MATH 400 Final Project

Group 2

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Section 1 Question 2:

$$\begin{aligned} f(x,y,z) &= x^2+y^2+z^2-10=0 \\ g(x,y,z) &= x+2y \quad -2=0 \\ h(x,y,z) &= x+ \quad 3z-9=0 \end{aligned}$$

k

$$x^{(0)} = [2 \ 0 \ 2]^T$$

$$\Rightarrow \underbrace{\begin{bmatrix} f \\ g \\ h \end{bmatrix}\Bigg|_{(x_n, y_n, z_n)}}_{f(x^{(k)})} + \underbrace{\begin{bmatrix} df/dx & df/dy & df/dz \\ dg/dx & dg/dy & dg/dz \\ dh/dx & dh/dy & dh/dz \end{bmatrix}\Bigg|_{(x_n, y_n, z_n)}}_{J} \underbrace{\begin{bmatrix} x_{n+1}-x_n \\ y_{n+1}-y_n \\ z_{n+1}-z_n \end{bmatrix}}_{\delta x^{(k)}} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Let $x=x_1$, $y=x_2$, $z=x_3$.

$$\Rightarrow J(x^{(k)})\,\delta x^{(k)} = -f(x^{(k)})$$

$$\Rightarrow \delta x^{(k)} = x^{(k+1)} - x^{(k)}$$

$$x^{(k+1)} = \delta x^{(k)} + x^{(k)}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix} \qquad J = \begin{bmatrix} 2x & 2y & 2z \\ 1 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} f \\ g \\ h \end{bmatrix}\Bigg|_{(x^{(0)})} = \begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix} = f(x^{(0)}), \quad J(x^{(0)}) = \begin{bmatrix} 4 & 0 & 4 \\ 1 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 0 & 4 \\ 1 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix} \delta x^{(0)} = \begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 4 & 0 & 4 & -2 \\ 1 & 2 & 0 & 0 \\ 1 & 0 & 3 & -1 \end{array}\right] \xrightarrow[R_3 \to R_3 - \frac{1}{4}R_1]{R_2 \to R_2 - \frac{1}{4}R_1} \left[\begin{array}{ccc|c} 4 & 0 & 4 & -2 \\ 0 & 2 & -1 & \frac{1}{2} \\ 0 & 0 & 2 & -\frac{1}{2} \end{array}\right]$$

$$\Rightarrow \begin{cases} 4x_0 + 4x_2 = -2 \\ 2x_1 - x_2 = \frac{1}{2} \\ 2x_2 = -\frac{1}{2} \end{cases} \Rightarrow \delta(x^{(0)}) = \begin{bmatrix} \frac{1}{4} \\ \frac{1}{8} \\ -\frac{1}{4} \end{bmatrix}$$

$$x^{(1)} = \delta x^{(0)} + x^{(0)}$$

$$x^{(1)} = \begin{bmatrix} \frac{1}{4} \\ \frac{1}{8} \\ -\frac{1}{4} \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{9}{4} \\ \frac{1}{8} \\ \frac{7}{4} \end{bmatrix}$$

$$x^{(1)} = \begin{bmatrix} 9/4 & 1/8 & 7/4 \end{bmatrix}^T$$

$$\vdots$$

We still struggle with coding part, but we think we have understand how each iteration should work, which is everything we wrote above.

2.

x1=$(-1+x2+2x4)/4$
x2=$(x1+x3+2x5)/4$
X3=$(1+x2+2x6)/4$
X4=$(-2+x5+x1)/4$
X5=$(1+x2+x4+x6)/4$
x6=$(2+x3+x5)/4$

Jacobi

k	x1^(k)	x2^(k)	x3^(k)	x4^(k)	x5^(k)	x6^(k)
0	0	0	0	0	0	0
1	-0.25	0	0.25	-0.5	0.25	0.5
2	-0.375	0.125	0.5	-0.5	0.25	0.625
3	-0.3438	0.1562	0.5938	-0.5312	0.3125	0.6875

Gauss-seidel

k	x1^(k)	x2^(k)	x3^(k)	x4^(k)	x5^(k)	x6^(k)
0	0	0	0	0	0	0
1	-0.25	-0.0625	0.2344	-0.5625	0.0938	0.582
2	-0.4062	0.0039	0.542	-0.5781	0.252	0.6985
3	-0.3936	0.1631	0.64	-0.5354	0.3315	0.7429

3.
a)
f(0,0) = p(0,0)= a00

f(1,0) = p(1,0) = a00 + a10 + a20 + a30

f(0,1) = p(0,1) = a00 + a01 + a02 + a03

$f(1,1) = p(1,1) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij}$

$f_x(0,0) = p_x(0,0) = a_{10}$

$f_x(1,0) = p_x(1,0) = a_{10} + 2a_{20} + 3a_{30}$

$f_x(0,1) = p_x(0,1) = a_{10} + a_{11} + a_{12} + a_{13}$

$f_x(1,1) = p_x(1,1) = \sum_{i=1}^{3} \sum_{j=0}^{3} a_{ij} \cdot i$
$f_y(0,0) = p_y(0,0) = a_{01}$

$f_y(1,0) = p_y(1,0) = a_{01} + a_{11} + a_{21} + a_{31}$

$f_y(0,1) = p_y(0,1) = a_{01} + 2a_{02} + 3a_{03}$

$f_y(1,1) = p_y(1,1) = \sum_{i=0}^{3} \sum_{j=1}^{3} a_{ij} \cdot j$

$f_{xy}(0,0) = p_{xy}(0,0) = a_{11}$

$f_{xy}(1,0) = p_{xy}(1,0) = a_{11} + 2a_{21} + 3a_{31}$

$f_{xy}(0,1) = p_{xy}(0,1) = a_{11} + 2a_{12} + 3a_{13}$

$f_{xy}(1,1) = p_{xy}(1,1) = \sum_{i=1}^{3} \sum_{j=1}^{3} a_{ij} \cdot ij$

b)