

$$\textcircled{3} \mathcal{L}^{\text{det}} = \log P(D|V) \quad P(D|V) = \prod_{n=1}^N \prod_{k=1}^K P_k(V_k^T X^{(n)}) |\det(V)|$$

$$= N \log |\det(V)| + \sum_{k=1}^K \sum_{n=1}^N \log(P_k(V_k^T X^{(n)}))$$

$$\textcircled{4} \frac{\partial \mathcal{L}}{\partial V_{ij}} = N \frac{V_{ij}}{\det(V)} + \sum_{n=1}^N \sum_{k=1}^K \log \frac{\text{sech}(V_k^T X^{(n)})}{\pi}$$

$$= N V_{ij} + \sum_{n=1}^N \frac{\tanh(V_i^T X^{(n)})}{\text{sech}(V_i^T X^{(n)})} \cdot \tanh(V_j^T X^{(n)}) \cdot \text{sech}(V_k^T X^{(n)}) \cdot X_j^{(n)}$$

$$= N V_{ij} + \sum_{n=1}^N \tanh(V_i^T X^{(n)}) X_j^{(n)}$$

$$\textcircled{5} \mathcal{L}^{\text{det}} = \log P(D|V) \quad P(D|V) = \prod_{n=1}^N \prod_{k=1}^K P_k(V_k^T X^{(n)}) |\det(V)|$$

$$\hookrightarrow \mathcal{L} = N \log |\det(V)| + \sum_{k=1}^K \sum_{n=1}^N \log [P_k(V_k^T X^{(n)})]$$

$$\textcircled{6} \frac{\partial \mathcal{L}}{\partial V_{ij}} = N \frac{V_{ij}}{|\det(V)|} + \frac{\sum_{n=1}^N \sum_{k=1}^K \log \frac{\text{sech}(V_i^T X^{(n)})}{\pi}}{\partial V_{ij}}$$

$$= N V_{ij} + \sum_{n=1}^N \frac{1}{\text{sech}(V_i^T X^{(n)})} \cdot \tanh(V_i^T X^{(n)}) (-\text{sech}(V_i^T X^{(n)})) \cdot X_j^{(n)}$$

$$= N \cdot V_{ij} + \sum_{n=1}^N \tanh(V_i^T X^{(n)}) \cdot X_j^{(n)}$$