Explanation: Transient response, which consists of natural and step responses, of an RL circuit with a DC voltage source V_S, an inductance L, and a resistance R, as shown in Figure 1, can be determined by solving the following ordinary differential equation (ODE) with the initial current i_0 as the initial condition:

$$V_S = L\frac{d}{dt}i(t) + Ri(t),\ i(t_0) = i_0, \tag{1}$$

where $i(t)$ is the current. In Figure 1, the switch is closed at $t = t_0$.

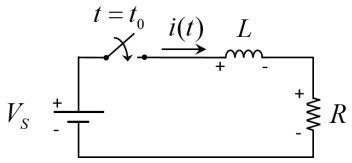

Figure 1: An RL circuit.

Question: Write a code to calculate the current $i(t)$ for $V_S = 12$ V, $L = 0.98$ H, $R = 14.2\ \Omega$, $t_0 = 0$ s, and $i_0 = 100$ mA by solving the equation (1) for the time interval $t \in [0, 600]$ ms using

(a) Euler's Method,

(b) Modified Euler's Method,

(c) Midpoint Method,

(d) Runge-Kutta Method Order Four,

and answer the following questions:

1. Write the problem given in equation (1) in the form of an initial value problem.
2. Find the analytical solution of the problem given in equation (1).
3. Solve the problem for the step size $\Delta t = 0.05$ s using the methods mentioned above.
4. Solve the problem for the step size $\Delta t = 0.025$ s using the methods mentioned above.
5. Plot and compare all results in terms of error and convergence.

Suggestions:

- You can present plots for different step sizes to compare the methods.
- You can calculate and tabulate the error with respect to the analytical solution obtained in Question 2.
- Do not forget to include your comments to your report.
- The convergence and error analyses do not have to be a mathematical analysis, it can be carried out using the plots and data. If you want to add an analytical error analysis for the error bound, it will be graded with extra points.

Notes:

- A report should be prepared as explained in **Homework and Project Report Preparation Guideline**.
- The codes and the report should be student's own work.
- A single, ready to run MATLAB script (m) file should be uploaded along with the pdf and docx files of your report.
- The figures presented in your report should have proper axis labels, legends, as well as figure numbers with proper citation in the report.
- Presenting only the code and plots will be graded with zero points.