0)} = \mathbf{x}_p^n, \mathbf{v}_p^{(0)} = \mathbf{v}_p^n, \mathbf{E}_p^{(0)} = \mathbf{E}_p^n, \mathbf{B}_p^{(0)} = \mathbf{B}_p^n$$

Faraday's Law with Average Field $\overline{\mathbf{E}}$

$$\mathbf{B}^{(k)} = \mathbf{B}^n - dt \nabla \times \overline{\mathbf{E}}, \quad \overline{\mathbf{E}} = \frac{\mathbf{E}^n + \mathbf{E}^{(k)}}{2}$$

Sub-stepping loop pushing particles with average fields $\overline{\mathbf{E}}, \overline{\mathbf{B}}$. Deposit current $\overline{\mathbf{J}}$

Ampere's Law

$$\mathbf{E}^{(k+1)} = \mathbf{E}^n + c^2 dt \nabla \times \overline{\mathbf{B}} - \frac{dt}{\epsilon_0} [\overline{\mathbf{J}} - \langle \overline{\mathbf{J}} \rangle]$$

Check Convergence

$$\|\mathbf{E}^{(k+1)} - \mathbf{E}^{(k)}\| < \text{tolerance}$$

Update Time and State

$$t_{n+1}: \mathbf{x}_p^{n+1}, \mathbf{v}_p^{n+1}, \mathbf{E}^{n+1}, \mathbf{B}^{n+1}$$

No

set $\mathbf{E}^{(k)} = \mathbf{E}^{(k+1)}$

Yes

Next Step

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