Microprocessors & Interfacing

Number Conversion

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Number Representation

- Any number can be represented in the form of

$$(a_n a_{n-1} \ldots a_1 a_0 . a_{-1} \ldots a_{-m})_r$$
$$= a_n \times r^n + a_{n-1} \times r^{n-1} + \ldots + a_1 \times r + a_0 + a_{-1} \times r^{-1} + \ldots + a_{-m} \times r^{-m}$$

r : radix, base

$$0 \le a_i < r$$

Example

- Decimal

$$\boxed{\begin{aligned}&(3597)_{10}\\&= 3\times 10^{3} + 5\times 10^{2} + 9\times 10 + 7\end{aligned}}$$

  - The place values, from right to left, are 1, 10, 100, 1000
  - The base or radix is 10
  - All digits must be less than the base, namely, 0~9

Example

- Binary

$$\boxed{\begin{aligned}&(1011)_2\\&= 1\times 2^3 + 0\times 2^2 + 1\times 2 + 1\end{aligned}}$$

  - The place values, from right to left, are 1, 2, 4, 8
  - The base or radix is 2
  - All digits must be less than the base, namely, 0~1

Example

- Hexadecimal

$$\boxed{\begin{aligned}&(\mathbf{F24B})_{16}\\&= \mathbf{F} \times \mathbf{16}^3 + \mathbf{2} \times \mathbf{16}^2 + \mathbf{4} \times \mathbf{16} + \mathbf{B}\\&= \mathbf{15} \times \mathbf{16}^3 + \mathbf{2} \times \mathbf{16}^2 + \mathbf{4} \times \mathbf{16} + \mathbf{11}\end{aligned}}$$

  - The place values, from right to left, are 1, 16, 16^2, 16^3
  - The base or radix is 16
  - All digits must be less than the base, namely, 0~9,**A,B,C,D,E,F**

Number Conversion

- From base r to base 10
  - Using

$$\boxed{\begin{aligned}&(a_n a_{n-1} \ldots a_1 a_0 . a_{-1} \ldots a_{-m})_r \\ &= a_n \times r^n + a_{n-1} \times r^{n-1} + \ldots + a_1 \times r + a_0 + a_{-1} \times r^{-1} + \ldots + a_{-m} \times r^{-m}\end{aligned}}$$

- Examples (next slide)

Examples

- From base 2

$$\boxed{(1011.1)_2 = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2 + 1 + 1 \times 2^{-1} = 11.5}$$

- From base 16

$$\boxed{(10A)_{16} = 1 \times 16^2 + 0 \times 16 + 10 = 266}$$

Number Conversion

- From base 10 to base r

Based on Formula

$$
\begin{aligned}
&(a_n a_{n-1} \ldots a_1 a_0 . a_{-1} \ldots a_{-m})_r \\
&= a_n \times r^n + a_{n-1} \times r^{n-1} + \ldots + a_1 \times r + a_0 + a_{-1} \times r^{-1} + \ldots + a_{-m} \times r^{-m}
\end{aligned}
$$

- For whole number
  - **Divide** the number/quotient repeatedly by **r** until the quotient is zero and the remainders are the digits of base r number, in reverse order
- For fraction
  - **Multiply** the number/fraction repeatedly by **r**, the whole numbers of the products are the digits of the base r fraction number

Examples

- To base 2
  - To convert $(11.25)_{10}$ to binary
    - For whole number $(11)_{10}$ – repeated division **(by 2)**

11	1
5	1
2	0
1	1
0	

    - For fraction $(0.25)_{10}$ – repeated multiplication **(by 2)**

0.25	
0.5	0
0.0	1

$(11.25)_{10}=(1011.01)_{2}$

Examples

- To base 16
  - To convert $(99.25)_{10}$ to hexadecimal
    - For whole number $(99)_{10}$ – division **(by 16)**

99	**3**
6	**6**
0	

    - For fraction $(0.25)_{10}$ – multiplication **(by 16)**

0.25	
0.0	**4**

$(99.25)_{10}=(63.4)_{hex}$

Number Conversion

- Between binary and octal
  - Direct mapping based on the observation:

$$
\begin{aligned}
&(abcdefgh.\ jklmn)_2 \\
&= (a \cdot 2 + b) \cdot 2^6 + (c \cdot 2^2 + d \cdot 2 + e) \cdot 2^3 + \\
&\quad (f \cdot 2^2 + g \cdot 2 + h) + (j \cdot 2^2 + k \cdot 2 + l) \cdot 2^{-3} + \\
&\quad (m \cdot 2^2 + n \cdot 2 + 0) \cdot 2^{-6} \\
&= (0ab_2) \cdot 8^2 + (cde_2) \cdot 8^1 + (fgh_2) \cdot 8^0 + \\
&\quad (jkl_2) \cdot 8^{-1} + (mn0_2) \cdot 8^{-2}
\end{aligned}
$$

  - The expressions in parentheses, being less than 8, are the octal digits.

Number Conversion

- Between binary and octal (cont.)
  - Binary to octal
    - The binary digits (“bits”) are grouped from the radix point, three digits a group. Each group corresponds to an octal digit.
  - Octal to binary
    - Each of octal digits is expanded to three binary digits

Examples

- Binary to octal
  - Convert $(10101100011010001000.10001)_2$ to octal :

    $010\ 101\ 100\ 011\ 010\ 001\ 000\ .\ 100\ 010_2$

    $= \quad 2 \quad 5 \quad 4 \quad 3 \quad 2 \quad 1 \quad 0 \quad . \quad 4 \quad 2_8$

    $= \quad 2543210.42_8$.
- Note:
  - Whole number parts are grouped from right to left. The leading 0 is optional
  - Fractional parts are grouped from left to right and padded with 0s

Examples

- Octal to binary
  - Convert 37425.62_8 to binary :

    $3 \quad 7 \quad 4 \quad 2 \quad 5 \;.\; 6 \quad 2_8$

    $= 011\ 111\ 100\ 010\ 101\ .\ 110\ 010_2$

    $= 11111100010101.11001_2$

- Note:
  - For whole number parts, the leading 0s can be omitted.
  - For fractional parts, the trailing 0s can be omitted.

Number Conversion

- Between binary and hexadecimal
  - Binary to hexadecimal
    - The binary digits (“bits”) are grouped from the radix point, **four** binary digits a group. Each group corresponds to a hexadecimal digit.
  - Hexadecimal to binary
    - Each of hexadecimal digits is expanded to four binary digits

Examples

- Binary to hexadecimal
  - Convert $101011000110100001000.10001_2$ to hexadecimal :

    $1010\ 1100\ 0110\ 1000\ 1000\ .\ 1000\ 1000_2$

    $= \quad A \quad C \quad 6 \quad 8 \quad 8 \quad . \quad 8 \quad 8 \quad _{16}$

    $= \quad AC688.88_{16}$.
- Note:
  - Whole number parts are grouped from right to left. The leading 0 is optional
  - Fractional parts are grouped from left to right and padded with 0s

Examples

- Hexadecimal to binary
  - Convert $2F6A.78_{16}$ to binary :

$$\begin{array}{cccccccc} & 2 & F & 6 & A & . & 7 & 8_{16} \\ = & 0010 & 1111 & 0110 & 1010 & . & 0111 & 1000_{2} \end{array}$$

$$= 10111101101010.01111_{2}$$

- Note:
  - For whole number parts, the leading 0s can be omitted.
  - For fractional parts, the trailing 0s can be omitted.

Conversion to binary via octal

The direct conversion of 2001_{10} to binary looks like this ...

2001	
1000	1
500	0
250	0
125	0
62	1
31	0
15	1
7	1
3	1
1	1
0	1

... and gives 11111010001.

It may be quicker to convert to octal first ...

2001	
250	1
31	2
3	7
0	3

... yielding **3721_8**, which can be instantly converted to **$11\ 111\ 010\ 001_2$**.