

Explicit Method

Implicit Method

Next Step

Start State at Step n: $\mathbf{x}_p^{n-\frac{1}{2}}, \mathbf{x}_p^n, \mathbf{x}_p^{n+\frac{1}{2}}, \mathbf{v}_p^n, \mathbf{E}^n, \mathbf{B}^n$

Interpolate Fields to Particle Positions
 $\mathbf{E}^n, \mathbf{B}^n \rightarrow \mathbf{E}_p^n, \mathbf{B}_p^n$

Push Particles with Boris Algorithm

$$\mathbf{v}_p^n \rightarrow \mathbf{v}_p^{n+1} \text{ then } \mathbf{x}_p^{n+\frac{1}{2}} \rightarrow \mathbf{x}_p^{n+\frac{3}{2}}$$

$$\mathbf{x}_p^{n+1} = \mathbf{x}_p^{n+\frac{3}{2}} - \frac{\Delta t}{2} \mathbf{v}_p^{n+1}$$

Deposit Current using $\nabla \cdot \mathbf{J} = \partial \rho / \partial t$
 \mathbf{J}^{n+1} from \mathbf{v}_p^{n+1} and $\mathbf{x}_p^{n+\frac{1}{2}}, \mathbf{x}_p^{n+1}, \mathbf{x}_p^{n+\frac{3}{2}}$

Update Fields with Maxwell's Eqs.
FDTD: $\mathbf{E}^n, \mathbf{B}^n, \mathbf{J}^{n+1} \rightarrow \mathbf{E}^{n+1}, \mathbf{B}^{n+1}$

Update Time and State
 $t_{n+1}: \mathbf{x}_p^{n+\frac{1}{2}}, \mathbf{x}_p^{n+1}, \mathbf{x}_p^{n+\frac{3}{2}}, \mathbf{v}_p^{n+1}, \mathbf{E}^{n+1}, \mathbf{B}^{n+1}$

Start State at Step n: $\mathbf{x}_p^n, \mathbf{v}_p^n, \mathbf{E}^n, \mathbf{B}^n$

Guess $(k) = (0)$ for state at $t + \Delta t$:

$$\mathbf{x}_p^{(0)} = \mathbf{x}_p^n, \mathbf{v}_p^{(0)} = \mathbf{v}_p^n, \mathbf{E}_p^{(0)} = \mathbf{E}_p^n, \mathbf{B}_p^{(0)} = \mathbf{B}_p^n$$

Faraday's Law with Average Field $\bar{\mathbf{E}}$
 $\mathbf{B}^{(k)} = \mathbf{B}^n - dt \nabla \times \bar{\mathbf{E}}, \bar{\mathbf{E}} = \frac{\mathbf{E}^n + \mathbf{E}^{(k)}}{2}$

Sub-stepping loop pushing particles with
average fields $\bar{\mathbf{E}}, \bar{\mathbf{B}}$. Deposit current $\bar{\mathbf{J}}$

No
set $\mathbf{E}^{(k)} = \mathbf{E}^{(k+1)}$

Ampere's Law
 $\mathbf{E}^{(k+1)} = \mathbf{E}^n + c^2 dt \nabla \times \bar{\mathbf{B}} - \frac{dt}{\epsilon_0} [\bar{\mathbf{J}} - \langle \mathbf{J} \rangle]$

Yes

Check Convergence
 $\|\mathbf{E}^{(k+1)} - \mathbf{E}^{(k)}\| < \text{tolerance}$

Update Time and State
 $t_{n+1}: \mathbf{x}_p^{n+1}, \mathbf{v}_p^{n+1}, \mathbf{E}^{n+1}, \mathbf{B}^{n+1}$

Next Step