

COMP341 Assignment 3

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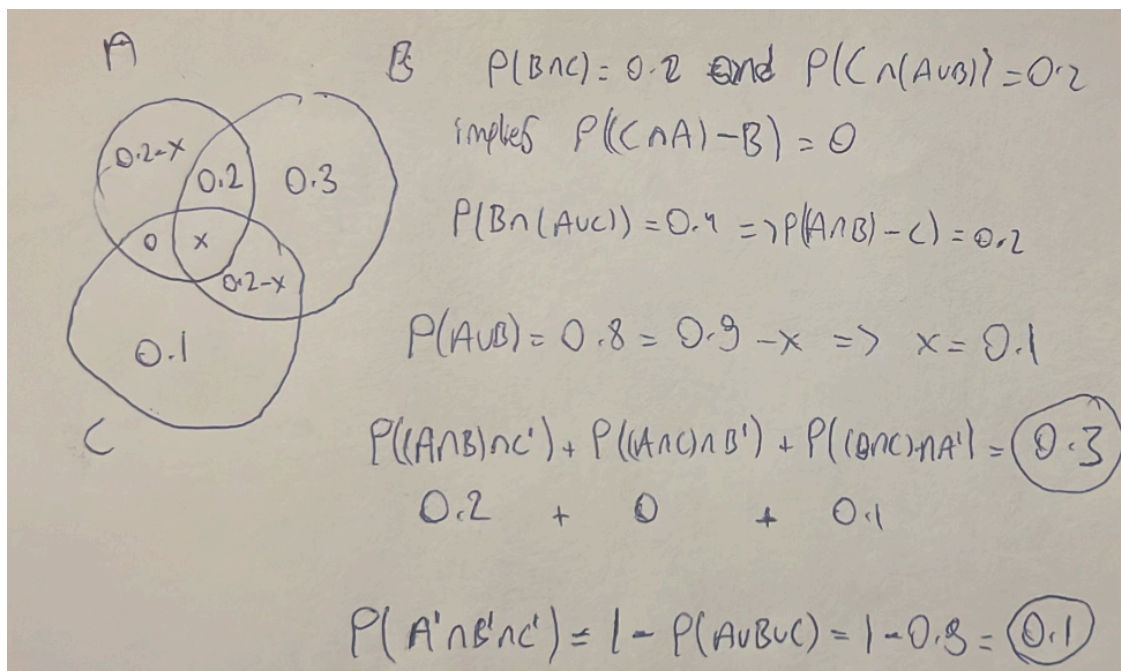
Q1)

1. **True:** Happy and smart are only connected with music so if music is given they do not affect themselves
2. **True:** Smart does not affect success since project and homework are the connections and they are given
3. **False:** Creative still affects success because of homework
4. **False:** Smart and Party are not only affected by success but also creative, homework, and music
5. **False:** Simply not enough for that conclusion, there are other dependencies,

Q2)

- 1-2) $P(\text{only two events}) = P(A \text{ and } B \text{ and } C') + P(A \text{ and } B' \text{ and } C) + P(A' \text{ and } B \text{ and } C)$
 $P(\text{no events occurring}) = 1 - P(\text{at least one event occurring})$

Solution:



As seen in above we have found $P(\text{only two events}) = 0.3$, $P(\text{no events occurring}) = 0.1$

Q3)

a) Product of all conditional probabilities is the joint probability distribution:

$$P(S, M, B, I, P, W, N, O) = P(S) P(M|S) P(B) P(I|S) P(P|M, B, I) P(W|P, I) P(N|W, I, S) P(O|N, W)$$

b) Numbers inside the parentheses denote the row number which the amount was extracted from.

$$\begin{aligned} \Theta = b &: 2(1) + 1(2) + 1(3) + 1(4) + 1(5) = 6 \\ \Theta = n &: 1(8) + 1(12) + 1(13) + 1(14) + \cancel{1(15)} = 4 \\ \Theta = g &: 1(6) + 1(7) + 1(9) + 1(10) + 1(11) = 5 \end{aligned}$$

$$P(\Theta | M = +m, I = +i) = \begin{cases} b: 6/15, \\ n: 4/15, \\ g: 5/15 \end{cases}$$

$$= \begin{cases} b: 2/5, \\ n: 4/15 \\ g: 1/3 \end{cases}$$

Q4)

a) Using the table provided for the political outcome we have I have constructed the required tables and the conditional probabilities of $P(O | M = +m, I = +I, W = +w)$ and $P(O | M = +m, I = +I, W = -w)$

W= +w								Summing Probabilites		
N	$P(N W=+w, I=+I, S)$	O	$P(O N, W=+w)$		$P(O M=+m, I=+i, W=+w)$			$P(O M=+m, I=+I, W=+w)$		
+n	0.75	b	0.3		0.225					
+n	0.75	n	0.3		0.225			b	0.45	
+n	0.75	g	0.4		0.30			n	0.25	
-n	0.25	b	0.9		0.225			g	0.30	
-n	0.25	n	0.1		0.025					
-n	0.25	g	0.0		0.0					
W= -w										
N	$P(N W=-w, I=+I, S)$	O	$P(O N, W=-w)$		$P(O M=+m, I=+i, W=-w)$			Summing Probabilites		
+n	0.75	b	0.0		0.0			$P(O M=+m, I=+I, W=-w)$		
+n	0.75	n	0.1		0.075					
+n	0.75	g	0.9		0.675			b	0.025	
-n	0.25	b	0.1		0.025			n	0.15	
-n	0.25	n	0.3		0.075			g	0.825	
-n	0.25	g	0.6		0.15					

b) Using the utility values from the given table we calculate:

For $W = +w$:

$$EU(W = +w) = 0.45 * -120 + 0.25 * 0 + 0.30 * 40 = -42$$

For $W = -w$:

$$EU(W = -w) = 0.025 * -80 + 0.15 * 20 + 0.825 * 120 = 100$$

Thus, $EU(W = +w) = -42$, $EU(W = -w) = 100$, so optimal action is to choose the minimum wage increase.

c)

For $S = +s$ using the above table we calculate:

$$EU(W = +w | S = +s) = 0.45 * -120 + 0.25 * 0 + 0.3 * 40 = -42$$

$$EU(W = -w | S = +s) = 0.025 * -80 + 0.15 * 20 + 0.825 * 120 = 100$$

$$\text{Thus } EU(S = +s) = \max(-42, 100) = 100$$

For $S = -s$ using the given table we calculate:

$$EU(W = +w | S = -s) = 0.225 * -120 + 0.025 * 0 + 0.75 * 40 = 3$$

$$EU(W = -w | S = -s) = 0.025 * -80 + 0.15 * 20 + 0.825 * 120 = 100$$

$$\text{Thus, } EU(S = -s) = \max(3, 100) = 100$$

We need to calculate the values of $P(S = +s \mid M = +m, I = +i)$ and $P(S = -s \mid M = +m, I = +i)$ to find the VPI. Applying Bayes theorem to find them:

$$P(S = +s \mid M = +m, I = +i) = P(M = +m, I = +i \mid S = +s) * P(S = +s) / P(M = +m, I = +i)$$

$$P(S = -s \mid M = +m, I = +i) = P(M = +m, I = +i \mid S = -s) * P(S = -s) / P(M = +m, I = +i)$$

$$P(M = +m, I = +i \mid S = +s) = P(M = +m \mid S = +s) * P(I = +i \mid S = +s) = 0.9 * 0.5 = 0.45$$

$$P(M = +m, I = +i \mid S = -s) = P(M = +m \mid S = -s) * P(I = +i \mid S = -s) = 0.2 * 0.75 = 0.15$$

We know that:

$$P(M = +m, I = +i) = P(M = +m, I = +i \mid S = +s) * P(S = +s) + P(M = +m, I = +i \mid S = -s) * P(S = -s)$$

$$P(M = +m, I = +i) = 0.45 * 0.25 + 0.15 * 0.75 = 0.225$$

Finally we have

$$P(S = +s \mid M = +m, I = +i) = 0.45 * 0.25 \text{ (since } P(S = +s) = 0.25) / 0.225 = 0.5$$

$$P(S = -s \mid M = +m, I = +i) = 0.15 * 0.75 \text{ (since } P(S = -s) = 0.75) / 0.225 = 0.5$$

$$\begin{aligned} EU(\text{with PI}) &= P(S = +s \mid M = +m, I = +i) * EU(S = +s) + P(S = -s \mid M = +m, I = +i) * EU(S = -s) \\ &= 0.5 * 100 + 0.5 * 100 = 100 \end{aligned}$$

$$\mathbf{VPI = EU(\text{with PI}) - EU = 100 - 100 = 0}$$

This means knowing S does not change the optimal action!