Section 1 Question 2:

$$f(x,y,z) = x^2+y^2+z^2-10=0 \qquad \overset{k}{x^{(0)}} = [2\ 0\ 2]^T$$
$$g(x,y,z) = x+2y \quad -2=0$$
$$h(x,y,z) = x+ \quad 3z-9=0$$

$$\Rightarrow \underbrace{\begin{bmatrix} f \\ g \\ h \end{bmatrix}\Bigg|_{(x_n,y_n,z_n)}}_{f(x^{(k)})} + \underbrace{\begin{bmatrix} df/dx & df/dy & df/dz \\ dg/dx & dg/dy & dg/dz \\ dh/dx & dh/dy & dh/dz \end{bmatrix}\Bigg|_{(x_n,y_n,z_n)}}_{J} \underbrace{\begin{bmatrix} x_{n+1}-x_n \\ y_{n+1}-y_n \\ z_{n+1}-z_n \end{bmatrix}}_{\delta x^{(k)}} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Let $x=x_1$, $y=x_2$, $z=x_3$.

$$\Rightarrow J(x^{(k)})\delta x^{(k)} = -f(x^{(k)})$$

$$\Rightarrow \delta x^{(k)} = x^{(k+1)} - x^{(k)}$$

$$x^{(k+1)} = \delta x^{(k)} + x^{(k)}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix} \qquad J = \begin{bmatrix} 2x & 2y & 2z \\ 1 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} f \\ g \\ h \end{bmatrix}\Bigg|_{(x^{(0)})} = \begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix} = f(x^{(0)}), \quad J(x^{(0)}) = \begin{bmatrix} 4 & 0 & 4 \\ 1 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 0 & 4 \\ 1 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix} \delta x^{(0)} = \begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 4 & 0 & 4 & -2 \\ 1 & 2 & 0 & 0 \\ 1 & 0 & 3 & -1 \end{array}\right] \xrightarrow[R_3 \to R_3 - \frac{1}{4}R_1]{R_2 \to R_2 - \frac{1}{4}R_1} \left[\begin{array}{ccc|c} 4 & 0 & 4 & -2 \\ 0 & 2 & -1 & \frac{1}{2} \\ 0 & 0 & 2 & -\frac{1}{2} \end{array}\right]$$

$$\Rightarrow \begin{cases} 4x_0 + 4x_2 = -2 \\ 2x_1 - x_2 = \frac{1}{2} \\ 2x_2 = -\frac{1}{2} \end{cases} \Rightarrow \delta(x^{(0)}) = \begin{bmatrix} \frac{1}{4} \\ \frac{1}{8} \\ -\frac{1}{4} \end{bmatrix}$$

$$x^{(1)} = \delta x^{(0)} + x^{(0)}$$

$$x^{(1)} = \begin{bmatrix} \frac{1}{4} \\ \frac{1}{8} \\ -\frac{1}{4} \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{9}{4} \\ \frac{1}{8} \\ \frac{7}{4} \end{bmatrix}$$

$$x^{(1)} = \begin{bmatrix} 9/4 & 1/8 & 7/4 \end{bmatrix}^T$$

$\vdots$

We still struggle with coding part, but we think we ~~are~~ have understand how each iteration ~~of~~ should work, which is everything we wrote above.