Prepared by
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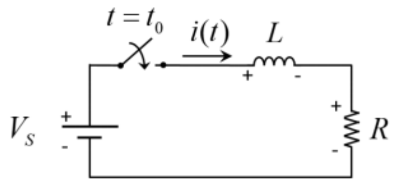


Figure 1: An RL circuit.

In this project, the 'i' current of the RL circuit given in Figure 1 will first be calculated analytically with the formula in equation 1 and find its real value, then numerically calculated using the **Euler, Modified Euler, Midpoint and Runge-Kutta methods** and the approximate value will be found. how close it will be observed. These methods used will also be compared among themselves and then compared again by changing the given time interval.

$$V_s = L\frac{di(t)}{dt} + Ri, \qquad i(t_0) = i_0 \qquad (1)$$

Where Vs is the voltage source, L is the inductance, R is the resistance and their values;

V_s=12 V

L=0.98 H

R=14.2 Ω given as.

If equation 1 is solved by the method of integral factor, first of all, the given values are written instead;

$$x = x(\infty) + (x_0 - x(\infty)) \times e^{-\frac{t}{z}}$$

Also;

$12 = 0.98 \times \frac{di(t)}{dt} + 14.2 \times i$ olur.

If µ, which is the integral factor of this equation, is found;

$\mu = e^{\int P(x)dt} = e^{\int 14.2dt} => e^{14.2t}$

If both sides of the equation are multiplied by the integral factor;

$e^{14.2t} \times 12 = e^{14.2t} \times (0.98 \times \frac{di(t)}{dt} + 14.2 \times i) => (e^{14.2t} \times i)' = e^{14.2t} \times 12$

Finally, if both sides are integrated, the real result is;

$(e^{14.2t} \times i) = \frac{(e^{14.2t} \times 12)}{14.2} => i = \frac{12}{14.2} = \mathbf{0.84500422}$ is found as.

The values to be calculated numerically are expected to approach this actual result.

Before starting the methods, the values to be used in the program are defined in figure 2.

```
i0=100*(10^-3);
L=0.98;
Vs=12;
R=14.2;
Z=L/R;
```

(Figure 2)

Then, if the methods are applied bu assuming the time interval Δt = 0.05

A) Euler's Method

In this section, the approximate value of the current will be calculated numerically by using the **Euler's method** whose formula is given in equation 2, which is the first method.

$\frac{dy}{dt} = f(t, y),\ a \leq t \leq b,\ y(a) = \alpha$. From here, the y value is found by the formula in equation 2.

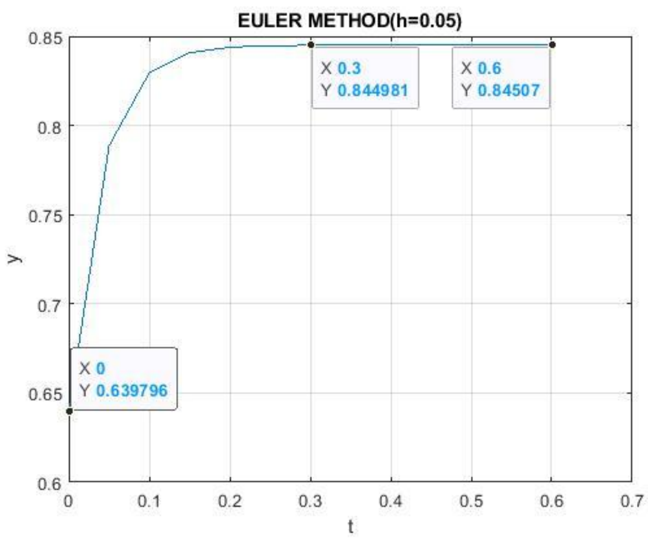
$$y = y_0 + f(t_0, y_0)(t - t_0) \qquad (2)$$

Since the value of h is accepted as 0.05 in this section, the program firstly enters a cycle that will last from 0 to 0.6, increasing by 0.05, as shown in Figure 3. Then, in the loop, the formula given in equation 2 is calculated and the i1e values found are thrown into the array. Then the value i0 given at the beginning is equal to the newly calculated variable i1e. The same steps are repeated until the cycle ends.

```
...EULER METHOD...
    i0=100*(10^-3);
    e=1;
for a=0:0.05:0.6
    i1e=i0+0.05*((Vs-R*i0)/L);
    b(e)=i1e;
    i0=i1e;
    t(e)=a;
    e=e+1;
end
```

(Figure 3)

According to these calculations, if the graph of Euler's method is plotted, graph 1 is obtained.



(Graph 1)

As seen in the graph, Euler's method found the current to be approximately **0.84507**. Relative error according to the analytical result found first;

$\frac{|0.84500422 - 0.84507|}{0.84500422} \times 100 = \%0.007784576508$ is found as.

B) Modified Euler's Method

In this section, the approximate value of the current will be calculated numerically by using the **Modified Euler Method** whose formula is given in equation 3, which is the second method.

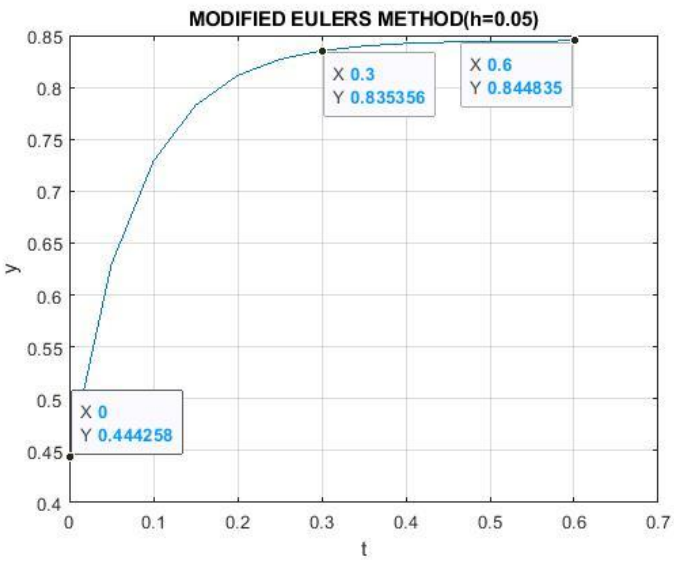
$$y_{i+1} = y_i + \frac{h}{2}\left(fi\ + f(t_{i+1}, y_i + hfi)\right) \qquad (3)$$

Since the value of h is accepted as 0.05 in this part, the program in figure 4 enters an increasing cycle from 0 to 0.6 with intervals of 0.05. Then, first in the loop, the derivative of the current is calculated with the value of i0 and equaled to the variable m, then the current value is calculated approximately by applying the formula given in equation 3 and thrown into the array. Then the value of i0 given at the beginning is equal to the variable i1me and the same operations continue until the end of the cycle.

```
...Modified Euler's Method...
    i0=100*(10^-3);
    e=1;
for a=0:0.05:0.6
    m=(Vs-R*i0)/L;
    i1me=i0+(0.05/2)*(m+(Vs-R*(i0+0.05*m))/L);
    c(e)=i1me;
    i0=i1me;
    e=e+1;
end
```

(Figure 4)

If the graph of the Modified Euler method is drawn according to these calculations, graph 2 is obtained.



(Graph 2)

As seen in the graph, the Modified Euler method calculated the current as **0.844835**. Relative error with respect to the analytically found real value;

$\frac{0.84500422-0.844835}{0.84500422} \times 100 = \%0.020025935$ is found as.

C) Midpoint Method

In this section, the approximate value of the current will be calculated numerically by using the **Midpoint metho**d whose formula is given in equation 4, which is the third method.

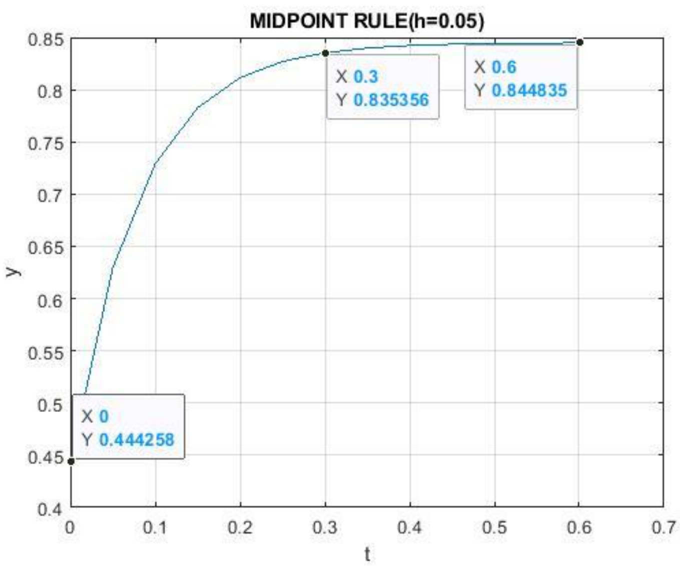
$$y_{i+1} = y_i + hf\left(t_{i+\frac{h}{2}}, y_i + \frac{h}{2} f(t_i, y_i)\right) \quad (4)$$

Since the value of h is accepted as 0.05 in this part, the program in figure 5 enters an increasing cycle from 0 to 0.6 with intervals of 0.05. Then, first in the loop, the derivative of the current is calculated with the value of i0 and equaled to the r variable, then the current value is calculated approximately by applying the formula given in equation 4 and thrown into the array. Then the value of i0 given at the beginning is equal to the variable i1m and the same processes continue until the end of the cycle.

```
...MİDPOİNT RULE...
    i0=100*(10^-3);
e=1;
for a=0:0.05:0.6
    r=(Vs-R*i0)/L;
    i1m=i0+0.05*((Vs-R*(i0+(0.05/2)*r))/L);
    d(e)=i1m;
    e=e+1;
    i0=i1m;
end
```

(Figure 5)

If the graph of the Midpoint method is plotted according to these calculations, graph 3 is obtained.



(Graph 3)

As seen in the graph, the Modified Euler's method also calculated the current as **0.844835**. Relative error with respect to the analytically found real value;

$\frac{0.84500422-0.844835}{0.84500422} \times 100 = \%0.020025935$ is found as.

D) Runge-Kutta Method Order Four

In this section, the approximate value of the current will be calculated numerically by using the **Runge-Kutta Method Order Four**, which is the fourth of the methods and given the formula in equation 5.

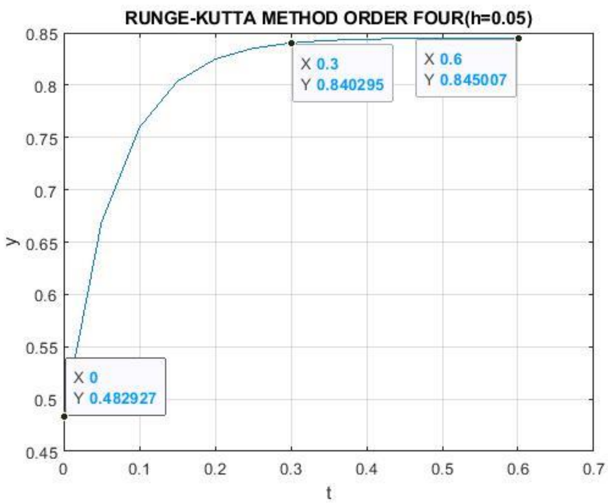
$$k_1 = hf(t_i, y_i)$$
$$k_2 = hf(t_{i+\frac{h}{2}}, y_i + \frac{k_1}{2})$$
$$k_3 = hf(t_{i+\frac{h}{2}}, y_i + \frac{k_2}{2})$$
$$k_4 = hf(t_{i+1}, y_i + k_3)$$
$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \qquad (5)$$

Since the value of h is accepted as 0.05 in this section, the program in figure 6 enters an increasing cycle from 0 to 0.6 with intervals of 0.05. Then, first k1, k2, k3, k4 values are calculated in the loop. Then, by applying the formula given in equation 5, the current value is calculated approximately and thrown into the array. Then, the i0 value given at the beginning is equal to the i1r variable and the same processes continue until the end of the cycle.

```
...Runge-Kutta Method Order Four...
    i0=100*(10^-3);
e=1;
for a=0:0.05:0.6
    k1=0.05*((Vs-R*i0)/L);
    k2=0.05*((Vs-R*(i0+k1/2))/L);
    k3=0.05*((Vs-R*(i0+k2/2))/L);
    k4=0.05*((Vs-R*(i0+k3))/L);
    i1r=i0+(1/6)*(k1+2*k2+2*k3+k4);
    f(e)=i1r;
    i0=i1r;
    e=e+1;
end
```

(Figure 6)

If a Runge-Kutta Method Order Four graph is drawn according to these calculations, graph 3 is obtained.

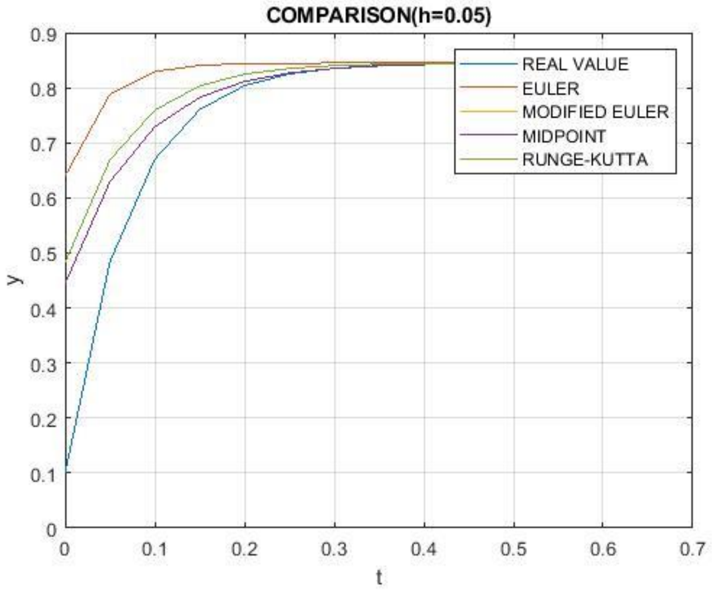


(Graph 4)

As seen in the graph, the Modified Euler method calculated the current as **0.845007**. Relative error according to the analytically found real value;

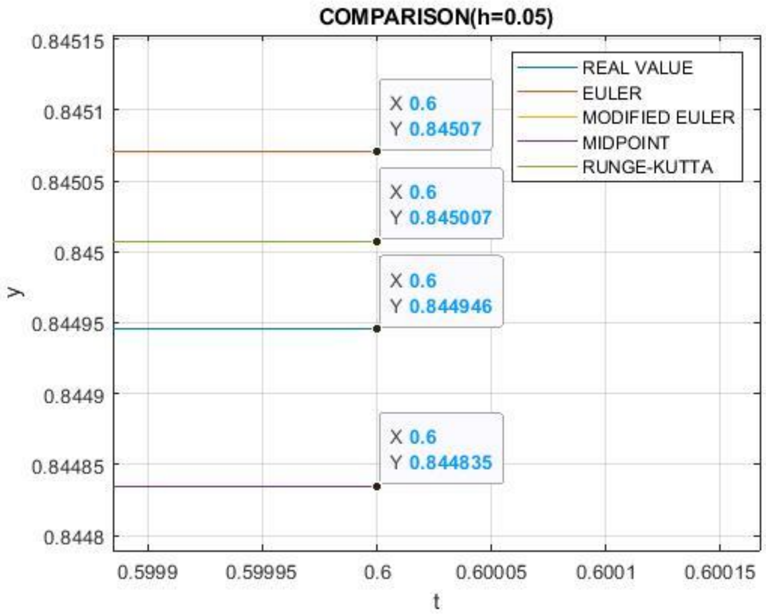
$\frac{|0.84500422-0.845007|}{0.84500422} \times 100 = \%0.0003289924398$ is found as.

If the graphs found according to these calculations are plotted on the same screen, graph 5 is obtained.



(Graph 5)

If this graphic drawn is examined more closely in graphic 6;



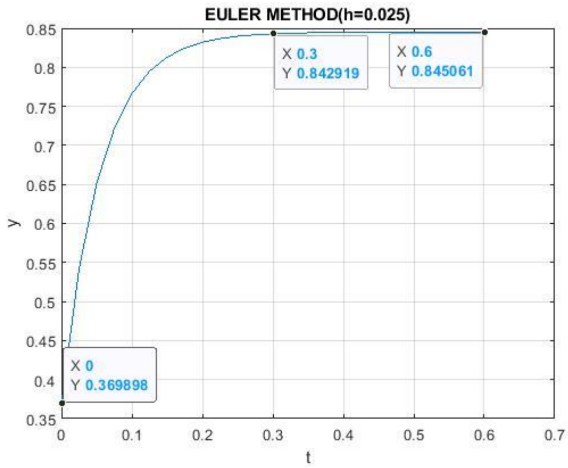
(Graph 6)

As seen in the calculations with the time interval as 0.05 and in graph 6, the method closest to the real value calculated analytically was **Runge-Kutta Method Order Four**. **The second method** that found the least error was **Euler's Method**. **Modified Euler's Method and Midpoint Method** were those that approached the result with equal error values and the most errors. Also, **Modified Euler's Method and Midpoint Method** found the same result.

If the same operations are repeated with the time interval assuming t = 0.025

E) Euler's Method(h=0.025)

In this section, the same operations were done with the time interval assuming t = 0.05. If the graph number 7 given below is examined, when the time interval is accepted as 0.025, **Euler's method** has found the current value analytically as **0.845061**.



(Graph 7)

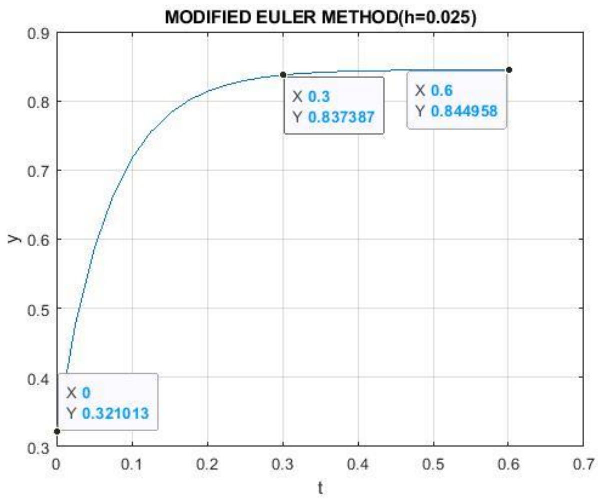
If this value is compared with the real value calculated analytically and the relative error is calculated;

$\frac{|0.84500422 - 0.845061\,|}{0.84500422} \times 100 = \%0.00671949307$ is found as.

When the time interval was accepted as 0.05, this value was **%0.007784576508**. As can be seen, when the time interval is reduced and more calculations are made, the error rate decreases.

F) Modified Euler's Method(h=0.025)

In this section, the same operations were done with the time interval assuming t = 0.05. If the graph number 8 given below is examined, when the time interval is accepted as 0.025, **the Modified Euler's method** has found the current value analytically as **0.844958**.



(Graph 8)

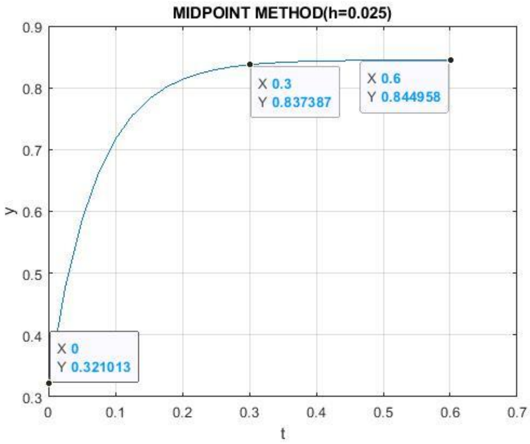
If this value is compared with the real value calculated analytically and the relative error is calculated;

$\frac{|0.84500422-0.844958\ |}{0.84500422} \times 100 = \%0.005469795169$ is found as.

When the time interval was accepted as 0.05, this value was **%0.020025935**. As can be seen, when the time interval is reduced and more calculations are made, the error rate decreases.

G) Midpoint Method(h=0.025)

In this section, the same operations were done with the time interval assuming t = 0.05. If the graph numbered 9 below is examined, when the time interval is accepted as 0.025, **the Midpoint method** has found the current value analytically as **0.844958**.



(Graph 9)

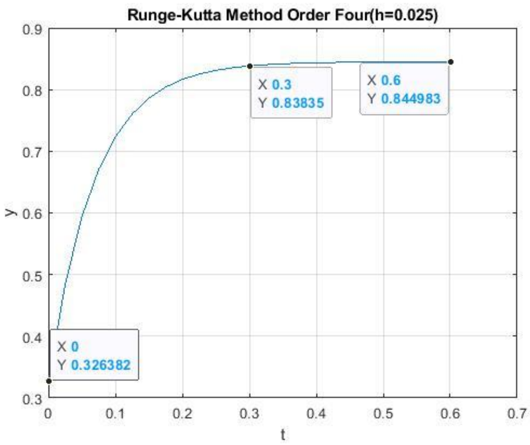
If this value is compared with the real value calculated analytically and the relative error is calculated;

$\frac{|0.84500422-0.844958\ |}{0.84500422} \times 100 = \%0.005469795169$ is found as.

When the time interval was accepted as 0.05, this value was **%0.020025935**. As can be seen, when the time interval is reduced and more calculations are made, the error rate decreases.

H) Runge-Kutta Method Order Four(h=0.025)

In this section, the same operations were done with the time interval assuming t = 0.05. If the graph number 10 given below is examined, when the time interval is accepted as 0.025, **Runge-Kutta Method Order Four** has analytically found the current value **0.844983**.



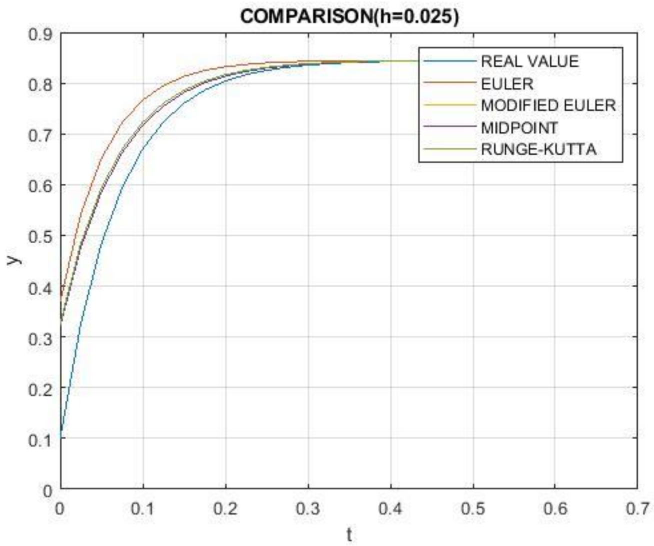
(Graph 10)

If this value is compared with the real value calculated analytically and the relative error is calculated;

$\frac{|0.84500422-0.844983\,|}{0.84500422} \times 100 = \%0.0002511230062$ is found as.

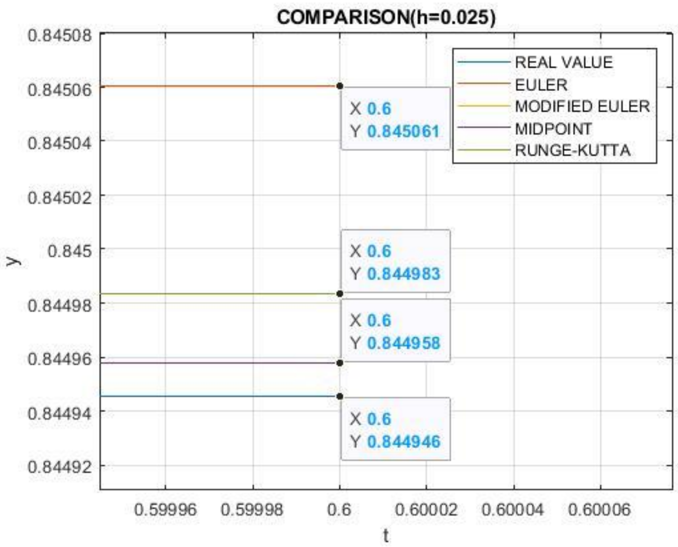
When the time interval was accepted as 0.05, this value was **%0.0003289924398**. As can be seen, when the time interval is reduced and more calculations are made, the error rate decreases.

If the graphs drawn with these values, assuming the time interval as 0.025, are collected on a single screen, graph 11 is obtained.



(Graph 11)

If this graph is examined more closely in graph 12;

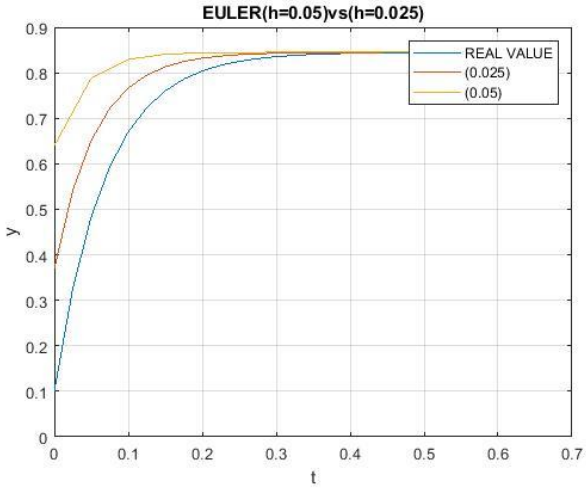


(Graph 12)

Runge-Kutta Method Order Four was the closest method to the real value calculated analytically, as can be seen in the calculations made by assuming the time interval as 0.025 and in graph 12. The second method that found the least error was **Euler's Method**. **Modified Euler's Method** and **Midpoint Method** were the ones that approached the result with equal error values and the most errors. Also, **Modified Euler's Method and Midpoint Method** found the same result.

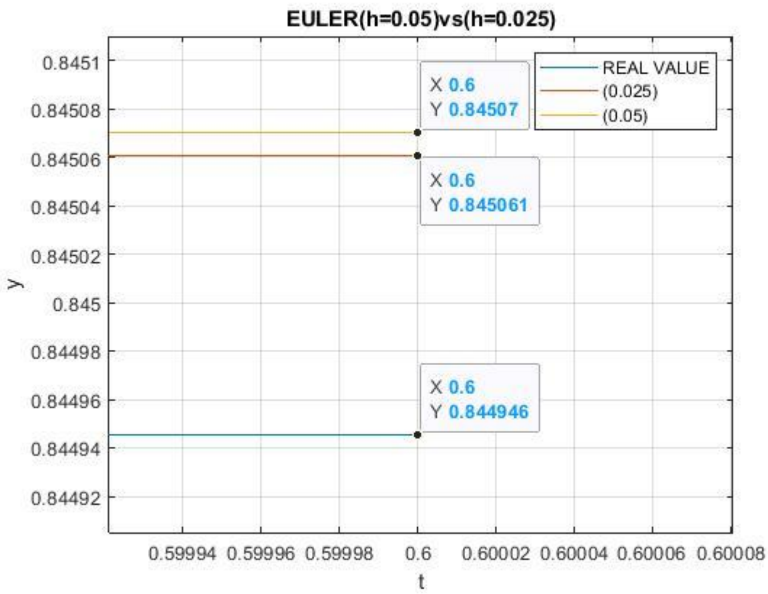
In this project, we examined 4 methods at different time intervals, had the graphs drawn and calculated the relative errors with the real value. The method that gave the most accurate results in both time intervals was **Runge-Kutta Method Order Four**.

In Graph 13, graphs of Euler's method in two time intervals are plotted.



(Graph 13)

If we take a closer look at this graph;



(Graph 14)

As seen in Graph 14, when we make more precise calculations in narrowing the time interval, we can approach the real result with less error.

The situation we mentioned above can be understood more clearly by showing the relative errors made on a table.

	H=0.05	H=0.025
Euler's Method	%0.007784576508	%0.00671949307
Modified Euler's Method	%0.020025935	%0.005469795169
Midpoint Method,	%0.020025935	%0.005469795169
Runge-Kutta Method Order Four	%0.0003289924398	%0.0002511230062

PROGRAM CODES:

```
clear;
clear all;


i0=100*(10^-3);
L=0.98;
Vs=12;
R=14.2;
Z=L/R;

.........ÇİZDİRMEK İSTEDİĞİNİZ GRAFİĞİN YORUMUNU KALDIRIN.........

................H DEĞERİ=0.05....................

....REAL VALUE...
    e=1;
    for a=0:0.05:0.6
        ie(e)=Vs/R+(i0-Vs/R)*exp(-a/Z);
        e=e+1;
    end

...EULER METHOD...
    i0=100*(10^-3);
    e=1;
for a=0:0.05:0.6
    i1e=i0+0.05*((Vs-R*i0)/L);
    b(e)=i1e;
    i0=i1e;
    t(e)=a;
    e=e+1;
end
% plot(t,b);
% title('EULER METHOD(h=0.05)');
% xlabel('t');
% ylabel('y');
% grid on

...Modified Euler's Method...
    i0=100*(10^-3);
    e=1;
for a=0:0.05:0.6
    m=(Vs-R*i0)/L;
    i1me=i0+(0.05/2)*(m+(Vs-R*(i0+0.05*m))/L);
    c(e)=i1me;
    i0=i1me;
    e=e+1;
end
% plot(t,c);
% title('MODIFIED EULERS METHOD(h=0.05)')
% grid on;
% xlabel('t');
% ylabel('y');

...MİDPOİNT RULE...
    i0=100*(10^-3);
 e=1;
 for a=0:0.05:0.6
    r=(Vs-R*i0)/L;
```

```
    i1m=i0+0.05*((Vs-R*(i0+(0.05/2)*r))/L);
    d(e)=i1m;
    e=e+1;
    i0=i1m;
 end
%  plot(t,d);
%  title('MIDPOINT RULE(h=0.05)');
%  grid on;
%  xlabel('t');
%  ylabel('y');

...Runge-Kutta Method Order Four...
    i0=100*(10^-3);
e=1;
for a=0:0.05:0.6
    k1=0.05*((Vs-R*i0)/L);
    k2=0.05*((Vs-R*(i0+k1/2))/L);
    k3=0.05*((Vs-R*(i0+k2/2))/L);
    k4=0.05*((Vs-R*(i0+k3))/L);
    i1r=i0+(1/6)*(k1+2*k2+2*k3+k4);
    f(e)=i1r;
    i0=i1r;
    e=e+1;
end
% plot(t,f);
% title('RUNGE-KUTTA METHOD ORDER FOUR(h=0.05)');
% grid on;
% xlabel('t');
% ylabel('y');


...ANALİZ GRAFİĞİ...
%      plot(t,ie,t,b,t,c,t,d,t,f);
%      title('COMPARISON(h=0.05)');
%      grid on;
%      legend('REAL VALUE','EULER','MODIFIED EULER','MIDPOINT','RUNGE-
KUTTA');
%      xlabel('t');
%      ylabel('y');


.............H DEĞERİ=0.025....................

    ....REAL VALUE...
    i0=100*(10^-3);
    e=1;
    for a=0:0.025:0.6
        iee(e)=Vs/R+(i0-Vs/R)*exp(-a/Z);
        e=e+1;
    end

    ...EULER METHOD...
    i0=100*(10^-3);
    e=1;
for a=0:0.025:0.6
    i1eh=i0+0.025*((Vs-R*i0)/L);
    z(e)=i1eh;
    i0=i1eh;
    t1(e)=a;
    e=e+1;
```

```
end
%     plot(t1,z)
%     title('EULER METHOD(h=0.025)');
%     grid on
%     xlabel('t');
%     ylabel('y');

...Modified Euler's Method...
    i0=100*(10^-3);
    e=1;
for a=0:0.025:0.6
    m=(Vs-R*i0)/L;
    i1meh=i0+(0.025/2)*(m+(Vs-R*(i0+0.025*m))/L);
    g(e)=i1meh;
    i0=i1meh;
    e=e+1;
end
%     plot(t1,g);
%     title('MODIFIED EULER METHOD(h=0.025)');
%     grid on
%     xlabel('t');
%     ylabel('y');

...MİDPOİNT RULE...
    i0=100*(10^-3);
    e=1;
 for a=0:0.025:0.6
    r=(Vs-R*i0)/L;
    i1mh=i0+0.025*((Vs-R*(i0+(0.025/2)*r))/L);
    h(e)=i1mh;
    e=e+1;
    i0=i1mh;
 end
%     plot(t1,h);
%     title('MIDPOINT METHOD(h=0.025)');
%     grid on
%     xlabel('t');
%     ylabel('y');

...Runge-Kutta Method Order Four...
    i0=100*(10^-3);
    e=1;
for a=0:0.025:0.6
    k1=0.025*((Vs-R*i0)/L);
    k2=0.025*((Vs-R*(i0+k1/2))/L);
    k3=0.025*((Vs-R*(i0+k2/2))/L);
    k4=0.025*((Vs-R*(i0+k3))/L);
    i1rh=i0+(1/6)*(k1+2*k2+2*k3+k4);
    j(e)=i1rh;
    i0=i1rh;
    e=e+1;
end
%     plot(t1,j)
%     title('Runge-Kutta Method Order Four(h=0.025)');
%     grid on
%     xlabel('t');
%     ylabel('y');

...ANALİZ GRAFİĞİ...
% plot(t1,iee,t1,z,t1,g,t1,h,t1,j);
% grid on;
```

```
% title('COMPARISON(h=0.025)');
% legend('REAL VALUE','EULER','MODIFIED EULER','MIDPOINT','RUNGE-KUTTA');
% xlabel('t');
% ylabel('y');

........EULER 0.025 VS 0.05...............
% plot(t1,iee,t1,z,t,b);
% grid on
% title('EULER(h=0.05)vs(h=0.025)');
% xlabel('t');
% ylabel('y');
% legend('REAL VALUE','(0.025)','(0.05)');
```