# Tutorial Week 3

COMP9418 – Advanced Topics in Statistical Machine Learning, 17s2, UNSW Sydney

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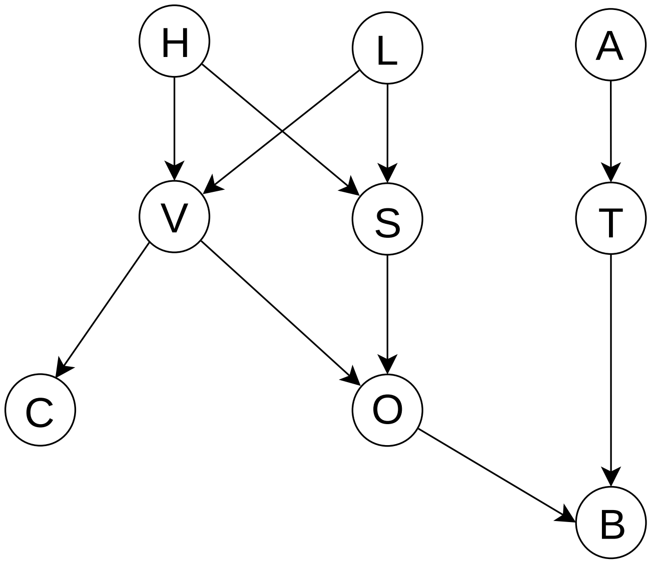
1. Consider the random variables $X, Y, Z$ which have the following joint distribution:

$$p(X, Y, Z) = p(X)p(Y|X)p(Z|Y). \tag{1}$$

   (a) Show that $X$ and $Z$ are conditionally independent given $Y$.

   (b) if $X$, $Y$ and $Z$ are binary variables, how many parameters are needed to specify a distribution of this form?

2. The Bayesian network shown below is a greatly simplified version of a network used for medical diagnosis in an intensive care unit. The diagnostic variables are the hypovolemia (H), the left ventricular failure (L) and the anaphylaxis (A). The intermediate variables are the left ventricular endiastolic volume (V), the stroke volume (S) and the total peripheral resistance (T). The measurement variables are the central venous pressure (C), the cardiac output (O) and the blood pressure (B). H, L and A take on the values $\{true, false\}$; V, S and T take on the values $\{low, high\}$; and C, O and B take on the values $\{low, medium, high\}$.




The Bayesian network is fully specified by its CPTs. We have:

$$
\begin{aligned}
p(H = true) &= 0.2 & & \\
p(L = true) &= 0.05 & p(A = true) &= 0.01 \\
p(V = low|H = false, L = false) &= 0.05 & p(S = low|H = false, L = false) &= 0.05 \\
p(V = low|H = false, L = true) &= 0.01 & p(S = low|H = false, L = true) &= 0.95 \\
p(V = low|H = true, L = false) &= 0.98 & p(S = low|H = true, L = false) &= 0.5 \\
p(V = low|H = true, L = true) &= 0.95 & p(S = low|H = true, L = true) &= 0.98 \\
p(T = low|A = false) &= 0.3 & p(T = low|A = true) &= 0.98 \\
p(C = low|V = low) &= 0.95 & p(C = medium|V = low) &= 0.04 \\
p(C = low|V = high) &= 0.01 & p(C = medium|V = high) &= 0.29 \\
p(O = low|V = low, S = low) &= 0.98 & p(O = medium|V = low, S = low) &= 0.01 \\
p(O = low|V = low, S = high) &= 0.3 & p(O = medium|V = low, S = high) &= 0.69 \\
p(O = low|V = high, S = low) &= 0.8 & p(O = medium|V = high, S = low) &= 0.19 \\
p(O = low|V = high, S = high) &= 0.01 & p(O = medium|V = high, S = high) &= 0.01 \\
p(B = low|O = low, T = low) &= 0.98 & p(B = medium|O = low, T = low) &= 0.01 \\
p(B = low|O = low, T = high) &= 0.3 & p(B = medium|O = low, T = high) &= 0.6 \\
p(B = low|O = medium, T = low) &= 0.98 & p(B = medium|O = medium, T = low) &= 0.01 \\
p(B = low|O = medium, T = high) &= 0.05 & p(B = medium|O = medium, T = high) &= 0.4 \\
p(B = low|O = high, T = low) &= 0.9 & p(B = medium|O = high, T = low) &= 0.09 \\
p(B = low|O = high, T = high) &= 0.01 & p(B = medium|O = high, T = high) &= 0.09
\end{aligned}
$$

(a) Write down the joint distribution defined by the Bayesian network.

(b) Show that the above joint distribution is correctly normalized.

(c) Let $X \perp\!\!\!\perp Y$ denote that $X$ and $Y$ are marginally independent and $X \perp\!\!\!\perp Y|Z$ denote that $X$ and $Y$ are *conditionally* independent given Z. Using the concept of *d-separation*, show or refute the following independence statements:

  i. $H \perp\!\!\!\perp L$
  ii. $H \perp\!\!\!\perp A$
  iii. $C \perp\!\!\!\perp L$
  iv. $C \perp\!\!\!\perp L|B$

(d) For the cases where independence holds in item 2c, prove these independences using the rules of probability.

(e) Compute $p(L|C = high)$ efficiently using variable elimination. Show all your working.

(f) Compute $p(O|H = true, L = true, A = true)$ using the Junction Tree Algorithm.