Shixin Wang
Henry Boateng
MATH 400-01
Homework 1
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Homework 1

1. I have my answer written in python in file “question_01.py”.

   To run the program, please run “python -u ./question_01.py” in terminal and make sure your current path is where “question_01.py” at.

   The program will print out the value of the integral, the average of the function and the total random points created.

   Here is output might have after running program:

```
PS D:\Math\MATH 400\02_Homework\homework_01> python -u .\question_01.py

========== Question 1 ==========

The value of the integral: 0.1018364807259941
The average of function: 1.018364807259941e-05
Total random points created: 1000000

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```
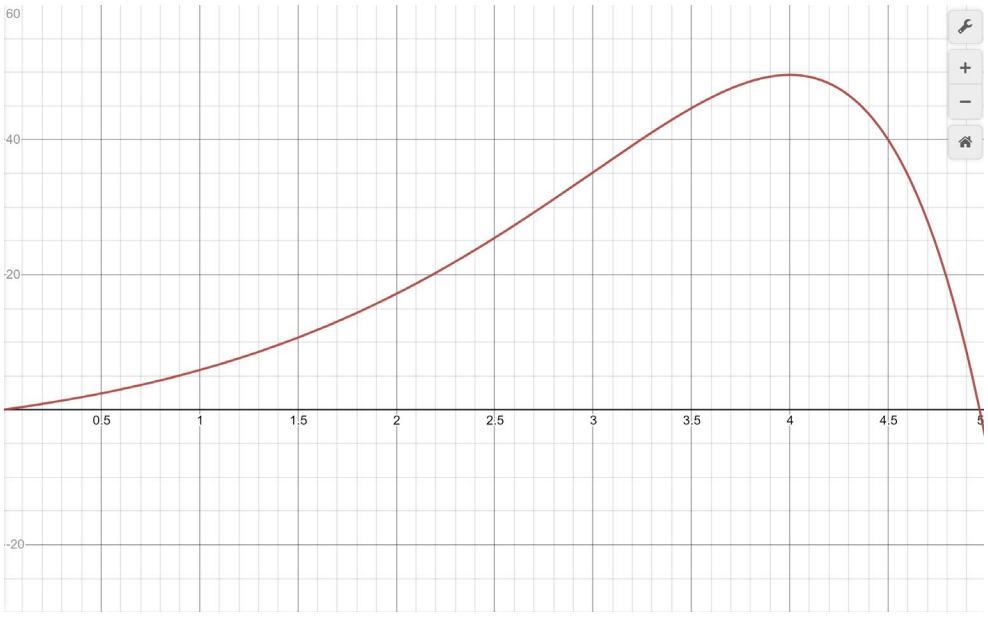

2. $f(x) = (5 - x)e^x - 5$



b. I wrote my answer in python in the file “question_02b.py”

To run the program, please run “python -u ./question_02b.py” in terminal and make sure your current path is where “question_02b.py” at.

The program will print out all values for each recursion and these values is round in 7 decimos, but you can feel free to change it by change the value of “format_output_decimos”. At the end of iteration, it will printout total amount of iterations and the estimate root of function in the given range.

Here is the output will have after running the program:

```
PS D:\Math\MATH 400\02_Homework\homework_01> python -u .\question_02b.py
initial interval: a = 4.0000000; b = 5.0000000; fa = 49.5981500;  fb = -5.0000000; Error_Bound = 0.5000000
n01: a = 4.5000000; b = 5.0000000; c = 4.5000000; fa =  40.0085657; fb = -5.0000000; fc = 40.0085657; max_possible_value = 0.2500000
n02: a = 4.7500000; b = 5.0000000; c = 4.7500000; fa =  23.8960711; fb = -5.0000000; fc = 23.8960711; max_possible_value = 0.1250000
n03: a = 4.8750000; b = 5.0000000; c = 4.8750000; fa =  11.3717692; fb = -5.0000000; fc = 11.3717692; max_possible_value = 0.0625000
n04: a = 4.9375000; b = 5.0000000; c = 4.9375000; fa =  3.7138288; fb = -5.0000000; fc = 3.7138288; max_possible_value = 0.0312500
n05: a = 4.9375000; b = 4.9687500; c = 4.9687500; fa =  3.7138288; fb = -0.5047823; fc = -0.5047823; max_possible_value = 0.0156250
n06: a = 4.9531250; b = 4.9687500; c = 4.9531250; fa =  1.6382887; fb = -0.5047823; fc = 1.6382887; max_possible_value = 0.0078125
n07: a = 4.9609375; b = 4.9687500; c = 4.9609375; fa =  0.5752945; fb = -0.5047823; fc = 0.5752945; max_possible_value = 0.0039062
n08: a = 4.9648438; b = 4.9687500; c = 4.9648438; fa =  0.0374041; fb = -0.5047823; fc = 0.0374041; max_possible_value = 0.0019531
n09: a = 4.9648438; b = 4.9667969; c = 4.9667969; fa =  0.0374041; fb = -0.2331505; fc = -0.2331505; max_possible_value = 0.0009766
n10: a = 4.9648438; b = 4.9658203; c = 4.9658203; fa =  0.0374041; fb = -0.0977388; fc = -0.0977388; max_possible_value = 0.0004883
n11: a = 4.9648438; b = 4.9653320; c = 4.9653320; fa =  0.0374041; fb = -0.0301338; fc = -0.0301338; max_possible_value = 0.0002441
n12: a = 4.9650879; b = 4.9653320; c = 4.9650879; fa =  0.0036435; fb = -0.0301338; fc = 0.0036435; max_possible_value = 0.0001221
n13: a = 4.9650879; b = 4.9652100; c = 4.9652100; fa =  0.0036435; fb = -0.0132430; fc = -0.0132430; max_possible_value = 0.0000610
n14: a = 4.9650879; b = 4.9651489; c = 4.9651489; fa =  0.0036435; fb = -0.0047992; fc = -0.0047992; max_possible_value = 0.0000305
n15: a = 4.9650879; b = 4.9651184; c = 4.9651184; fa =  0.0036435; fb = -0.0005777; fc = -0.0005777; max_possible_value = 0.0000153
n16: a = 4.9651031; b = 4.9651184; c = 4.9651031; fa =  0.0015329; fb = -0.0005777; fc = 0.0015329; max_possible_value = 0.0000076
n17: a = 4.9651108; b = 4.9651184; c = 4.9651108; fa =  0.0004776; fb = -0.0005777; fc = 0.0004776; max_possible_value = 0.0000038
n18: a = 4.9651108; b = 4.9651146; c = 4.9651146; fa =  0.0004776; fb = -0.0000500; fc = -0.0000500; max_possible_value = 0.0000019
n19: a = 4.9651127; b = 4.9651146; c = 4.9651127; fa =  0.0002138; fb = -0.0000500; fc = 0.0002138; max_possible_value = 0.0000010
Number of iterations =  19
An estimate of the root is  4.965112686157227
```

We can see the number of iterations is 19. If we take the size of interval 10^{-6}, we can calculate the value of n(how many steps needed):

$$Error = \delta = \left(\frac{1}{2}\right)^{n+1} |5-4| \leq 10^{-6}$$

$$2^{n+1} = 10^{6}$$

$$n = \log_2 10^{6} - 1$$

$$n = 18.931 \approx 19$$

Therefore, the program is running exactly right steps to getting the size of interval 10^{-6}

c. $Error = \delta = \left(\frac{1}{2}\right)^{n+1} |5-4| \leq 10^{-12}$

$2^{n+1} = 10^{12}$

$n = \log_2 10^{12} - 1$

$n = 38.86 \approx 39$

Therefore, we need 39 steps to reduce the size of the interval of this interval to 10^{-12}.

According to the formula: $Error = |p - x_{n+1}| \leq \left(\frac{1}{2}\right)^{n+1} |b_0 - a_0|$

We can get: $Error = \delta = \left(\frac{1}{2}\right)^{n+1} |5 - 4| \leq 10^{-12}$

So next we just need to solve the equation

Here is the formula explanation documented on my note: