APPENDIX A

Reliability Test Systems

A.1. IEEE RELIABILITY TEST SYSTEM (IEEE RTS)

The IEEE Reliability Test System (RTS) was developed by the Subcommittee on the Application of Probability Methods in the IEEE Power Engineering Society to provide a common test system which could be used for comparing the results obtained by different methods. The details of the RTS can be found in Reference 1. Since the IEEE RTS was created, there have been many new developments in power system reliability evaluation. Additional data or modifications are therefore required to conduct extended studies.[2–8] This appendix presents the basic data and the additional data for the IEEE RTS.

A.1.1. Load Model

The basic annual peak load for the test system is 2850 MW. Table A.1 gives data on weekly peak loads in percentage of the annual peak load. If week 1 is taken as January, Table A.1 describes a winter peaking system. If week 1 is taken as a summer month, a summer peaking system can be described. Table A.2 gives a daily peak load cycle, in percentage of the weekly peak. The same weekly peak load cycle is assumed to apply for all seasons. The data in Tables A.1 and A.2 together with the annual peak load define a daily peak load model of $52 \times 7 = 364$. days, with Monday as the first day of the year. Table A.3 gives weekday and weekend hourly load models for each of three seasons. Combination of Tables A.1, A.2, and A.3 with the annual peak load defines an hourly load model of $364 \times 24 = 8736$ hr.

Table A.1. Weekly Peak Load in Percentage of Annual Peak

Week	Peak	Week	Peak	Week	Peak	Week	Peak
1	86.2	14	75.0	27	75.5	40	72.4
2	90.0	15	72.1	28	81.6	41	74.3
3	87.8	16	80.0	29	80.1	42	74.4
4	83.4	17	75.4	30	88.0	43	80.0
5	88.0	18	83.7	31	72.2	44	88.1
6	84.1	19	87.0	32	77.6	45	88.5
7	83.2	20	88.0	33	80.0	46	90.9
8	80.6	21	85.6	34	72.9	47	94.0
9	74.0	22	81.1	35	72.6	48	89.0
10	73.7	23	90.0	36	70.5	49	94.2
11	71.5	24	88.7	37	78.0	50	97.0
12	72.7	25	89.6	38	69.5	51	100.0
13	70.4	26	86.1	39	72.4	52	95.2

Table A.2. Daily Peak Load in Percent of Weekly Peak

Day	Peak load
Monday	93
Tuesday	100
Wednesday	98
Thursday	96
Friday	94
Saturday	77
Sunday	75

A.1.2. Generating System

Table A.4 shows the generating unit ratings and reliability data. The 50-MW units are assumed to be hydro units and their capacity and energy limitations are given in Table A.5. Table A.6 gives operating cost data for the generating units. Power production data are given in terms of heat rates at selected output levels, since fuel costs are subject to considerable variation due to geographical location and other factors. The following fuel costs were suggested for general use (1979 base):

#6 oil	$2.30/MBtu
#2 oil	$3.00/MBtu
Coal	$1.20/MBtu
Nuclear	$0.60/MBtu

Table A.3. Hourly Peak Load in Percentage of Daily Peak

Hour	Winter week 1–8 & 44–52		Summer weeks 18–30		Spring/fall weeks 9–17 & 31–43	
	Wkdy[a]	Wknd[a]	Wkdy	Wknd	Wkdy	Wknd
12–1 am	67	78	64	74	63	75
1– 2	63	72	60	70	62	73
2– 3	60	68	58	66	60	69
3– 4	59	66	56	65	58	66
4– 5	59	64	56	64	59	65
5– 6	60	65	58	62	65	65
6– 7	74	66	64	62	72	68
7– 8	86	70	76	66	85	74
8– 9	95	80	87	81	95	83
9–10	96	88	95	86	99	89
10–11	96	90	99	91	100	92
11–Noon	95	91	100	93	99	94
Noon–1 pm	95	90	99	93	93	91
1– 2	95	88	100	92	92	90
2– 3	93	87	100	91	90	90
3– 4	94	87	97	91	88	86
4– 5	99	91	96	92	90	85
5– 6	100	100	96	94	92	88
6– 7	100	99	93	95	96	92
7– 8	96	97	92	95	98	100
8– 9	91	94	92	100	96	97
9–10	83	92	93	93	90	95
10–11	73	87	87	88	80	90
11–12	63	81	72	80	70	85

[a]Wkdy = weekday, Wknd = weekend.

Table A.4. Generating Unit Reliability Data

Unit size (MW)	Number of units	Forced outage rate	MTTF (hr)	MTTR (hr)	Scheduled maintenance (weeks/yr)
12	5	0.02	2940	60	2
20	4	0.10	450	50	2
50	6	0.01	1980	20	2
76	4	0.02	1960	40	3
100	3	0.04	1200	50	3
155	4	0.04	960	40	4
197	3	0.05	950	50	4
350	1	0.08	1150	100	5
400	2	0.12	1100	150	6

Table A.5. Hydro Capacity and Energy

Quarter	Capacity available[a] (%)	Energy distribution[b] (%)
1	100	35
2	100	35
3	90	10
4	90	20

[a]100% capacity = 50 MW,
[b]100% energy = 200 GWh.

A.1.3. Transmission System

The transmission network consists of 24 bus locations connected by 38 lines and transformers, as shown in Figure A.1. The transmission lines are at two voltages, 138 kV and 230 kV. The 230-kV system is the top part of Figure A.1, with 230/138 kV tie stations at Buses 11, 12, and 24. The locations of the generating units are shown in Table A.7. Table A.8 gives data on generating unit MVAr capacities. The system has voltage corrective devices at Bus 14 (synchronous condenser) and Bus 6 (reactor). Table A.9 gives the MVAr capacities of these devices. Bus load data at the time of system peak are shown in Table A.10. Transmission line forced outage data are given in Table A.11. Impedance and rating data for lines and transformers are given in Table A.12. The "B" values in the impedance data are the total, not the values in one leg of an equivalent circuit.

Outages on station components which are not switched as a part of a line are not included in the outage data in Table A.11. The following data are provided for bus sections:

	138 kV	230 kV
Faults per bus section/yr	0.027	0.021
Percentage of permanent faults	42	43
Outage duration for permanent faults (hr)	19	13

The following statistics are provided for circuit breakers:

Physical failures/breaker/yr	0.0066
Breaker operational failure per breaker/yr	0.0031
Outage duration (hr)	72

Table A.6. Generating Unit Operating Cost Data

Unit size (MW)	Type	Fuel	Output (%)	Heat rate (BTu/kWh)	O & M Cost Fixed ($/kW/yr)	O & M Cost Variable ($/MWh)
12	Fossil steam	#6 oil	20	15600	10.0	0.90
			50	12900		
			80	11900		
			100	12000		
20	Combus. turbine	#2 oil	80	15000	0.3	5.00
			100	14500		
50	Hydro					
76	Fossil steam	Coal	20	13000	10.5	0.90
			50	12900		
			80	11900		
			100	12000		
100	Fossil steam	#6 oil	25	13000	8.5	0.80
			55	10600		
			80	10100		
			100	10000		
155	Fossil steam	Coal	35	11200	7.0	0.80
			60	10100		
			80	9800		
			100	9700		
197	Fossil steam	#6 oil	35	10750	5.0	0.70
			60	9850		
			80	9840		
			100	9600		
350	Fossil steam	Coal	40	10200	4.5	0.70
			65	9600		
			80	9500		
			100	9500		
400	Nuclear steam	LWR	25	12550	5.0	0.30
			50	10825		
			80	10170		
			100	10000		

A physical failure is a mandatory unscheduled removal from service for repair or replacement. An operational failure is a failure to clear a fault within the breaker's normal protection zone.

There are several lines which are assumed to be on a common right-of-way or common tower for at least a part of their length. These line pairs are indicated in Figure A.1 by circles around the line pair, and an associated letter identification. Table A.13 gives the actual length of common right-of-way or common tower facility.

Table A.7. Generating Unit Locations

Bus	Unit 1 (MW)	Unit 2 (MW)	Unit 3 (MW)	Unit 4 (MW)	Unit 5 (MW)	Unit 6 (MW)
1	20	20	76	76		
2	20	20	76	76		
7	100	100	100			
13	197	197	197			
15	12	12	12	12	12	155
16	155					
18	400					
21	400					
22	50	50	50	50	50	50
23	155	155	350			

Table A.8. Generating Unit MVAr Capacities

Size (MW)	MVAr	
	Minimum	Maximum
12	0	6
20	0	10
50	−10	16
76	−25	30
100	0	60
155	−50	80
197	0	80
350	−25	150
400	−50	200

Table A.9. Voltage Correction Devices

Device	Bus	MVAr capacity
Synchronous Condenser	14	50 Reactive 200 Capacitive
Reactor	6	100 Reactive

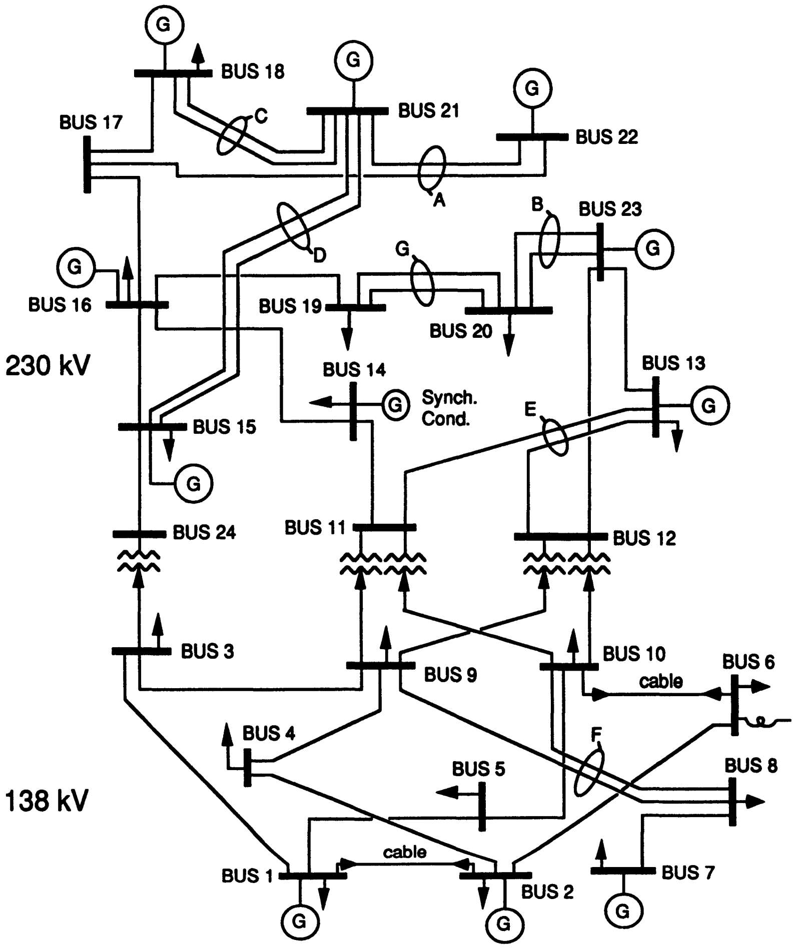

Figure A.1. IEEE Reliability Test System.

A.1.4. Additional Data

The extended studies using the IEEE RTS require additional data. The following additional data are given in the text.

1. Table 4.2 gives derated state data for the 400-MW and 350-MW generating units.
2. Table 4.4 gives data on additional 25-MW gas turbine units.
3. Data associated with nonexponential distribution repair times of generating units are shown in Section 4.2.4(d).
4. Data associated with bus load uncertainty and correlation are shown in Section 5.7.3.

Table A.10. Bus Load Data

Bus	Load MW	Load MVAr
1	108	22
2	97	20
3	180	37
4	74	15
5	71	14
6	136	28
7	125	25
8	171	35
9	175	36
10	195	40
13	265	54
14	194	39
15	317	64
16	100	20
18	333	68
19	181	37
20	128	26
Total	2850	580

5. Table 7.1 gives sector customer damage functions for seven customer categories. Table 7.12 gives customer sector allocations (in %) at the load buses. The bus peak load MW values in Table 7.12 are for the Modified IEEE RTS. These values are 125% of the bus peak loads in the original RTS. The percentage loads for the customer sectors remain unchanged.
6. Table 7.8 gives modified generating unit operating cost data. Table 7.22 gives investment-cost-related data.
7. The IEEE RTS has an oversized transmission network. Several modifications have been suggested for the purpose of transmission planning studies. The Modified IEEE RTS is described in Section 7.4.2.

A.2. ROY BILLINTON TEST SYSTEM (RBTS)

The RBTS is a six-bus composite system developed at the University of Saskatechewan for educational purpose. It is sufficiently small to permit the conduct of a large number of reliability studies with reasonable solution time but sufficiently detailed to reflect the actual complexities involved in practical reliability analysis and can be used to examine a newly developed technique or method. The details of the RBTS are given in Billinton *et al.*[9]

Table A.11. Transmission Line Length and Forced Outage Data

From bus	To bus	Length (miles)	Permanent Outage rate (occ/yr)	Permanent Outage duration (hr)	Transient outage rate (occ/yr)
1	2	3	0.24	16	0.0
1	3	55	0.51	10	2.9
1	5	22	0.33	10	1.2
2	4	33	0.39	10	1.7
2	6	50	0.48	10	2.6
3	9	31	0.38	10	1.6
3	24	0	0.02	768	0.0
4	9	27	0.36	10	1.4
5	10	23	0.34	10	1.2
6	10	16	0.33	35	0.0
7	8	16	0.30	10	0.8
8	9	43	0.44	10	2.3
8	10	43	0.44	10	2.3
9	11	0	0.02	768	0.0
9	12	0	0.02	768	0.0
10	11	0	0.02	768	0.0
10	12	0	0.02	768	0.0
11	13	33	0.40	11	0.8
11	14	29	0.39	11	0.7
12	13	33	0.40	11	0.8
12	23	67	0.52	11	1.6
13	23	60	0.49	11	1.5
14	16	27	0.38	11	0.7
15	16	12	0.33	11	0.3
15	21	34	0.41	11	0.8
15	21	34	0.41	11	0.8
15	24	36	0.41	11	0.9
16	17	18	0.35	11	0.4
16	19	16	0.34	11	0.4
17	18	10	0.32	11	0.2
17	22	73	0.54	11	1.8
18	21	18	0.35	11	0.4
18	21	18	0.35	11	0.4
19	20	27.5	0.38	11	0.7
19	20	27.5	0.38	11	0.7
20	23	15	0.34	11	0.4
20	23	15	0.34	11	0.4
21	22	47	0.45	11	1.2

Table A.12. Impedance and Rating Data

From bus	To bus	Impedance p.u. (100 MVA base) R	X	B	Rating (MVA) Normal	Short term	Long term	Equipment
1	2	0.0026	0.0139	0.4611	175	200	193	138-kV cable
1	3	0.0546	0.2112	0.0572	175	220	208	138-kV line
1	5	0.0218	0.0845	0.0229	175	220	208	138-kV line
2	4	0.0328	0.1267	0.0343	175	220	208	138-kV line
2	6	0.0497	0.1920	0.0520	175	220	208	138-kV line
3	9	0.0308	0.1190	0.0322	175	220	208	138-kV line
3	24	0.0023	0.0839		400	600	510	Transformer
4	9	0.0268	0.1037	0.0281	175	220	208	138-kV line
5	10	0.0228	0.0883	0.0239	175	220	208	138-kV line
6	10	0.0139	0.0605	2.4590	175	200	193	138-kV cable
7	8	0.0159	0.0614	0.0166	175	220	208	138-kV line
8	9	0.0427	0.1651	0.0447	175	220	208	138-kV line
8	10	0.0427	0.1651	0.0447	175	220	208	138-kV line
9	11	0.0023	0.0839		400	600	510	Transformer
9	12	0.0023	0.0839		400	600	510	Transformer
10	11	0.0023	0.0839		400	600	510	Transformer
10	12	0.0023	0.0839		400	600	510	Transformer
11	13	0.0061	0.0476	0.0999	500	625	600	230-kV line
11	14	0.0054	0.0418	0.0879	500	625	600	230-kV line
12	13	0.0061	0.0476	0.0999	500	525	600	230-kV line
12	23	0.0124	0.0966	0.2030	500	525	600	230-kV line
13	23	0.0111	0.0865	0.1818	500	525	600	230-kV line
14	26	0.0050	0.0389	0.0818	500	625	600	230-kV line
15	16	0.0022	0.0173	0.0364	500	625	600	230-kV line
15	21	0.0063	0.0490	0.1030	500	525	600	230-kV line
15	21	0.0063	0.0490	0.1030	500	525	600	230-kV line
15	24	0.0067	0.0519	0.1091	500	525	600	230-kV line
16	17	0.0033	0.0259	0.0545	500	525	600	230-kV line
16	19	0.0030	0.0231	0.0485	500	525	600	230-kV line
17	18	0.0018	0.0144	0.0303	500	525	600	230-kV line
17	22	0.0135	0.1053	0.2212	500	625	600	230-kV line
18	21	0.0033	0.0259	0.0545	500	625	600	230-kV line
18	21	0.0033	0.0259	0.0545	500	625	600	230-kV line
19	20	0.0051	0.0396	0.0833	500	525	600	230-kV line
19	20	0.0051	0.0396	0.0833	500	525	600	230-kV line
20	23	0.0028	0.0216	0.0455	500	525	600	230-kV line
20	23	0.0028	0.0216	0.0455	500	525	600	230-kV line
21	22	0.0087	0.0678	0.1424	500	525	600	230-kV line

Table A.13. Circuits on Common Right-of-Way or Common Structure

Right-of-way identification	From bus	To bus	Common ROW (miles)	Common structure (miles)
A	22	21	45.0	
	22	17	45.0	
B	23	20		15.0
	23	20		15.0
C	21	18		18.0
	21	18		18.0
D	15	21	34.0	
	15	21	34.0	
E	13	11		33.0
	13	12		33.0
F	8	10		43.0
	8	9		43.0
G	20	19	27.5	
	20	19	27.5	

A.2.1. Brief Description of the RBTS

The single line diagram of the test system is shown in Figure A.2. The system has two generator buses, four load buses, nine transmission lines, and eleven generating units. The system voltage level is 230 kV and the voltage limits for system buses are assumed to be between 1.05 p.u. and 0.97 p.u. The system peak load is 185 MW and the total installed generating capacity is 240 MW. The transmission network shown in Figure A.2 has been drawn to give a geographical representation. The line lengths are shown in proportion to their actual lengths. Customer sectors at the load buses are given in Figure A.2.

A.2.2. Load Model

The annual peak load for the system is 185 MW. The data on weekly peak loads in percentage of the annual peak load, daily peak load in percentage of the weekly peak, and hourly peak load in percentage of the daily peak are the same as those given in Tables A.1, A.2, and A.3.

A.2.3. Generating System

The generating unit ratings and reliability data for the RBTS are shown in Table A.14. The two 40-MW thermal units have been given an optional three-state representation. The derated model is shown in Figure A.3. It

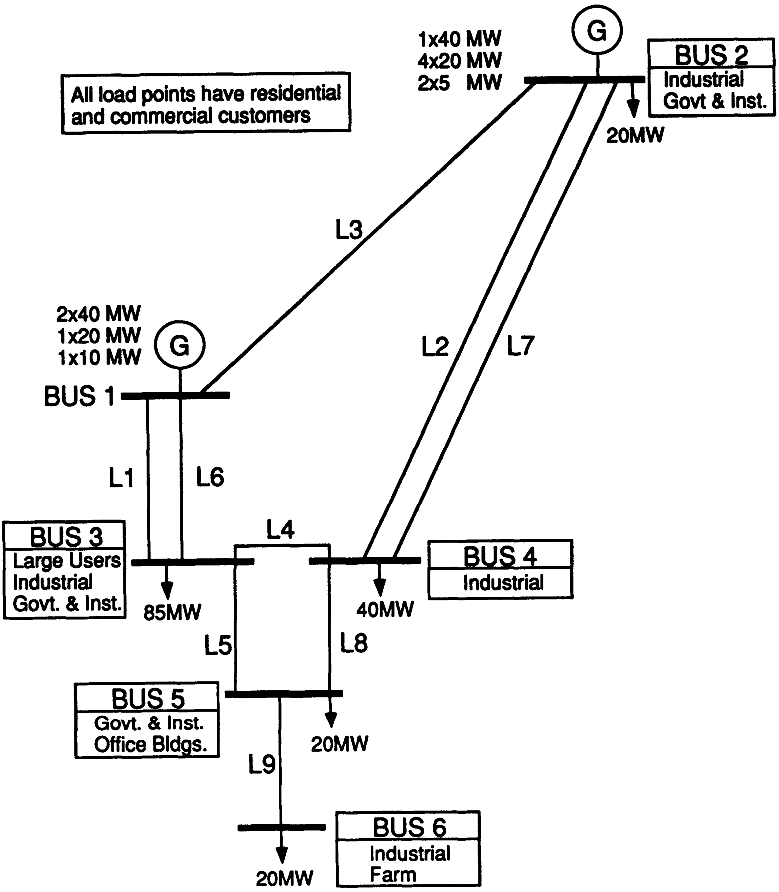

Figure A.2. Single line diagram of the RBTS.

has been assumed that there are no transitions between the derated state and the down state. The state probabilities and transition rates of the derated model are such that the derating-adjusted two-state model data are identical to those given in Table A.14. The two-state model is also shown in Figure A.3.

The generating unit cost data are shown in Table A.15. The operating costs include materials, supplies, manpower, etc. The fuel costs are those directly associated with energy production. The fuel cost for a hydro unit includes water rental charges. The fixed costs include the annual charges which continue as long as capital is tied up in the enterprise and whether or not the equipment is operating. The capital cost is the total cost to install

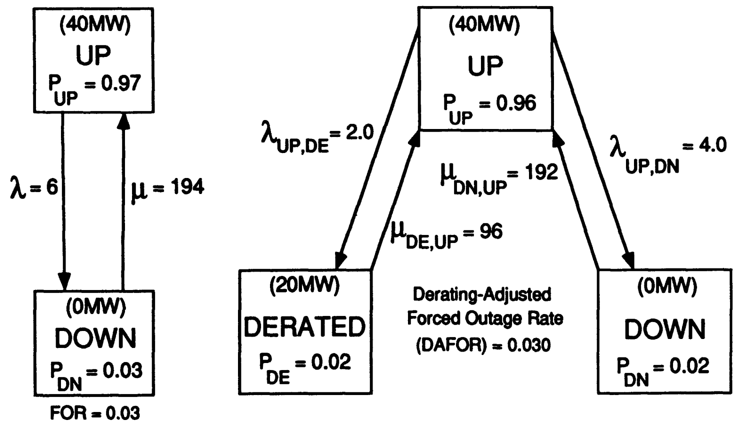

Figure A.3. Two- and three-state models for a 40-MW thermal generating unit.

Table A.14. Generating Unit Rating and Reliability Data

Unit size (MW)	Type	No. of units	Forced outage rate	Failure rate (1/yr)	Repair rate (1/yr)	Scheduled maintenance (weeks/yr)
5	Hydro	2	0.010	2.0	198.0	2
10	Thermal	1	0.020	4.0	196.0	2
20	Hydro	4	0.015	2.4	157.6	2
20	Thermal	1	0.025	5.0	195.0	2
40	Hydro	1	0.020	3.0	147.0	2
40	Thermal	2	0.030	6.0	194.0	2

Table A.15. Generating Unit Cost Data

Unit size (MW)	Type	No. of units	Loading order		Variable cost ($/MWh)			Fixed cost (k$/yr)	Capital cost (M$)
			1st	2nd	Fuel	Operat.	Total		
40	(Hydro)	1	1	1	0.45	0.05	0.50	100.0	160.0
20	(Hydro)	2	2–3	2–3	0.45	0.05	0.50	50.0	80.0
40	(Thermal)	2	8–9	4–5	9.50	2.50	12.00	790.0	80.0
20	(Thermal)	1	10	6	9.75	2.50	12.25	680.0	60.0
10	(Thermal)	1	11	7	10.00	2.50	12.50	600.0	40.0
20	(Hydro)	2	4–5	8–9	0.45	0.05	0.50	50.0	80.0
5	(Hydro)	2	6–7	10–11	0.45	0.05	0.50	12.5	40.0

Table A.16. Generation, Outage, and Cost Data for Additional Gas Turbines

Capacity (MW)	FOR	MTTF (hr)	MTTR (hr)	Fuel cost ($/MWh)	Operating cost ($/MWh)	Fixed cost (k$/yr)	Capital cost (M$)
10	0.12	550	75	52.0	4.5	40.0	5.0

a generating unit. Two loading orders are given in Table A.15. The first loading order is on a purely economic basis. The second loading order allocates some hydro units as peaking units which could reflect limited energy considerations. Either of the loading orders can be selected depending upon the operating philosophy in conducting reliability studies. Additional gas turbine units can be added to the RBTS. The generation, outage, and cost data for these units are given in Table A.16.

A.2.4. Transmission System

The transmission network consists of six buses and nine transmission lines. The locations of the generating units are shown in Table A.17. Table A.18 gives data on generating unit MVAr capacity. Bus load data at the time of system peak in MW and in percentage of the total system load are shown in Table A.19. At 0.98 power factor, the reactive load MVAr

Table A.17. Generating Unit Locations

Bus No.	Unit 1 (MW)	Unit 2 (MW)	Unit 3 (MW)	Unit 4 (MW)	Unit 5 (MW)	Unit 6 (MW)	Unit 7 (MW)
1	(Thermal plant)						
	40	40	10	20			
2	(Hydro plant)						
	5	5	40	20	20	20	20

Table A.18. Generating Unit MVAr Capacities

Unit size (MW)	MVAr	
	Minimum	Maximum
5	0	5
10	0	7
20	−7	12
40	−15	17

Table A.19. Bus Load Data

Bus	Load (MW)	Bus load in % of system load
2	20.0	10.81
3	85.0	45.95
4	40.0	21.62
5	20.0	10.81
6	20.0	10.81
Total	185.0	100.00

requirements at each bus is 20% of the corresponding MW load. The load forecast uncertainty can be assumed to follow a normal distribution having a standard deviation from 2.5% to 10%.

Table A.20 shows the transmission line lengths and outage data. The permanent outage rate of a given line is obtained using a value of 0.02 outages/yr/km. Line transient outage rates are calculated using a value of 0.05 outages/yr/km. The outage duration of a transient outage is assumed to be less than 1 min and is, therefore, not included in Table A.20. Outages of substation components which are not switched as a part of a line are not included in the outage data given in Table A.20. Two pairs of transmission lines are assumed to be on a common right-of-way or common tower for their entire length. The common mode data for these two line pairs are given in Table A.21. The line impedance and rating data are given in Tables A.22.

A.2.5. Station Data

The station configurations for the load and generator buses are given in the extended single line diagram shown in Figure A.4.

Table A.20. Transmission Line Length and Outage Data

Line	From	To	Length (kM)	Permanent Outage rate (occ/yr)	Permanent Duration (hr)	Transient outage rate (occ/yr)
1	1	3	75	1.5	10.0	3.75
2	2	4	250	5.0	10.0	12.50
3	1	2	200	4.0	10.0	10.00
4	3	4	50	1.0	10.0	2.50
5	3	5	50	1.0	10.0	2.50
6	1	3	75	1.5	10.0	3.75
7	2	4	250	5.0	10.0	12.50
8	4	5	50	1.0	10.0	2.50
9	5	6	50	1.0	10.0	2.50

Table A.21. Common Mode Data for the Circuits on a Common Right-of-Way or a Common Tower

Line pair	Buses From	Buses To	Line No.	Common length (kM)	Outage rate (occ/yr)	Outage duration (hr)
A	1	3	1 & 6	75	0.15	16.0
B	2	4	2 & 7	250	0.50	16.0

Table A.22. Line Impedance and Rating Data (100-MVA Base and 230-kV Base)

Line	Buses		Impedance (p.u.)			Current rating (p.u.)
	From	To	R	X	$B/2$	
1, 6	1	3	0.0342	0.180	0.0106	0.85
2, 7	2	4	0.1140	0.600	0.0352	0.71
3	1	2	0.0912	0.480	0.0282	0.71
4	3	4	0.0228	0.120	0.0071	0.71
5	3	5	0.0028	0.120	0.0071	0.71
8	4	5	0.0228	0.120	0.0071	0.71
9	5	6	0.0028	0.120	0.0071	0.71

The station equipment data are as follows:

Circuit Breaker

Active failure rate = 0.0066 failure/yr
Passive failure rate = 0.0005 failure/yr
Average outage duration = 72 hr
Maintenance outage rate = 0.2 outages/yr
Maintenance time = 108 hr
Switching time = 1 hr

Bus Section

Failure rate = 0.22 failures/yr
Outage duration = 10 hr

Station Transformer

Failure rate = 0.02 failures/yr
Outage duration = 768 hr
Maintenance outage rate = 0.2 outages/yr
Maintenance time = 72 hr
Switching time = 1 hr

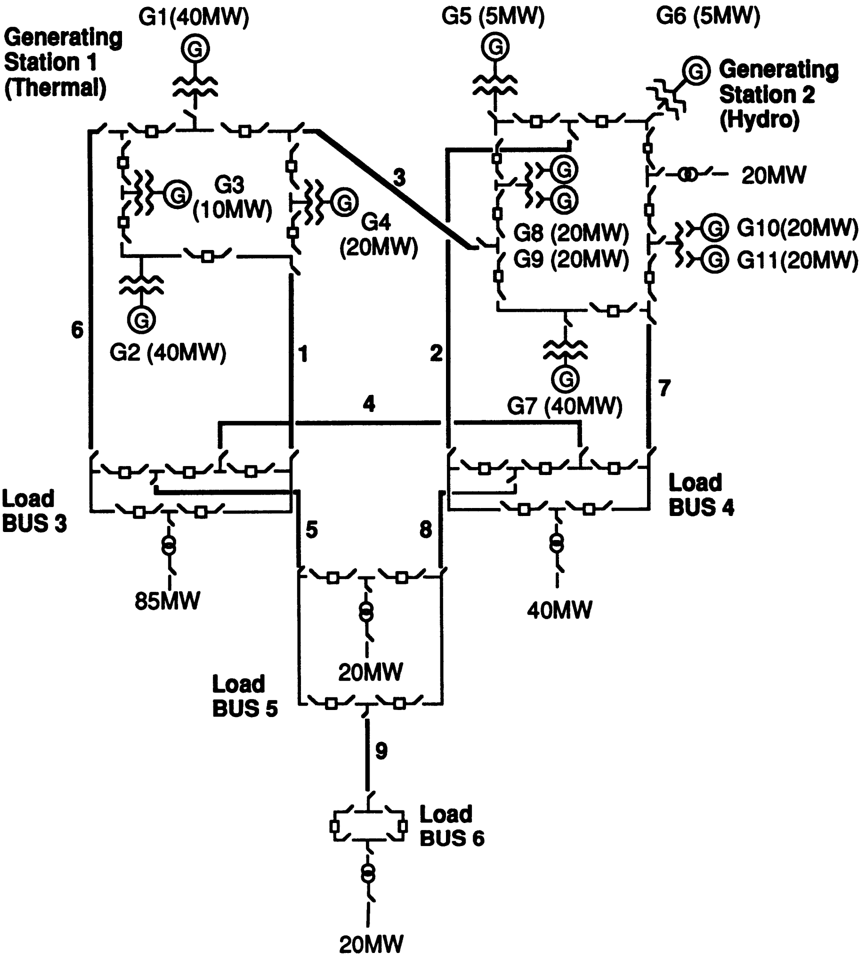

Figure A.4. Extended single line diagram of the RBTS.

A.2.6. Reliability Worth Assessment Data

The sector customer damage function data are the same as those given in Table 7.1. Customer sector allocations at buses are shown in Table A.23. The system customer mix by energy consumption and peak demand and system composite customer damage function for the customer mix are the same as those given in Tables 7.2 and 7.3, respectively. Bus composite customer damage functions can be calculated using the data given in Tables 7.1 and A.23.

Table A.23. Customer Sector Allocations at Load Buses

Bus No.	Peak load (MW)	Load percentage of customer sector (in %)						
		Agri.	Large user	Resid.	Gover.	Indus.	Commer.	Office
2	20	0.0	0.0	50.95	22.20	12.95	13.90	0.0
3	85	0.0	65.29	23.16	0.0	4.58	4.35	2.62
4	40	0.0	0.0	37.12	0.0	42.08	20.80	0.0
5	20	0.0	0.0	50.05	33.30	0.0	9.25	7.4
6	20	37.0	0.0	40.8	0.0	12.95	9.25	0.0

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APPENDIX B

Elements of Probability and Statistics

B.1. PROBABILITY CONCEPT AND CALCULATION RULES

B.1.1. Probability Concept

A random event can be considered as a phenomenon which may or may not occur in a given trial, time, or space. Assume that an experiment is repeated N times under the same conditions and that the number of occurrences of event A is M. The ratio M/N approaches a determined value when N becomes very large. This value is defined as the probability of event A occurring, i.e.,

$$P(\mathrm{A}) = \lim_{N \to \infty} \frac{M}{N} \tag{B.1}$$

Probability is a numerical measure of likelihood of a random event occurring and its value lies between 0.0 and 1.0.

B.1.2. Probability Calculation Rules

Some important and basic rules of probability theory are as follows:

$$P(\mathrm{A} \cup \mathrm{B}) = P(\mathrm{A}) + P(\mathrm{B}) - P(\mathrm{A} \cap \mathrm{B}) \tag{B.2}$$

$$P(\mathrm{A} \cap \mathrm{B}) = P(\mathrm{A})P(\mathrm{B}/\mathrm{A}) \tag{B.3}$$

where $P(A \cup B)$ is the probability of the occurrence of either A or B or both, $P(A \cap B)$ is the probability of the simultaneous occurrence of events A and B, and $P(B/A)$ is the conditional probability of B occurring given that A has occurred. If events A and B are mutually exclusive, $P(A \cap B) = 0$. This can be generalized to N events. If N events are mutually exclusive, the following probability equation applies:

$$P(A_1 \cup A_2 \cup \cdots \cup A_N) = P(A_1) + P(A_2) + \cdots + P(A_N) \qquad \text{(B.4)}$$

If events A and B are independent, $P(A \cap B) = P(A)P(B/A) = P(A)P(B)$. This can also be generalized to N events. If N events are independent of each other, the following probability equation holds:

$$P(A_1 \cap A_2 \cap \cdots \cap A_N) = P(A_1)P(A_2) \ldots P(A_N) \qquad \text{(B.5)}$$

If the events $\{B_1, B_2, \ldots, B_N\}$ represent a full and mutually exclusive set, i.e., $P(B_1) + P(B_2) + \cdots + P(B_N) = 1.0$ and $P(B_i \cap B_j) = 0.0$ $(i \neq j;\ i, j = 1, 2, \ldots, N)$, then for any event A,

$$P(A) = \sum_{i=1}^{N} P(B_i)P(A/B_i) \qquad \text{(B.6)}$$

B.2. PROBABILITY DISTRIBUTIONS OF RANDOM VARIABLES

B.2.1. Probability Distribution Function and Density Function

A random event or phenomenon can be represented by a random variable. Given a continuous random variable X, the probability of X being not larger than a real number x is a function of x. This function is defined as the cumulative distribution function $F(x)$ of random variable X, i.e.,

$$F(x) = P(X \leq x) \qquad (-\infty < x < \infty) \qquad \text{(B.7)}$$

The cumulative distribution function indicates the probabilities of all possible values of X. Function $F(x)$ can be expressed in the following form:

$$F(x) = \int_{-\infty}^{x} f(x)\, dx \qquad \text{(B.8)}$$

where $f(x)$ is the probability density function, and

$$f(x) = \frac{dF(x)}{dx} \qquad \text{(B.9)}$$

The probability of X lying between a and b can be calculated by

$$P(a \leq X \leq b) = \int_a^b f(x)\, dx \tag{B.10}$$

When $a = b$, the integration value given by equation (B.10) is zero. This indicates that the probability of a continuous random variable being equal to a single point value is zero.

Assume that the random variable Y is a function of a random variable X, i.e., $y = y(x)$. The following relationship exists between the probability density functions of Y and X:

$$f(y) = f(x) \left| \frac{dx}{dy} \right| \tag{B.11}$$

The concepts and the relationship shown in equation (B.11) can be generalized to the case of multiple random variables (random vector). Consider a two-dimensional random vector and assume that the random vector $\xi = (X, Y)$ has a density function $f(x, y)$ and the random vector $\eta = (Z_1, Z_2)$ is a function of ξ, i.e., $z_1 = z_1(x, y)$ and $z_2 = z_2(x, y)$. The inverse functions can be expressed as $x = x(z_1, z_2)$ and $y = y(z_1, z_2)$. The density function of the random vector η is given by

$$f(z_1, z_2) = f[x(z_1, z_2), y(z_1, z_2)]|J| \tag{B.12}$$

where J is the Jacobian determinant of the inverse functions:

$$J = \begin{vmatrix} \dfrac{\partial x}{\partial z_1} & \dfrac{\partial x}{\partial z_2} \\ \dfrac{\partial y}{\partial z_1} & \dfrac{\partial y}{\partial z_2} \end{vmatrix} \tag{B.13}$$

When a random variable is discrete, its probability density function can be expressed as

$$p_k = P(X = x_k) \qquad (k = 1, 2, \ldots) \tag{B.14}$$

and its cumulative probability distribution function as

$$F(x_k) = P(X \leq x_k) \tag{B.15}$$

The relationship between the density function and the cumulative distribution function of a discrete random variable is described by

$$F(x_k) = \sum_{i \leq k} p_i \tag{B.16}$$

and

$$p_k = F(x_k) - F(x_{k-1}) \tag{B.17}$$

B.2.2. Important Distributions in Reliability Evaluation

The following five distributions are used in the text:

1. Exponential Distribution

The density function:

$$f(t) = \lambda \exp(-\lambda t) \qquad (t \geq 0) \tag{B.18}$$

The cumulative distribution function:

$$F(t) = 1 - \exp(-\lambda t) \tag{B.19}$$

The mean and variance of the exponential distribution are $1/\lambda$ and $1/\lambda^2$, respectively.

2. Normal Distribution

The density function:

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(t-\mu)^2}{2\sigma^2}\right] \qquad (-\infty \leq t \leq \infty) \tag{B.20}$$

where μ and σ^2 are the mean and variance of the normal distribution.

3. Log-Normal Distribution

The density function:

$$f(t) = \frac{1}{t\sigma\sqrt{2\pi}} \exp\left[-\frac{(\ln t-\mu)^2}{2\sigma^2}\right] \qquad (t > 0) \tag{B.21}$$

It should be noted that μ and σ^2 in equation (B.21) are not the mean and variance of the log-normal distribution. The mean and variance of the log-normal distribution are given, respectively, by

$$E(t) = \exp\left(\mu + \frac{\sigma^2}{2}\right) \tag{B.22}$$

and

$$V(t) = \exp(2\mu + \sigma^2)[\exp(\sigma^2) - 1] \tag{B.23}$$

If the mean and variance of the log-normal distribution (i.e., E and V) are specified, the parameters μ and σ^2 in equation (B.21) are given by

$$\mu = \ln\left[\frac{E^2}{(V+E^2)^{1/2}}\right] \tag{B.24}$$

and

$$\sigma^2 = \ln\left[\frac{V+E^2}{E^2}\right] \tag{B.25}$$

4. Gamma Distribution

The density function:

$$f(t) = \frac{t^{\beta-1}}{\alpha^{\beta}\Gamma(\beta)}\exp\left[-\frac{t}{\alpha}\right] \qquad (t \geq 0, \beta > 0, \alpha > 0) \tag{B.26}$$

where α and β are the scale and shape parameters of the gamma distribution.

There are no explicit analytical expressions for the cumulative distribution functions for normal, log-normal, and gamma distributions.

5. Weibull Distribution

The density function:

$$f(t) = \frac{\beta t^{\beta-1}}{\alpha^{\beta}}\exp\left[-\left(\frac{t}{\alpha}\right)^{\beta}\right] \qquad (\infty > t \geq 0, \beta > 0, \alpha > 0) \tag{B.27}$$

The cumulative distribution function:

$$F(t) = 1 - \exp\left[-\left(\frac{t}{\alpha}\right)^{\beta}\right] \tag{B.28}$$

where α and β are the scale and shape parameters of the Weibull distribution.

B.3. NUMERICAL CHARACTERISTICS OF RANDOM VARIABLES

Random variables can be generally described by one or more parameters rather than as a specific distribution. These parameters are known as numerical characteristics. The most useful numerical characteristics are mathematical expectation (mean), variance, covariance, and correlation functions.

B.3.1. Expectation and Variance

If a random variable X has a probability density function $f(x)$ and the random variable Y is a function of X, i.e., $y = y(x)$, then

$$E(Y) = \int_{-\infty}^{\infty} y(x)f(x)\,dx \tag{B.29}$$

where $E(Y)$ is called the expected or mean value of the random variable Y. The expected value of the random variable X is

$$E(X) = \int_{-\infty}^{\infty} xf(x)\,dx \tag{B.30}$$

where $E(X)$ indicates the mean of all possible values of X.

The variance of X is the expected value of the function $[x - E(X)]^2$, i.e.,

$$E([X - E(X)]^2) = \int_{-\infty}^{\infty} [x - E(X)]^2 f(x)\,dx \tag{B.31}$$

The variance indicates the degree of dispersion of the possible values of X from its mean. The variance is often expressed by the notation $V(X)$ or $\sigma^2(X)$. The square root of the variance, $\sigma(X)$, is known as the standard deviation.

In the case of a discrete random variable, equations (B.29), (B.30), and (B.31) become equations (B.32), (B.33), and (B.34):

$$E(Y) = \sum_{i=1}^{n} y(x_i)p_i \tag{B.32}$$

$$E(X) = \sum_{i=1}^{n} x_i p_i \tag{B.33}$$

$$V(X) = \sum_{i=1}^{n} [x_i - E(X)]^2 p_i \tag{B.34}$$

B.3.2. Covariance and Correlation Function

Given an N-dimension random vector $\boldsymbol{\eta} = (X_1, X_2, \ldots, X_N)$, the covariance between any two elements X_i and X_j is defined as

$$\begin{aligned} c_{ij} &= E\{[X_i - E(X_i)][X_j - E(X_j)]\} \\ &= E(X_i X_j) - E(X_i)E(X_j) \end{aligned} \tag{B.35}$$

The covariance is often expressed by the notation $\mathrm{cov}(X_i, X_j)$. The covariance between an element and itself is its variance, i.e.,

$$\mathrm{cov}(X_i, X_i) = V(X_i) \tag{B.36}$$

The covariances of all elements of random vector $\boldsymbol{\eta}$ form the covariance matrix:

$$(c_{ij}) = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1N} \\ c_{21} & c_{22} & \cdots & c_{2N} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ c_{N1} & c_{N2} & \cdots & c_{NN} \end{bmatrix} \tag{B.37}$$

This is a symmetrical matrix whose diagonal components are variances of each element of $\boldsymbol{\eta}$.

The correlation function (coefficient) of X_i and X_j is defined as

$$\rho_{ij} = \frac{\mathrm{cov}(X_i, X_j)}{\sqrt{V(X_i)}\sqrt{V(X_j)}} \tag{B.38}$$

The correlation functions of all elements of random vector $\boldsymbol{\eta}$ form the correlation matrix:

$$(\rho_{ij}) = \begin{bmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1N} \\ \rho_{21} & \rho_{22} & \cdots & \rho_{2N} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \rho_{N1} & \rho_{N2} & \cdots & \rho_{NN} \end{bmatrix} \tag{B.39}$$

The absolute value of ρ_{ij} is smaller or equal to 1.0. When $\rho_{ij} = 0$, X_i and X_j are not correlated; when $\rho_{ij} > 0$, X_i and X_j are positively correlated; when $\rho_{ij} < 0$, X_i and X_j are negatively correlated.

B.4. LIMIT THEOREMS

The limit theorems are the theoretical bases of Monte Carlo simulation.

B.4.1. Law of Large Numbers

The following is one of several different representations of the law of large numbers:

If N independent random variables X_1, $X_2, \ldots, X_N$ follow the same distribution and $E(X_i) = \mu$, then for a sufficiently small positive number ε

$$\lim_{N\to\infty} P\left[\left|\frac{1}{N}\sum_{i=1}^{N} X_i - \mu\right| < \varepsilon\right] = 1.0 \tag{B.40}$$

The law of large numbers indicates that when N is very large, the arithmetic mean of a group of random variables approaches its expectation with a very large probability.

B.4.2. Central Limit Theorem

If N independent random variables $X_1, X_2, \ldots, X_N$ follow the same distribution and $E(X_i) = \mu$, $V(X_i) = \sigma_2$, then

$$\lim_{N\to\infty} P\left[\frac{|1/N\sum_{i=1}^{N} X_i - \mu|}{\sigma/\sqrt{N}} \le x\right] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-t^2/2}\, dt \tag{B.41}$$

This theorem indicates that when N is sufficiently large, the arithmetic mean approximately follows a normal distribution. If the effect of each variable X_i is "uniformly small," the central limit theorem still holds even if these random variables do not follow the same distribution.

B.5. PARAMETER ESTIMATION

Mathematically, the calculation of a reliability index using the Monte Carlo method is a parameter estimation problem.

B.5.1. Basic Definitions

1. Population (X): set of all possible observed outcomes.
2. Samples $(X_1, X_2, \ldots, X_N)$: a subset of the population.
3. Sample size (N): the number of samples in the subset.
4. Statistic: a function of the sample not containing any unknown parameter.
5. Sampling distribution: distribution of a statistic.

6. Estimate: Assume that a population has density function $f(x, \theta_1, \ldots, \theta_M)$ where $\theta_1, \ldots, \theta_M$ are unknown parameters, and that the statistic of the samples $\theta_i = \theta_i(X_1, \ldots, X_N)$ can be used to estimate the parameter θ_i of the population; θ_i is called the estimate of the parameter θ_i.

B.5.2. Sample Mean and Sample Variance

Quantities $X_1, X_2, \ldots, X_N$ are samples of a population X. The sample mean is defined as

$$\bar{X} = \frac{1}{N} \sum_{i=1}^{N} X_i \tag{B.42}$$

where $\bar{X}$ is an unbiased estimate of the population mean. The sample mean is a random variable and its variance is $1/N$ of the population variance, i.e.,

$$V(\bar{X}) = \frac{1}{N} V(X) \tag{B.43}$$

If the population follows a distribution with mean μ and variance σ^2, then the sample mean approximately follows a normal distribution with mean μ and variance σ^2/N when the sample size is sufficiently large.

The sample variance is defined as

$$s^2 = \frac{1}{N-1} \sum_{i=1}^{N} (X_i - \bar{X})^2 \tag{B.44}$$

where s^2 is an unbiased estimate of the population variance. The sample variance is also a random variable and its variance is given by

$$V(s^2) = \frac{1}{N}\left[v_4 - \frac{N-3}{N-1} V^2(X) \right] \tag{B.45}$$

where v_4 is the fourth-order central moment of the population X.

In Monte Carlo simulation, the sample mean and the sample variance are calculated repeatedly as the number of samples increases. The recursive equations for the sample mean and the sample variance are as follows:

$$\bar{X}_N = \frac{1}{N} [(N-1)\bar{X}_{N-1} + X_N] \tag{B.46}$$

and

$$s_N^2 = \frac{1}{N-1}\left[(N-2)s_{N-1}^2 + (N-1)\bar{X}_{N-1}^2 - N\bar{X}_N^2 + X_N^2\right] \qquad \text{(B.47)}$$

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APPENDIX C

Power System Analysis Techniques

C.1. AC LOAD FLOW MODELS

C.1.1. Load Flow Equations

The basic load flow equations in polar coordinate form are as follows:

$$P_i = V_i \sum_{j=1}^{n} V_j(G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) \qquad (i=1, \ldots, n) \qquad \text{(C.1)}$$

$$Q_i = V_i \sum_{j=1}^{n} V_j(G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) \qquad (i=1, \ldots, n) \qquad \text{(C.2)}$$

where P_i and Q_i are bus real and reactive power injections at Bus i; V_i and δ_i are the magnitude and angle of the voltage at Bus i; $\delta_{ij} = \delta_i - \delta_j$; G_{ij} and B_{ij} are the real and imaginary parts of the element of the bus admittance matrix; n is the number of system buses.

Each bus has four variables (V_i, δ_i, P_i, and Q_i) and therefore n buses have $4n$ variables in total. (C.1) and (C.2) consists of $2n$ equations. In order to solve the load flow equations, two of the four variables for each bus have to be prespecified. In general, P_i and Q_i of the load buses are known and they are called PQ buses; P_i and V_i of generator buses are specified and they are called PV buses; V_i and δ_i of one bus in the system must be specified to adjust the power balance of whole system, and this bus is called the swing bus.

C.1.2. Newton–Raphson Model

The load flow equations can be solved by the successive linearization method. This is the well-known Newton–Raphson model. Equations (C.1) and (C.2) are linearized to yield the following matrix equation:

$$\begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix} = \begin{pmatrix} H & N \\ J & L \end{pmatrix} \begin{pmatrix} \Delta\delta \\ \Delta V/V \end{pmatrix} \tag{C.3}$$

The Jacobian coefficient matrix is a $(n+m-1)$-dimensional square matrix where n and m are the numbers of all buses and load buses, respectively; $\Delta V/V$ implies that its elements are $\Delta V_i/V_i$. The elements of the Jacobian matrix are calculated by

$$\begin{aligned} H_{ij} &= \frac{\partial P_i}{\partial \delta_j} = V_i V_j (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) \\ H_{ii} &= \frac{\partial P_i}{\partial \delta_i} = -Q_i - B_{ii} V_i^2 \end{aligned} \tag{C.4}$$

$$\begin{aligned} N_{ij} &= \frac{\partial P_i}{\partial V_j} V_j = V_i V_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) \\ N_{ii} &= \frac{\partial P_i}{\partial V_i} V_i = P_i + G_{ii} V_i^2 \end{aligned} \tag{C.5}$$

$$\begin{aligned} J_{ij} &= \frac{\partial Q_i}{\partial \delta_j} = -N_{ij} \\ J_{ii} &= \frac{\partial Q_i}{\partial \delta_i} = P_i - G_{ii} V_i^2 \end{aligned} \tag{C.6}$$

$$\begin{aligned} L_{ij} &= \frac{\partial Q_i}{\partial V_j} V_j = H_{ij} \\ L_{ii} &= \frac{\partial Q_i}{\partial V_i} V_i = Q_i - B_{ii} V_i^2 \end{aligned} \tag{C.7}$$

C.1.3. Fast Decoupled Model

Branch reactances are normally much larger than branch resistances and angle differences between two buses are very small in power systems. This results in strong dominance of the diagonal matrix block, i.e., the values of matrix blocks N and J are much smaller than those of H and L. Equation (C.3) can be decoupled by assuming $N=0$ and $J=0$. Considering $|G_{ij}\sin\delta_{ij}| \ll |B_{ij}\cos\delta_{ij}|$ and $|Q_i| \ll |B_{ii}V_i^2|$, the decoupled equation can be further simplified to

$$\begin{aligned}[\Delta P/V]&=[B'][V\Delta\delta]\\ [\Delta Q/V]&=[B''][\Delta V]\end{aligned} \tag{C.8}$$

where $[\Delta P/V]$ and $[\Delta Q/V]$ are vectors whose elements are $\Delta P_i/V_i$ and $\Delta Q_i/V_i$, respectively; $[V\Delta\delta]$ is a vector whose elements are $V_i\Delta\delta_i$. The elements of the constant matrices $[B']$ and $[B'']$ are obtained using

$$\begin{aligned}B'_{ij}&=\frac{-1}{x_{ij}}\\ B'_{ii}&=-\sum_{j\in R_i}B'_{ij}\end{aligned} \tag{C.9}$$

$$\begin{aligned}B''_{ij}&=-\frac{x_{ij}}{r_{ij}^2+x_{ij}^2}\\ B''_{ii}&=-2b_{i0}-\sum_{j\in R_i}B''_{ij}\end{aligned} \tag{C.10}$$

where r_{ij} and x_{ij} are the branch resistances and reactances, respectively; b_{i0} is the branch susceptance between Bus i and the ground; R_i is the set of buses directly connected to Bus i.

C.2. DC LOAD FLOW MODELS

C.2.1. Basic Equations

DC load flow equations relate the real power to bus voltage angles. DC load flow based models are widely used in composite system adequacy evaluation because: (a) Many of the important reliability indices are associated with real power load curtailments and calculating these indices only requires real power related information. (b) Load flow calculations in practical application indicate that in many systems there are relatively small (3%–10%) differences between AC and DC load flows. These are small compared to possible errors due to uncertainties in basic reliability data such as component failure rates and outage times. (c) A large number of system states have to be evaluated to guarantee accuracy of probability indices. DC load flow based models including optimal power flow type can be rapidly calculated.

It should be clearly appreciated that if voltage and reactive power considerations are important requirements in a particular system study, then DC load flow is not an acceptable approach.

DC load flow is based on the following four assumptions:

1. Branch resistances are much smaller than branch reactances. Branch susceptances can be approximated by

$$b_{ij} \approx -\frac{1}{x_{ij}} \tag{C.11}$$

2. Voltage angle difference between two buses of a line is small and therefore

$$\begin{aligned} \sin \delta_{ij} &\approx \delta_i - \delta_j \\ \cos \delta_{ij} &\approx 1.0 \end{aligned} \tag{C.12}$$

3. Susceptances between the buses and the ground can be neglected, i.e.,

$$b_{i0} = b_{j0} = 0 \tag{C.13}$$

4. All bus voltage magnitudes are assumed to be 1.0 p.u.

Based on the above assumptions, the real line flow in a branch can be calculated by

$$P_{ij} = \frac{\delta_i - \delta_j}{x_{ij}} \tag{C.14}$$

and therefore bus real power injections are

$$P_i = \sum_{j \in R_i} P_{ij} = B'_{ii}\delta_i + \sum_{j \in R_i} B'_{ij}\delta_j \qquad (i = 1, \ldots, n) \tag{C.15}$$

where

$$B'_{ij} = -\frac{1}{x_{ij}} \qquad \text{and} \qquad B'_{ii} = -\sum_{j \in R_i} B'_{ij} \tag{C.16}$$

Equation (C.15) can be expressed in a matrix form:

$$P = [B'][\delta] \tag{C.17}$$

If Bus n is selected as the swing bus and we let $\delta_n = 0$, $[B']$ is a $(n-1)$ dimensional square matrix. It is exactly the same as $[B']$ in equation (C.8).

C.2.2. Relationship between Power Injections and Line Flows

Based on the DC load flow equation (C.17), a linear relationship between power injections and line flows can be obtained. Combining equations (C.14) and (C.17) yields

$$[T_P] = [A][P] \tag{C.18}$$

where T_P is the line flow vector and its elements are line flows $\{P_{ij}\}$. Matrix $[A]$ is the relationship matrix between power injections and line flows and its dimension is $L \times (n-1)$, where L is the number of lines and n the number of buses; $[A]$ can be calculated directly from $[B']$. Assume that two buses of line k are i and j. For $k = 1, \ldots, L$, the kth row of matrix $[A]$ is the solution of the following linear equations:

$$[B'][X] = [b] \tag{C.19}$$

where

$$b = \left[0, \ldots, 0, \underset{\substack{\uparrow \\ i\text{th element}}}{\frac{1}{x_{ij}}}, 0, \ldots, 0, -\underset{\substack{\uparrow \\ j\text{th element}}}{\frac{1}{x_{ij}}}, 0, \ldots, 0 \right]^T \tag{C.20}$$

C.3. OPTIMAL POWER FLOW

Optimal power flow (OPF) can be defined as finding a solution of the power system operating state including control variables and state variables to optimize a given objective and satisfy load flow equations (equality constraints) and all given feasibility and security requirements (inequality constraints).

There are a number of possible types or forms of OPF depending on different objectives and constraints. Generally, OPF is a nonlinear optimization problem. Many linearized OPF models, however, are used for different purposes, particularly in composite system reliability evaluation. The minimization models given in Chapters 5 and 7 are essentially linearized OPF. This section provides a general representation of OPF.

OPF is an optimization problem and can be mathematically described as follows:

$$\min F = f(P, Q, V, \delta) \tag{C.21}$$

subject to

$$P_i(V, \delta) = \mathrm{PD}_i \qquad (i \in \mathrm{ND}) \tag{C.22}$$

$$Q_i(V, \delta) = \mathrm{QD}_i \qquad (i \in \mathrm{ND}) \tag{C.23}$$

$$\mathrm{PG}_i^{\min} \le P_i(V, \delta) \le \mathrm{PG}_i^{\max} \qquad (i \in \mathrm{NG}) \tag{C.24}$$

$$\mathrm{QG}_i^{\min} \le Q_i(V, \delta) \le \mathrm{QG}_i^{\max} \qquad (i \in \mathrm{NG}) \tag{C.25}$$

$$|T_k(V, \delta)| \le T_k^{\max} \qquad (k \in L) \tag{C.26}$$

$$V_i^{\min} \le V_i \le V_i^{\max} \qquad (i \in N) \tag{C.27}$$

where P and Q are bus real and reactive power injection vectors, respectively; P_i and Q_i are their elements; V and δ are bus voltage magnitude and angle vectors; V_i is an element of V; PD_i and QD_i are real and reactive loads at Bus i; $\mathrm{PG}_i^{\min}$, $\mathrm{PG}_i^{\max}$, $\mathrm{QG}_i^{\min}$, and $\mathrm{QG}_i^{\max}$ are lower and upper limits of real and reactive generations at Bus i, respectively; T_k is the line power flow on line k and $T_k^{\max}$ is its capacity limit; $V_i^{\min}$ and $V_i^{\max}$ are lower and upper limits of bus voltage magnitude at Bus i; ND, NG, N, and L are sets of load buses, generator buses, all buses, and all lines in the system, respectively. The P, Q, V, and δ in the objective function F may be different subsets of these variables, depending on the selection of control and state variables.

In composite system reliability evaluation, it is necessary to introduce bus load curtailment variables into the OPF model. This has been done in the linearized OPF models given in Chapters 5 and 7. It is possible to consider more constraints in an OPF model which can reflect system security, such as operating limits on voltage and transient stability.

C.4. CONTINGENCY ANALYSIS

The purpose of contingency analysis in composite system adequacy evaluation is to calculate line flows following one or more line outages and judge if there are overloads in other lines. The most accurate method is to rebuild the bus admittance matrix and resolve the load flow equations for each line outage state. This is not practical when tens of thousands or even hundreds of thousands of outage states need to be evaluated in a composite system adequacy assessment. Outage state line flows can be approximately obtained from information from the normal state using contingency analysis techniques without resolving the load flow equations. A linearized load-flow-based contingency analysis method is described in Section 5.2.2. This section presents a supplementary AC load-flow-based sensitivity method for contingency analysis.

Figures C.1a and C.1b show the the preoutage and the postoutage states for the line i–j outage. Figure C.1c shows the case in which additional power $\Delta P_i+j\Delta Q_i$ and $\Delta P_j+j\Delta Q_j$ are injected at Buses i and j, respectively, in the preoutage state. If the additional power injections can produce power flow increments so that power flows on the rest of the system are the same as those in the postoutage state, the effect of the additional power injections is equivalent to the outage of line i–j. In the postoutage state,

$$\begin{aligned} P_i+jQ_i &= P'_{i\alpha}+jQ'_{i\alpha} \\ P_j+jQ_j &= P'_{j\beta}+jQ'_{j\beta} \end{aligned} \tag{C.28}$$

In the equivalent power injection case,

$$\begin{aligned} (P_i+\Delta P_i)+j(Q_i+\Delta Q_i) &= (P'_{i\alpha}+P_{ij}+\Delta P_{ij})+j(Q'_{i\alpha}+Q_{ij}+\Delta Q_{ij}) \\ (P_j+\Delta P_j)+j(Q_j+\Delta Q_j) &= (P'_{j\beta}+P_{ji}+\Delta P_{ji})+j(Q'_{j\beta}+Q_{ji}+\Delta Q_{ji}) \end{aligned} \tag{C.29}$$

where $P_{ij}+jQ_{ij}$ and $P_{ji}+jQ_{ji}$ are the power flows on line i–j in the preoutage state and $\Delta P_{ij}+j\Delta Q_{ij}$ and $\Delta P_{ji}+j\Delta Q_{ji}$ are power increments on line i–j due to additional power injections to the preoutage state.

Subtracting equation (C.28) from equation (C.29) yields

$$\begin{aligned} \Delta P_i+j\Delta Q_i &= (P_{ij}+\Delta P_{ij})+j(Q_{ij}+\Delta Q_{ij}) \\ \Delta P_j+j\Delta Q_j &= (P_{ji}+\Delta P_{ji})+j(Q_{ji}+\Delta Q_{ji}) \end{aligned} \tag{C.30}$$

Equation (C.30) can be rewritten in matrix form:

$$\begin{bmatrix} P_{ij} \\ Q_{ij} \\ P_{ji} \\ Q_{ji} \end{bmatrix} = \begin{bmatrix} \Delta P_i \\ \Delta Q_i \\ \Delta P_j \\ \Delta Q_j \end{bmatrix} - \begin{bmatrix} \Delta P_{ij} \\ \Delta Q_{ij} \\ \Delta P_{ji} \\ \Delta Q_{ji} \end{bmatrix} \tag{C.31}$$

By introducing the sensitivity relationship between the power injection increments and power flow increments on line i–j, equation (C.32) can be obtained from equation (C.31):

$$\begin{bmatrix} P_{ij} \\ Q_{ij} \\ P_{ji} \\ Q_{ji} \end{bmatrix} = \left[\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} \dfrac{\partial P_{ij}}{\partial P_i} & \dfrac{\partial P_{ij}}{\partial Q_i} & \dfrac{\partial P_{ij}}{\partial P_j} & \dfrac{\partial P_{ij}}{\partial Q_j} \\ \dfrac{\partial Q_{ij}}{\partial P_i} & \dfrac{\partial Q_{ij}}{\partial Q_i} & \dfrac{\partial Q_{ij}}{\partial P_j} & \dfrac{\partial Q_{ij}}{\partial Q_j} \\ \dfrac{\partial P_{ji}}{\partial P_i} & \dfrac{\partial P_{ji}}{\partial Q_i} & \dfrac{\partial P_{ji}}{\partial P_j} & \dfrac{\partial P_{ji}}{\partial Q_j} \\ \dfrac{\partial Q_{ji}}{\partial P_i} & \dfrac{\partial Q_{ji}}{\partial Q_i} & \dfrac{\partial Q_{ji}}{\partial P_j} & \dfrac{\partial Q_{ji}}{\partial Q_j} \end{bmatrix} \right] \begin{bmatrix} \Delta P_i \\ \Delta Q_i \\ \Delta P_j \\ \Delta Q_j \end{bmatrix} \tag{C.32}$$

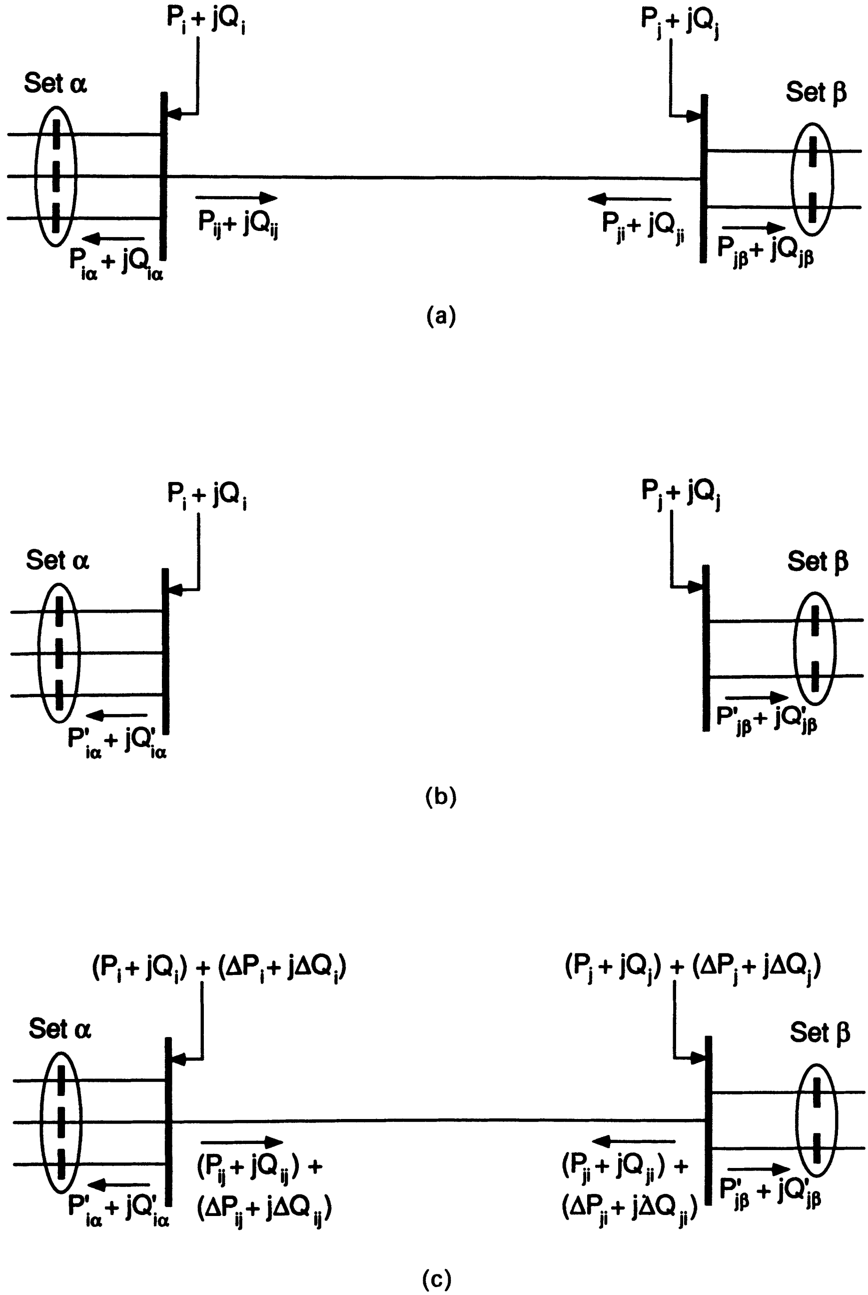

Figure C.1. Equivalent power injections for a line outage.

The sensitivity matrix S can be calculated by decomposition into two factors:

$$S=\begin{bmatrix} \frac{\partial P_{ij}}{\partial \delta_i} & \frac{\partial P_{ij}}{\partial \delta_j} & \frac{\partial P_{ij}}{\partial V_i} & \frac{\partial P_{ij}}{\partial V_j} \\ \frac{\partial Q_{ij}}{\partial \delta_i} & \frac{\partial Q_{ij}}{\partial \delta_j} & \frac{\partial Q_{ij}}{\partial V_i} & \frac{\partial Q_{ij}}{\partial V_j} \\ \frac{\partial P_{ji}}{\partial \delta_i} & \frac{\partial P_{ji}}{\partial \delta_j} & \frac{\partial P_{ji}}{\partial V_i} & \frac{\partial P_{ji}}{\partial V_j} \\ \frac{\partial Q_{ji}}{\partial \delta_i} & \frac{\partial Q_{ji}}{\partial \delta_j} & \frac{\partial Q_{ji}}{\partial V_i} & \frac{\partial Q_{ji}}{\partial V_j} \end{bmatrix} \begin{bmatrix} \frac{\partial \delta_i}{\partial P_i} & \frac{\partial \delta_i}{\partial Q_i} & \frac{\partial \delta_i}{\partial P_j} & \frac{\partial \delta_i}{\partial Q_j} \\ \frac{\partial \delta_j}{\partial P_i} & \frac{\partial \delta_j}{\partial Q_i} & \frac{\partial \delta_j}{\partial P_j} & \frac{\partial \delta_j}{\partial Q_j} \\ \frac{\partial V_i}{\partial P_i} & \frac{\partial V_i}{\partial Q_i} & \frac{\partial V_i}{\partial P_j} & \frac{\partial V_i}{\partial Q_j} \\ \frac{\partial V_j}{\partial P_i} & \frac{\partial V_j}{\partial Q_i} & \frac{\partial V_j}{\partial P_j} & \frac{\partial V_j}{\partial Q_j} \end{bmatrix} \tag{C.33}$$

The first matrx is obtained from the explicit expression of the line power as a function of the two terminal bus voltages. The elements of the second matrix are included in the inverse matrix of the Newton–Raphson load flow Jacobian matrix and can be obtained by solving the linear equations. For example, the elements of the first row in the second matrix are included in the solution of the following linear equation:

$$[J][X]=[b] \tag{C.34}$$

where

$[J]$ = Jacobian matrix prior to the outage

$$[b]=[0,\ldots,0,\underset{\substack{\uparrow \\ \text{(corresponding to } \delta_i)}}{1},0,\ldots,0]^{\mathrm{T}}$$

Having obtained the equivalent power injection increments from solving equation (C.32), the increments of bus voltage magnitudes and angles due to the line outage can be obtained by resolution of the equation

$$J\begin{bmatrix} \Delta\delta \\ \Delta V/V \end{bmatrix}=[\Delta I] \tag{C.35}$$

where

$$[\Delta I]=[0,\ldots 0,\Delta P_i,0,\ldots 0,\Delta P_j,0,\ldots 0,\Delta Q_i,0,\ldots 0,\Delta Q_j,0,\ldots 0]^T$$

The line power flows following the line outage can be calculated using bus voltages.

The concept and the procedure given above can also be applied to multiple line outages. The procedure is similar for the fast decoupled load flow model. The equivalent real and reactive power injections can be calculated separately in this case.

C.5. REFERENCES

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APPENDIX D

Optimization Techniques

D.1. LINEAR PROGRAMMING

The following is a brief summary of the optimization techniques used in this book.

D.1.1. Basic Concepts

The basic linear programming problem is to minimize (or maximize) a linear function while satisfying a set of linear equality and inequality constraints and has the following standard form:

$$\min cx \tag{D.1}$$

subject to

$$Ax=b \tag{D.2}$$

$$x \geq 0 \tag{D.3}$$

where c is an n-dimensional row vector, x an n-dimensional column vector, b an m-dimensional column vector, $b \geq 0$, and A is an $m \times n$ dimensional matrix.

A linear programming problem in nonstandard form can be converted into the standard form by equivalent transformation. In the following two LP problems, for example, the left one can be converted into the right one in the standard form by adding relaxation variable vector y:

$$\begin{array}{ll} & \min cx \\ \text{subject to} & \\ & Ax \leq 0 \\ & x \geq 0 \end{array} \qquad\qquad \begin{array}{ll} & \min cx \\ \text{subject to} & \\ & Ax+y=0 \\ & x \geq 0 \\ & y \geq 0 \end{array}$$

Basic definitions associated with linear programming are as follows:

1. Feasible solution: A solution x satisfying (D.2) and (D.3). The set of all feasible solutions is called a feasible zone.
2. Optimal and feasible solution: A feasible solution satisfying (D.1).
3. Base: If the rank of Matrix A is m, a nonsingular $m \times m$ dimensional matrix block B in Matrix A is known as a base. The columns of B are known as basic vectors. The elements of x corresponding to the basic vectors are basic variables and the remaining elements of x are nonbasic variables.
4. Basic solution: A solution of (D.2) with all nonbasic variables being zero.
5. Basic feasible solution: A basic solution satisfying (D.2) and (D.3). The base corresponding to the basic feasible solution is called the feasible base.
6. Optimal basic solution: A basic feasible solution satisfying (D.1). The base corresponding to the optimal basic solution is called the optimal base.

D.1.2. Generalized Simplex Method

When variable x has both lower and upper bounds, the constraints of x can be converted into a form $0 \leq x \leq h$. A more general LP problem therefore is

$$\min cx$$

subject to

$$Ax=b$$

$$0 \leq x \leq h$$

The approach to solving such a general LP problem is called the generalized simplex method. Its basic steps are as follows.

Step 1: Determine an initial basic feasible solution using the artificial variable technique and create an initial simplex tableau:

	x_1	·	$\cdots$	x_m	x_{m+1}	$\cdots$	x_n	b
	c_1	·	$\cdots$	c_m	c_{m+1}	$\cdots$	c_n	
x_1	1	0	$\cdots$	0	$y_{1,m+1}$	$\cdots$	$y_{1,n}$	y_{10}
x_2	0	1	$\cdots$	0				
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$		$\vdots$	$\vdots$
x_i	0	0	$\cdots 1 \cdots$	0	$y_{i,m+1}$	$\cdots$	$y_{i,n}$	y_{i0}
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$		$\vdots$	$\vdots$
x_m	0	0	$\cdots$	1	$y_{m,m+1}$	$\cdots$	$y_{m,n}$	y_{m0}
	0	0	$\cdots$	0	r_{m+1}	$\cdots$	r_n	$-z_0$
	e_1	e_2	$\cdots$	e_m	e_{m+1}	$\cdots$	e_n	

In the tableau, y_{ij} and y_{i0} are coefficients corresponding to Matrix A and Vector b, respectively, following each Gaussian elimination step; $r_j = c_j - z_j$ $(j = m+1, \ldots, n)$, where c_j are the coefficients in the objective function of the original problem which are known as direct cost coefficients; $z_j = \sum c_i y_{ij}$ are known as composite cost coefficients and r_j are known as relative cost coefficients; $z_0 = \sum c_i y_{i0}$ is the value of the objective function at the present step; $e_j = +$ or $-$ $(j = 1, \ldots, n)$ which is called the sign row. For the initial basic feasible solution

$$x_i = \begin{cases} y_{i0} & \text{(if } x_i \text{ is a basic variable)} \\ 0 & \text{(if } x_i \text{ is a nonbasic variable)} \end{cases}$$

the e_j take $+$ signs.

Step 2: Select $r_k = \min\{r_j < 0, j = m+1, \ldots, n\}$. The kth column is called the pivotal column. If there is no negative r_j, the present solution is already an optimal and feasible solution and the simplex process ends. The values of the variables are determined according to the signs of e_j. If $e_j = +$, $x_j = x_j$, and if $e_j = -$, $x_j = h_j - x_j$.

Step 3: Calculate the following three values for the selected pivotal column:

- h_k
- $\theta_1 = \min\{y_{i0}/y_{ik}\}$ for all $y_{ik} > 0$ (if there is no positive y_{ik}, $\theta_1 = \infty$)
- $\theta_2 = \min\{(y_{i0} - h_i)/y_{ik}\}$ for all $y_{ik} < 0$ (if there is no negative y_{ik}, $\theta_2 = \infty$)

Step 4: Modify the simplex tableau according to the magnitude of the three values in Step 3:

- If h_k is minimum, the last column is subtracted by a column which is obtained from the kth column multiplying h_k and then the kth column is multiplied by -1 (including the change of the sign for e_k). The base remains unchanged.
- If θ_1 is minimum and θ_1 appears in the qth row, then y_{qk} is selected as a pivot.
- If θ_2 is minimum and θ_2 appears in the qth row, then $y_{q0(\text{new})} = y_{q0(\text{old})} - h_q$, y_{qq} is multiplied by -1, and the sign of e_q is changed; y_{qk} is selected as a pivot.

Step 5: With the selected pivot element y_{qk}, conduct Gaussian elimination in the simplex tableau so that the pivot becomes 1; the other elements in the pivotal column become 0. An updated simplex tableau is obtained and go to Step 2.

D.1.3. Duality Principle

For each minimum-value linear programming problem, there is a corresponding dual maximum-value linear programming problem. The coefficients of the objective functions and the right-side terms of the constraints in the two LP problems exchange and they have an equal optimal objective function value. This is known as the duality of linear programming.

Two pairs of dual LP problems are given in the following. The first pair is called the symmetrical form of the dual LP and the second pair the nonsymmetrical form.

	Primary problem	Dual problem
Symmetrical form		
	$\min cx$	$\min \lambda b$
	subject to	subject to
	$Ax \geq b$	$\lambda A \leq c$
	$x \geq 0$	$\lambda \geq 0$
Nonsymmetrical form		
	$\min cx$	$\min \lambda b$
	subject to	subject to
	$Ax = b$	$\lambda A \leq c$
	$x \geq 0$	

where x and λ are variable vectors for the primary and dual problems, respectively.

The optimal basic solution of the dual problem can be obtained from the final simplex tableau of the primary problem. This solution is

$$\lambda = c_B B^{-1} \tag{D.4}$$

where B is the optimal base and c_B a subvector of c coefficient row in the final simplex tableau, which corresponds to the basic variables. At optimal solution, the nonbasic variables are zero and therefore the optimal value of the objective function is

$$F = c_B x_B = c_B B^{-1} b \tag{D.5}$$

where x_B is the basic variable vector. Differentiating equation (D.5) yields

$$\Delta F = c_B B^{-1} \Delta b \tag{D.6}$$

Combining equations (D.4) and (D.6) yields

$$\Delta F = \lambda \Delta b \tag{D.7}$$

This equation indicates that the optimal dual variable λ_i is a partial differential of the objective function with respect to the right-side value b_i of the constraints. This is the fundamental to calculate sensitivity indices in reliability assessment.

D.1.4. Dual Simplex Method

The simplex method given in Section D.1.2 is known as the primal simplex algorithm. Starting from an initial feasible solution, an optimal solution is gradually obtained while retaining feasibility in the primal algorithm. The dual simplex method starts with an initial basic solution satisfying optimality of the objective function but not satisfying feasibility. Feasibility is gradually obtained under the condition that optimality is retained. Which method is used depends upon the features of the problems to be solved. If an initial feasible solution can be easily obtained, the primal algorithm is used. If an initial optimal but nonfeasible solution can be obtained, the dual algorithm is used. The primal simplex method is used in the LP models for multi-area reliability evaluation in Chapter 4 and the increment-type LP model for voltage adjustment in Section 5.9.2, while all other LP models in Chapters 5 and 7 utilize the dual simplex method. It should be noted that the dual algorithm is used to perform dual treatment on the simplex tableau of the primal problem but not to solve a dual problem of the primal problem. The basic steps of the dual simplex method can be summarized as follows:

Step 1: Create the initial simplex tableau of the primal problem.

Step 2: Find a dual basic feasible solution X_B, i.e., in the simplex tableau corresponding to this solution, $r_j \geq 0$ for $j = m+1, \ldots, n$. If $x_B \geq 0$, i.e., there is no negative element in the column **b** of the simplex tableau, then an optimal and feasible solution is already obtained. If there are any negative elements in the column **b**, go to Step 3.

Step 3: Select the smallest value in the negative elements of x_B, i.e.,

$$\min_i \{(x_B)_i | (x_B)_i < 0\} = x_q$$

The x_q is the leaving base variable. This means that the qth row is the pivotal row.

Step 4: Check all elements of the pivotal row y_{qj} ($j = 1, \ldots, n$). If all $y_{qj} \geq 0$, there is no feasible solution. If there are negative elements in the pivotal row, then

$$\theta = \min_j \{(z_j - c_j)/y_{qj} | y_{qj} \leq 0\} = (z_k - c_k)/y_{qk}$$

where z_j, c_j, and y_{qj} are as defined in Section D.1.2; x_k is the entering base variable, which means that the kth column is the pivotal column.

Step 5: With the pivot element y_{qk}, conduct Gaussian elimination to update the simplex tableau. An updated base is obtained and then a new dual basic feasible solution is calculated: $x_B = B^{-1}b$. Go back to Step 2.

D.1.5. Linear Programming Relaxation Technique

In composite system adequacy assessment, the number of constraints in linear programming OPF models are very large, particularly when preventive outage constraints are considered. Active constraints at an optimal solution or in the process of resolution, however, are relatively few. If the active constraints are satisfied, all constraints will be satisfied. The basic characteristic of the linear programming relaxation technique is that a large-scale LP problem is decomposed into a sequence of small-scale linear programming problems and only a few active constraints are considered in each small LP problem.

Consider the linear programming problem:

$$\text{LP1}\begin{cases} & \min cx \\ \text{subject to} & \\ & A_1x=b \\ & \underline{y}\leq A_2x\leq \bar{y} \\ & \underline{x}\leq x\leq \bar{x} \end{cases}$$

where A_1 is a $J\times S$ dimensional matrix; A_2 is an $M\times S$ dimensional matrix; x is an S dimensional column vector; b is a J dimensional column vector; c is an S dimensional row vector.

Let

$$A_2x=y \qquad \text{and} \qquad h=[0, c]$$

$$z=\begin{bmatrix} y \\ x \end{bmatrix}, \qquad \underline{z}=\begin{bmatrix} \underline{y} \\ \underline{x} \end{bmatrix}, \qquad \bar{z}=\begin{bmatrix} \bar{y} \\ \bar{x} \end{bmatrix}$$

the linear programming problem LP1 can be rewritten as LP2:

$$\text{LP2}\begin{cases} & \min hz \\ \text{subject to} & \\ & Az=d \\ & \underline{z}\leq z\leq \bar{z} \end{cases}$$

where

$$A=\begin{bmatrix} -I & A_2 \\ 0 & A_1 \end{bmatrix} \qquad \text{and} \qquad d=\begin{bmatrix} 0 \\ b \end{bmatrix}$$

Let $K\subset M$, i.e., K is a subset of M. The following linear programming problem is constructed:

$$\text{LP2}(K)\begin{cases} & \min h_Hz_h \\ \text{subject to} & \\ & A_G^Hz_H=d_G \\ & \underline{z_H}\leq z_H\leq \overline{z_H} \end{cases}$$

where h_H denotes a subvector of h and the subscript set of its elements is H. The definitions of z_H and d_G are similar; A_G^H denotes a submatrix of A. The subscript set of its columns is H and the subscript set of its rows is G; $G=K\cup J$ and $H=K\cup S$.

The LP2(K) is a small-scale linear programming problem which contains the partial inequalities and all equalities in the original problem LP1.

Subset K can be quite small and even an empty set initially. Consequently, LP2(K) can be a very small problem despite the scale of the original problem. A solution of LP2(K) satisfies optimality of the original problem but does not necessarily satisfy its feasibility (not all inequality constraints are necessarily satisfied). The LP2(K) is solved using the dual simplex method.

Let

$$z_H(K) = \begin{bmatrix} y_K(K) \\ x(K) \end{bmatrix}$$

denote the optimal basic solution of LP2(K) and $I(K)$ is the subscript set of the basic variable components of the solution. Assume that

$$R = M - K, \qquad y(K) = \begin{bmatrix} y_K(K) \\ y_R(K) \end{bmatrix}, \qquad z(K) = \begin{bmatrix} y(K) \\ x(K) \end{bmatrix}$$

There are two cases:

(a) If

$$\underline{y} \leq y(K) \leq \bar{y}$$

is satisfied, then $z(K)$ is an optimal basic solution of the original problem LP1.

(b) If subset

$$F(K) = \{i \in R \mid y_i(K) > \bar{y}_i, \text{ or } y_i(K) < \underline{y}_i\}$$

is not empty, let

$$E(K) = K \cap I(K)$$

and

$$K^* = K - E(K) + F(K)$$

and K^* is used to replace K to construct a new small-scale linear programming LP2(K^*). This LP2(K^*) is solved using the dual simplex method. The initial basic solution of LP2(K^*) is z_{H^*} where $H^* = K^* \cup S$. The subscript set of the basic variable components of z_{H^*} is $I^* = I(K) - E(K) + F(K)$.

The above process is repeated (the old K^* is replaced by a new K^*) until Case (a) is achieved.

D.2. MAXIMUM FLOW METHOD

The maximum flow method is applied to multi-area generating system adequacy evaluation in Chapter 4. This section presents its basic concepts and procedure.

D.2.1. Basic Concepts

(a) Definitions

Network (oriented graph):	A set composed of node subset P and oriented branch subset B, denoted by $G(P,B)$
Arc:	An oriented branch between two nodes i and j, denoted by a_{ij}.
Arc parameters:	Arc length w_{ij}, arc flow f_{ij}, and arc capacity c_{ij}.
Source:	A node producing flow, denoted by s.
Sink:	A node collecting flow, denoted by t.
Path:	An oriented sequence of nodes and arcs. An arc having the same direction as the path is called a forward arc and an arc having the opposite direction to the path is called a backward arc.
Augmented path:	If all arc flows on a path satisfy the following conditions, this path is called an augmented path: $0 \leq f_{ij} < c_{ij}$ if $a_{ij} \in \mu^+$; $0 < f_{ij} \leq c_{ij}$ if $a_{ij} \in \mu^-$; where μ^+ and μ^- denote forward and backward arc sets, respectively.
Cut set:	A set of arcs connecting two mutually complementary node subsets in a network, denoted by $\{a_{ij} \mid i \in P_i, j \in P_j\}$. The sum of all arc capacities in a cut set is called the cut capacity. The one having minimum cut capacity in all possible cut sets is called the minimum cut set.

(b) Two Basic Theorems

Theorem 1: *A feasible flow $f = \{f_{ij}\}$ is the maximum flow if and only if there is no augmented path between the source and the sink.*

Theorem 2 (Ford–Fulkerson theorem): *The flow capacity of a maximum flow from the source to the sink equals the cut capacity of the minimum cut set separating the source and the sink.*

D.2.2. Maximum Flow Problem

The maximum flow problem is to find a feasible flow $f=\{f_{ij}\}$ which has maximum flow capacity. It is described mathematically as follows:

$$\max F(f_{ij})$$

subject to

$$0 \le f_{ij} \le c_{ij} \qquad (\text{for all } a_{ij})$$

$$\sum_j f_{sj} = F$$

$$\sum_j f_{jt} = F$$

$$\sum_j f_{ij} - \sum_j f_{ji} = 0 \qquad (i \ne s, t)$$

where s and t indicate the source and the sink, respectively; F is the flow capacity.

There are different methods for solving the maximum flow problem. The most commonly used method is the labeling algorithm. This algorithm starts from a feasible flow and includes the two processes of labeling and adjusting.

(a) Labeling. In this process, nodes are classified into the labeled and the unlabeled. The labeled nodes are further divided into the checked and the unchecked. Each label includes two numbers: The first number indicates from which node the label is obtained. The second number gives the possible maximum flow increment from the prior node to the present node.

Step 1: Label the source with $(0, +\infty)$ so that the source is a labeled but unchecked node and other nodes are unlabeled.

Step 2: Consider any labeled but unchecked node i. There are three cases for all nodes j which are connected to i but unlabeled:

- If $f_{ij}=c_{ij}$ for arc a_{ij}, node j is ignored.
- If $f_{ij}<c_{ij}$ for arc a_{ij}, node j is labeled with $(+i, \delta(j))$, where $\delta(j)=\min[\delta(i), c_{ij}-f_{ij}]$. $+i$ indicates that the flow can be increased from i to j and $\delta(j)$ is the possible magnitude to be increased.
- If $f_{ji}>0$ for arc a_{ji}, node j is labeled with $(-i, \delta(j))$, where $\delta(j)=\min[\delta(i), f_{ji}]$. $-i$ indicates that the flow can be decreased from j to i and $\delta(j)$ is the possible magnitude to be decreased.

After all possible j are labeled, i becomes a labeled and checked node and all j become the labeled but unchecked nodes.

Step 3: Repeat the above process until the sink is labeled and then go to the adjusting process. If all possible nodes are checked and the labeling process cannot proceed further to label the sink, the algorithm ends. The present flow is a maximum flow.

(b) Adjusting

Step 1: Find an augmented path μ from the source to the sink in terms of the first number in the labels. Assume that the first number in the label for the sink t is $+k$ (or $-k$). Arc a_{kt} (or a_{tk}) is an arc on the augmented path. Then the first number in the label for node k is checked. If it is $+i$ (or $-i$), a_{ik} (or a_{ki}) is an arc on the augmented path. The pursuit is conducted until the source is checked.

Step 2: Adjust the flow capacities of the arcs on the augmented path:

$$f'_{ij} = \begin{cases} f_{ij} & \text{if } a_{ij} \in \mu \\ f_{ij} + \delta(t) & \text{if } a_{ij} \in \mu^{+} \\ f_{ij} - \delta(t) & \text{if } a_{ij} \in \mu^{-} \end{cases}$$

where $\delta(t)$ is the second number in the label for the sink.

Step 3: Erase all labels and return to the labeling process with the new feasible flow $\{f'_{ij}\}$.

A minimum cut is also obtained using the presented labeling method. When labeling ends, the cut set between labeled and unlabeled node subsets is the minimum cut set. It locates the "bottle neck" from the source to the sink.

D.3. REFERENCES

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