Exercise-1.8: First question asks us to verify $E[x] = \int_{-\infty}^{\infty} N(X|\mu, \sigma^2)xdx = \mu$. Where $N(X|\mu, \sigma^2)$ is gaussian distribution. To do this we will calculate the integral by using integration by parts. When you do the calculations, you will reach to the result.

Second part of the question asks us to verify $E[x^2] = \int_{-\infty}^{\infty} N(X|\mu, \sigma^2)x^2dx = \mu^2 + \sigma^2$. To do this question already gave us a hint. We must differentiate equation (1.49) by σ^2 (you may want to remember how to differentiate integrals). As result we obtain:

$$(\frac{1}{2\pi\sigma^2})^2 \int_{-\infty}^{\infty} \exp\{-\frac{1}{2\sigma^2}(x-\mu)^2\}(x-\mu)^2 dx = \sigma^2$$

Remember the definition of expectancy variance and formula of the gaussian distribution. This equation is:

$$E[(x-\mu)^2] = var[x] = \sigma^2$$

Now let's distribute square inside the expectancy.

$$E[x^2 - 2x\mu + \mu^2] = E[x^2] - 2\mu E[x] + u^2 = \sigma^2$$

Re-arrange the equation

$$E[x^2] - 2\mu^2 + \mu^2 = \sigma^2$$

This gives us

$$E[x^2] = \sigma^2 + \mu^2$$

Which is equation (1.50). It follows directly from the equation above that

$$var[x] = E[x^2] - E[x]^2 = \sigma^2$$

Holds which is the equation (1.51), the last part of the question.