Shixin Wang
Henry Boateng
2/23/2020

Homework 3

1. $f(x,y) = y - \cos^2 x = 0$; $g(x,y) = x^2 + y^2 - x - 2 = 0$

$(x_0, y_0) = (2, 0.2)$

$df/dx = -2(\cos x)\sin x = -\sin 2x$ $\qquad df/dy = 1$

$dg/dx = 2x - 1$ $\qquad dg/dy = 2y$

$$\Rightarrow J(x_n, y_n) = \begin{bmatrix} -\sin 2x & 1 \\ 2x-1 & 2y \end{bmatrix}$$

STEP 1:
$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} - \begin{bmatrix} -\sin 2x_0 & 1 \\ 2x_0 - 1 & 2y_0 \end{bmatrix}^{-1} \begin{bmatrix} f(x_0, y_0) \\ g(x_0, y_0) \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0.2 \end{bmatrix} - \begin{bmatrix} -\sin 4 & 1 \\ 3 & 0.4 \end{bmatrix}^{-1} \begin{bmatrix} 0.026821804 \\ 0.04 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 0.2 \end{bmatrix} - \begin{pmatrix} -0.14829 & 0.37074 \\ 1.11223 & -0.28057 \end{pmatrix} \begin{bmatrix} 0.026821 \\ 0.04 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 1.98914785 \\ 0.1813911236 \end{bmatrix}$$

STEP 2:
$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1.98914785 \\ 0.1813911236 \end{bmatrix} - \begin{bmatrix} 0.7424384829 & 1 \\ 2.9782957 & 0.3627822472 \end{bmatrix}^{-1} \begin{bmatrix} 0.01634824774 \\ 4.640588805 \times 10^{-4} \end{bmatrix}$$

$$= \begin{bmatrix} 1.991165898 \\ 0.1635445999 \end{bmatrix}$$

Hence:

when $\gamma = 0$: $(x_0, y_0) = (2, 0.2)$
$f(x_0, y_0) = 0.026821804$
$g(x_0, y_0) = 0.04$

when $\gamma = 1$: $(x_1, y_1) = (1.98914785, 0.1813911236)$
$f(x_1, y_1) = 0.01634824774$
$g(x_1, y_1) = 4.640588805 \times 10^{-4}$

when $\gamma = 2$: $(x_2, y_2) = (1.991165898, 0.1635445999)$
$f(x_2, y_2) = -0.002999276973 5$
$g(x_2, y_2) = 3.225715146 \times 10^{-4}$

2. $(0.1_{10})_{10} \approx (0.0001100110011001100110)_2$

a) $(0.00011001100110011001100)_2 = 2^{-4} + 2^{-5} + 2^{-8} + 2^{-9} + 2^{-12} + 2^{-13} + 2^{-16} + 2^{-17}$
$+ 2^{-20} + 2^{-21}$
$= 0.099999904_6$

$x = 9.99999046 \times 10^{-2}$

b) Absolute Error $= |0.1 - x|$
$= 9.5367432 \times 10^{-8}$

Hence, the absolute error is 9.5367432×10^{-8}

c) Time Error $= |360000 - 3600000x|$
$= 0.34344$

Hence, the time error after 100 hours is 0.343744

d) 3750 mph $= 3750$ m/h $\approx 1.041666666666666667$ m/s

Distance $=$ Time Error $\times 1.041666666666666667$

$= 0.35775$ m

Hence, the distance will be 0.35775 mile during the time error 0.34344.