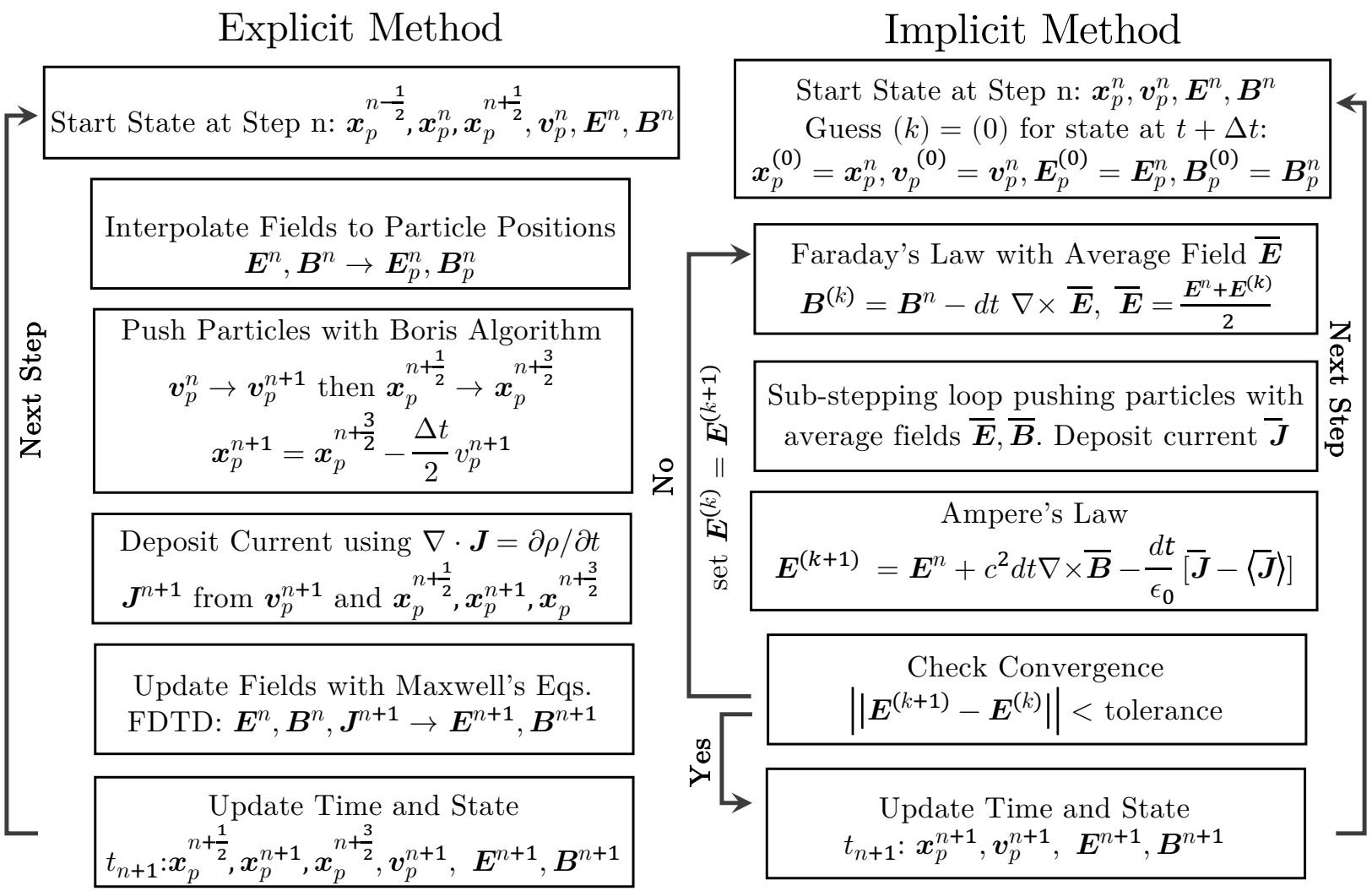
Explicit Method
Start State at Step n: $\boldsymbol{x}_p^{n-\frac{1}{2}}, \boldsymbol{x}_p^n, \boldsymbol{x}_p^{n+\frac{1}{2}}, \boldsymbol{v}_p^n, \boldsymbol{E}^n, \boldsymbol{B}^n$
Interpolate Fields to Particle Positions
$\boldsymbol{E}^n, \boldsymbol{B}^n \rightarrow \boldsymbol{E}_p^n, \boldsymbol{B}_p^n$
Push Particles with Boris Algorithm
$\boldsymbol{v}_p^n \rightarrow \boldsymbol{v}_p^{n+1}$ then $\boldsymbol{x}_p^{n+\frac{1}{2}} \rightarrow \boldsymbol{x}_p^{n+\frac{3}{2}}$
$\boldsymbol{x}_p^{n+1} = \boldsymbol{x}_p^{n+\frac{3}{2}} - \frac{\Delta t}{2} v_p^{n+1}$
Deposit Current using $\nabla \cdot \boldsymbol{J} = \partial\rho/\partial t$
$\boldsymbol{J}^{n+1}$ from $\boldsymbol{v}_p^{n+1}$ and $\boldsymbol{x}_p^{n+\frac{1}{2}}, \boldsymbol{x}_p^{n+1}, \boldsymbol{x}_p^{n+\frac{3}{2}}$
Update Fields with Maxwell's Eqs.
FDTD: $\boldsymbol{E}^n, \boldsymbol{B}^n, \boldsymbol{J}^{n+1} \rightarrow \boldsymbol{E}^{n+1}, \boldsymbol{B}^{n+1}$
Update Time and State
t_{n+1}:$\boldsymbol{x}_p^{n+\frac{1}{2}}, \boldsymbol{x}_p^{n+1}, \boldsymbol{x}_p^{n+\frac{3}{2}}, \boldsymbol{v}_p^{n+1}, \boldsymbol{E}^{n+1}, \boldsymbol{B}^{n+1}$
Next Step
Implicit Method
Start State at Step n: $\boldsymbol{x}_p^n, \boldsymbol{v}_p^n, \boldsymbol{E}^n, \boldsymbol{B}^n$
Guess $(k) = (0)$ for state at $t + \Delta t$:
$\boldsymbol{x}_p^{(0)} = \boldsymbol{x}_p^n, \boldsymbol{v}_p^{(0)} = \boldsymbol{v}_p^n, \boldsymbol{E}_p^{(0)} = \boldsymbol{E}_p^n, \boldsymbol{B}_p^{(0)} = \boldsymbol{B}_p^n$
Faraday's Law with Average Field $\overline{\boldsymbol{E}}$
$\boldsymbol{B}^{(k)} = \boldsymbol{B}^n - dt\ \nabla\times\ \overline{\boldsymbol{E}},\ \overline{\boldsymbol{E}} = \frac{\boldsymbol{E}^n + \boldsymbol{E}^{(k)}}{2}$
Sub-stepping loop pushing particles with average fields $\overline{\boldsymbol{E}}, \overline{\boldsymbol{B}}$. Deposit current $\overline{\boldsymbol{J}}$
Ampere's Law
$\boldsymbol{E}^{(k+1)} = \boldsymbol{E}^n + c^2 dt \nabla\times\overline{\boldsymbol{B}} - \frac{dt}{\epsilon_0}[\overline{\boldsymbol{J}} - \langle\overline{\boldsymbol{J}}\rangle]$
Check Convergence
$\left|\left|\boldsymbol{E}^{(k+1)} - \boldsymbol{E}^{(k)}\right|\right| <$ tolerance
No
set $\boldsymbol{E}^{(k)} = \boldsymbol{E}^{(k+1)}$
Yes
Update Time and State
t_{n+1}: $\boldsymbol{x}_p^{n+1}, \boldsymbol{v}_p^{n+1}, \boldsymbol{E}^{n+1}, \boldsymbol{B}^{n+1}$
Next Step