Tower of Hanoi

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University of Miami

CSC220 Programming II – Spring 2022

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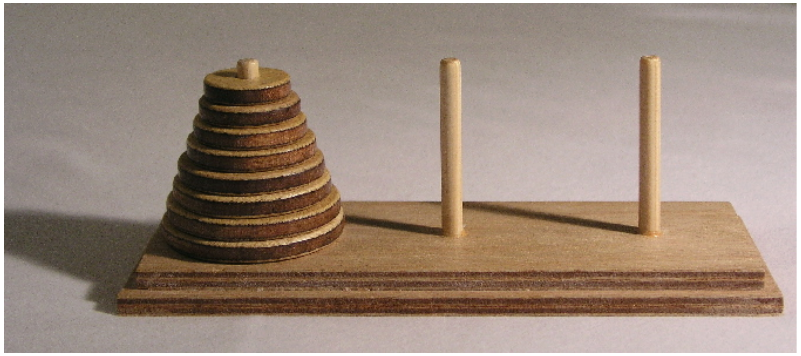

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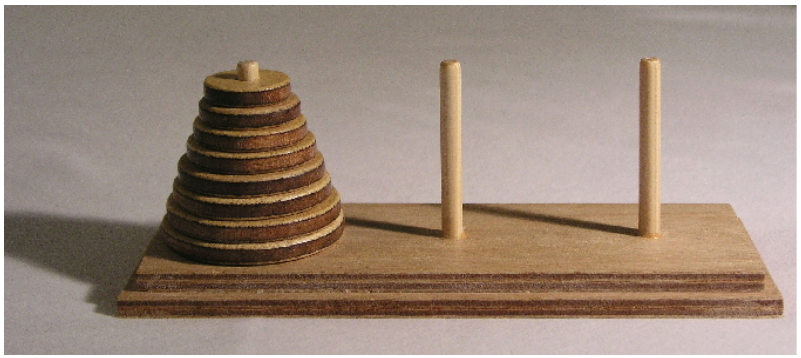

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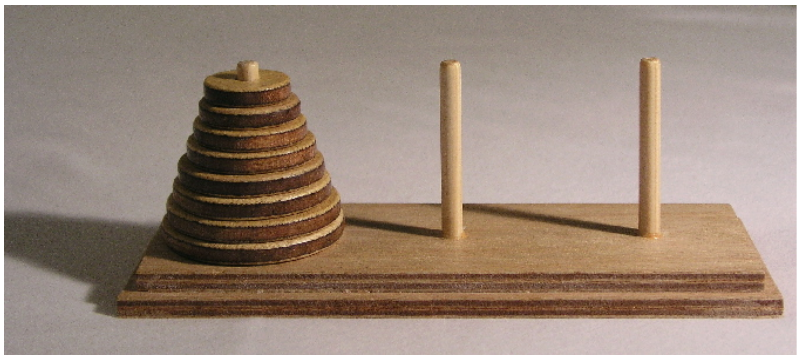

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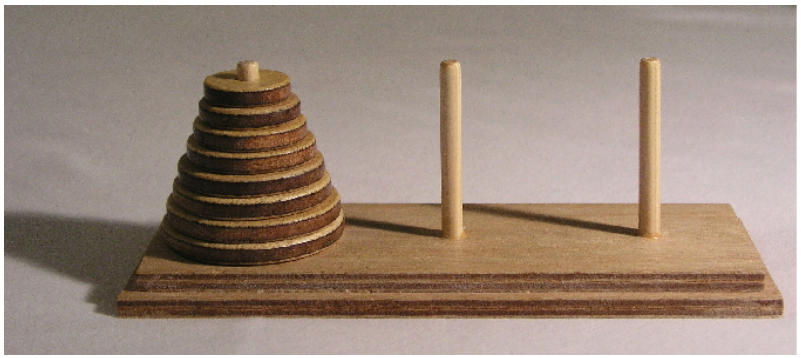

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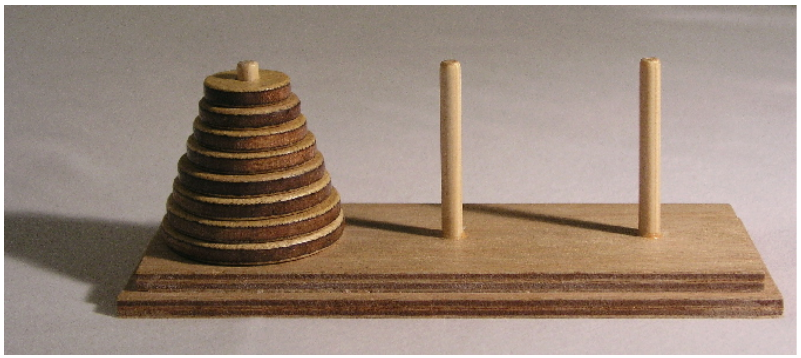

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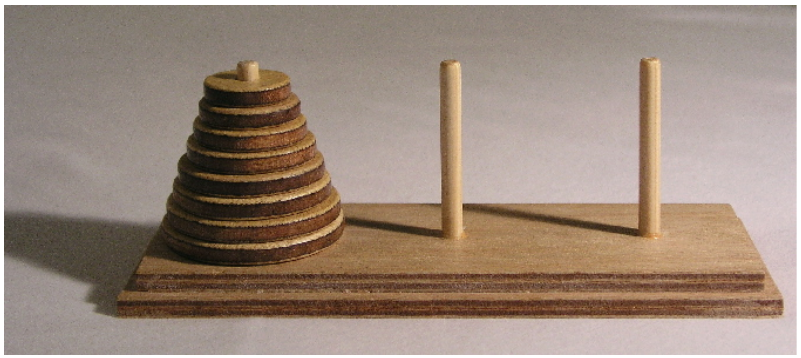

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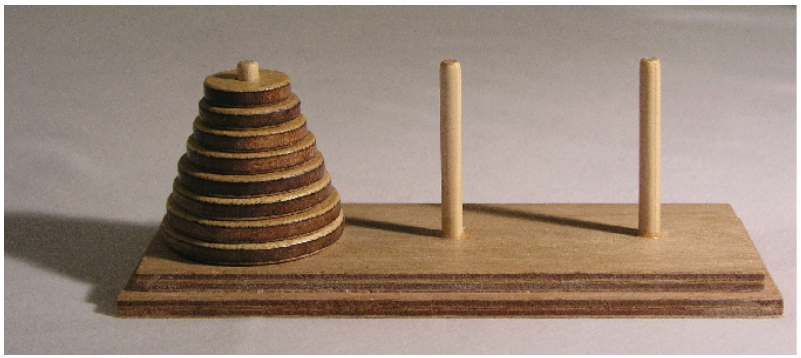

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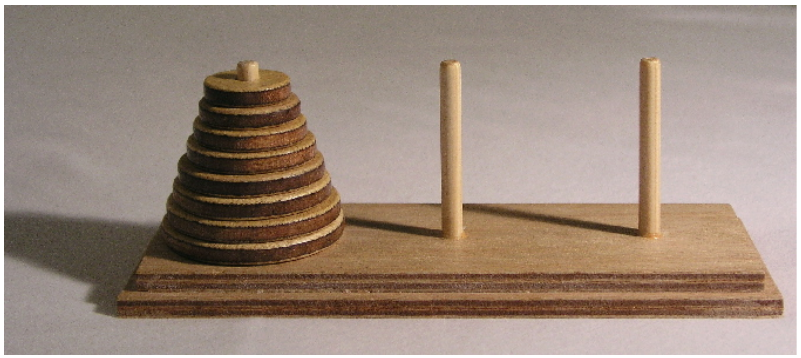

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Here is a good place to practice.

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  - pop from helper, print it, push on a

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    - and push the submoves onto the stack.

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- Any move with one disk, such as 1ab, is easy, and we can do it right away.
- All others can be broken into three submoves.

4-disk Solution

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- Move stack:

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4-disk Solution

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- And the stack is StackInterface<Move>.
- Since a MoveN is a Move, it can go into the stack.

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  - 292 billion years.

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Summary

- Tower of Hanoi has a natural representation using three StackInterface<Integer>.
- Printing or displaying uses a *helper* stack.
- Solving Tower of Hanoi requires *top-down* planning.
- Which we can implement using a StackInterface<Move>.
- This solution uses $O(2^n)$ time.