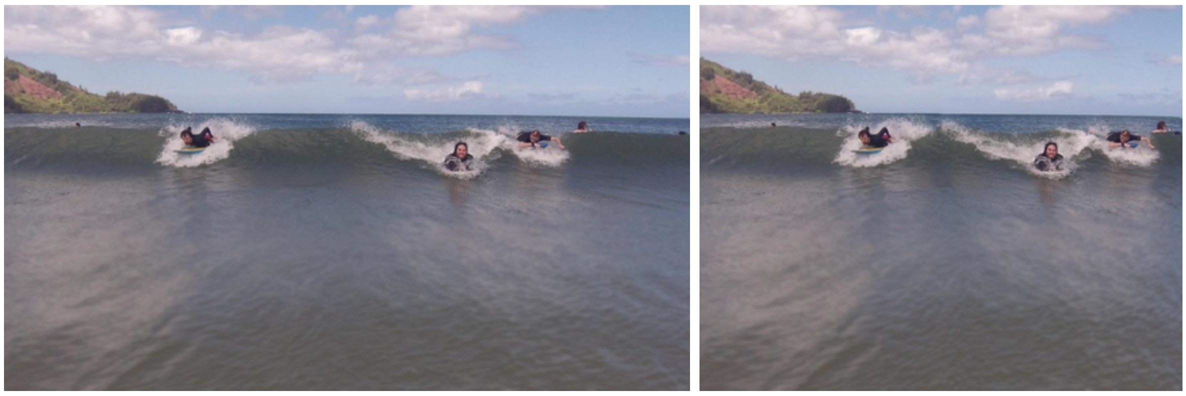
CSE317 Design & Analysis of Algorithms (Spring'19)

Assignment #1: Seam Carving

Max Marks: 20 *Due:* Mar 29

You are allowed to collaborate with other students provided that you do not exchange any written code. Write the solution on your own and mention the names of students you have collaborated with. Failure to do so may earn you a zero in this assignment.

Seam-carving is a content-aware image resizing technique where the image is reduced in size by one pixel of height (or width) at a time. A vertical seam in an image is a path of pixels connected from the top to the bottom with one pixel in each row; a horizontal seam is a path of pixels connected from the left to the right with one pixel in each column. Below left is the original 505-by-287 pixel image; below right is the result after removing 150 vertical seams, resulting in a 30% narrower image. Unlike standard content-agnostic resizing techniques (such as cropping and scaling), seam carving preserves the most interest features (aspect ratio, set of objects present, etc.) of the image.
Although the underlying algorithm is simple and elegant, it was not discovered until 2007 by Shai Avidan and Ariel Shamir. It is now a core feature in Adobe Photoshop and other computer graphics applications.



In this assignment, you will create a data type that resizes a W-by-H image using the seam-carving technique. Finding and removing a seam involves three parts and a tiny bit of notation:

0. *Notation.* In image processing, pixel (x, y) refers to the pixel in column x and row y, with pixel $(0, 0)$ at the upper-left corner and pixel $(W - 1, H - 1)$ at the lower-right corner. This is consistent with the `Picture` data type supplied as `Picture.java`.

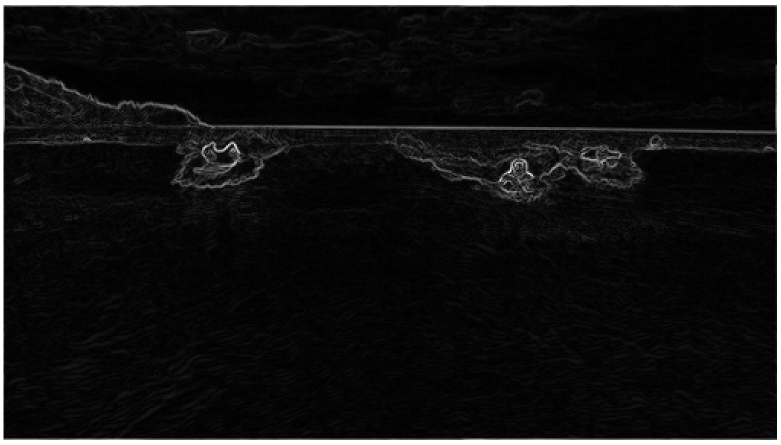
a 3-by-4 image

(0,0)	(1,0)	(2,0)
(0,1)	(1,1)	(2,1)
(0,2)	(1,2)	(2,2)
(0,3)	(1,3)	(2,3)

Warning: this is the opposite of the standard mathematical notation used in linear algebra, where (i, j) refers to row i and column j and $(0, 0)$ is at the lower-left corner. We also assume that the color of each pixel is represented in RGB space, using three integers between 0 and 255. This is consistent with the `java.awt.Color` data type

1. *Energy calculation.* The first step is to calculate the *energy* of a pixel, which is a measure of its importance–the higher the energy, the less likely that the pixel will be included as part of a seam (as you will see in the next step). In this assignment, you will use the *dual-gradient energy function*, which is described below.

   Here is the dual-gradient energy function of the surfing image above:

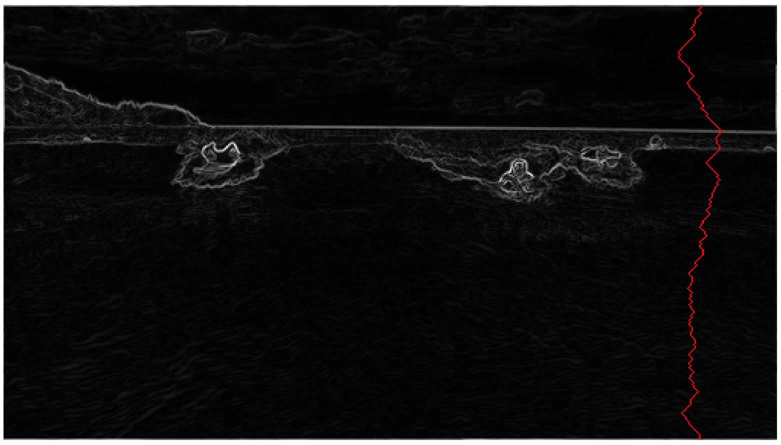
   

   The energy is high (white) for pixels in the image where there is a rapid color gradient (such as the boundary between the sea and sky and the boundary between the surfing Josh Hug on the left and the ocean behind him). The seam-carving technique avoids removing such high-energy pixels.

2. *Seam identification.* The next step is to find a vertical seam of minimum total energy. (Finding a horizontal seam is analogous.) This is similar to the classic shortest path problem in an edge-weighted digraph, but there are three important differences:

   - The weights are on the vertices instead of the edges.
   - The goal is to find the shortest path from any of the W pixels in the top row to any of the W pixels in the bottom row.
   - The digraph is acyclic, where there is a downward edge from pixel (x, y) to pixels $(x-1, y+1)$, $(x, y+1)$, and $(x+1, y+1)$; assuming that the coordinates are in the prescribed ranges.

   Seams cannot wrap around the image (e.g., a vertical seam cannot cross from the leftmost column of the image to the rightmost column).

3. *Seam removal.* The final step is remove from the image all of the pixels along the vertical or horizontal seam.

The `SeamCarver` API. Your task is to implement the following mutable data type:

```
public class SeamCarver {
   public SeamCarver(Picture picture)    // create a seam carver object
   public Picture picture()              // current picture
   public int width()                    // width of current picture
   public int height()                   // height of current picture
   public double energy(int x, int y)    // energy of pixel at column x and row y
   public int[] findHorizontalSeam()     // sequence of indices for horizontal seam
   public int[] findVerticalSeam()       // sequence of indices for vertical seam
   public void removeHorizontalSeam(int[] seam) //remove horizontal seam from current
      picture
   public void removeVerticalSeam(int[] seam)   //remove vertical seam from current
      picture
   public static void main(String[] args)       // do unit testing of this class
}
```

- **Corner cases.** Your code should throw an exception when a constructor or method is called with an invalid argument, as documented below:
  - By convention, the indices x and y are integers between 0 and $W-1$ and between 0 and $H-1$, respectively, where W is the width and H is the height of the current image.
    Throw a `java.lang.IndexOutOfBoundsException` if `energy()` is called with either an x-coordinate or y-coordinate outside its prescribed range.
  - Throw a `java.lang.NullPointerException` if the constructor, `removeVerticalSeam()`, or `removeHorizontalSeam()` is called with a null argument.
  - Throw a `java.lang.IllegalArgumentException` if either `removeVerticalSeam()` or `removeHorizontalSeam()` is called with an array of the wrong length or if the array is not a valid seam (either an entry is outside the height/width bounds or two adjacent entries differ by more than 1).
  - Throw a `java.lang.IllegalArgumentException` if either `removeVerticalSeam()` or `removeHorizontalSeam()` is called when the width or height of the current picture is 1, respectively.
- **Constructor.** The data type may not mutate the Picture argument to the constructor.
- **Computing the energy of a pixel.** You will use the dual-gradient energy function: The energy of pixel (x, y) is $\sqrt{\Delta_x^2(x,y) + \Delta_y^2(x,y)}$, where the square of the x-gradient $\Delta_x^2(x,y) = R_x(x,y)^2 + G_x(x,y)^2 + B_x(x,y)^2$, and where the central differences $R_x(x,y)$, $G_x(x,y)$, and $B_x(x,y)$ are the differences in the red, green, and blue components between pixel $(x+1, y)$ and pixel $(x-1, y)$, respectively. The square of the y-gradient $\Delta_y^2(x,y)$ is defined in an analogous manner. To handle pixels on the

borders of the image, calculate energy by defining the leftmost and rightmost columns as adjacent and the topmost and bottommost rows as adjacent. For example, to compute the energy of a pixel $(0, y)$ in the leftmost column, use its right neighbor $(1, y)$ and its "left" neighbor $(W - 1, y)$.

As an example, consider the 3-by-4 image (supplied as `3x4.png`) with RGB values—each component is an integer between 0 and 255 – as shown in the table below:

RGB values for pixel (1, 2)

(255, 101, 51)	(255, 101,153)	(255, 101, 255)
(255, 153, 51)	(255, 153, 153)	(255, 153, 255)
(255, 203, 51)	$\boxed{(255, 204, 153)}$	(255, 205, 255)
(255, 255, 51)	(255, 255, 153)	(255, 255, 255)

a 3-by-4 image (RGB values)

energy of pixel (1, 2)

$\sqrt{20808}$	$\sqrt{52020}$	$\sqrt{20808}$
$\sqrt{20808}$	$\sqrt{52225}$	$\sqrt{21220}$
$\sqrt{20809}$	$\sqrt{52024}$	$\sqrt{20809}$
$\sqrt{20808}$	$\sqrt{52225}$	$\sqrt{21220}$

dual-gradient energies

– *Non-border pixel example.* The energy of pixel $(1, 2)$ is calculated from pixels $(0, 2)$ and $(2, 2)$ for the x-gradient

$$R_x(1,2) = 255 - 255 = 0,$$
$$G_x(1,2) = 205 - 203 = 2,$$
$$B_x(1,2) = 255 - 51 = 204,$$

yielding $\Delta_x^2(1,2) = 2^2 + 204^2 = 41620$; and pixels $(1, 1)$ and $(1, 3)$ for the y-gradient

$$R_y(1,2) = 255 - 255 = 0,$$
$$G_y(1,2) = 255 - 153 = 102,$$
$$B_y(1,2) = 153 - 153 = 0,$$

yielding $\Delta_y^2(1,2) = 102^2 = 10404$.

Thus, the energy of pixel $(1, 2)$ is $\sqrt{41620 + 10404} = \sqrt{52024}$. Similarly, the energy of pixel $(1, 1)$ is $\sqrt{204^2 + 103^2} = \sqrt{52225}$.

– *Border pixel example.* The energy of the border pixel $(1, 0)$ is calculated by using pixels $(0, 0)$ and $(2, 0)$ for the x-gradient

$$Rx(1,0) = 255 - 255 = 0,$$
$$Gx(1,0) = 101 - 101 = 0,$$
$$Bx(1,0) = 255 - 51 = 204,$$

yielding $\Delta_x^2(1,0) = 204^2 = 41616$; and pixels $(1,3)$ and $(1,1)$ for the y-gradient

$$\begin{aligned} Ry(1,0) &= 255 - 255 = 0, \\ Gy(1,0) &= 255 - 153 = 102, \\ By(1,0) &= 153 - 153 = 0, \end{aligned}$$

yielding $\Delta_y^2(1,2) = 102^2 = 10404$.
Thus, the energy of pixel $(1,0)$ is $\sqrt{41616 + 10404} = \sqrt{52020}$.

- **Finding a vertical seam.** The `findVerticalSeam()` method returns an array of length H such that entry y is the column number of the pixel to be removed from row y of the image. For example, the dual-gradient energies of a 6-by-5 image (supplied as `6x5.png`) are shown in the table below.

energy of seam = 159.43 + 107.89 + 133.07 + 174.01 + 70.06 = 644.47

240.18	225.59	302.27	**159.43**	181.81	192.99
124.18	237.35	151.02	234.09	**107.89**	159.67
111.10	138.69	228.10	**133.07**	211.51	143.75
130.67	153.88	**174.01**	284.01	194.50	213.53
179.82	175.49	**70.06**	270.80	201.53	191.20

the minimum energy vertical seam in a 6-by-5 image

The minimum energy vertical seam is highlighted in blue. In this case, the method `findVerticalSeam()` should return the array $\{3,4,3,2,2\}$ because the pixels in the minimum energy vertical seam are $(3,0)$, $(4,1)$, $(3,2)$, $(2,3)$, and $(2,4)$. Remember that seams cannot wrap around the image. When there are multiple vertical seams with minimal total energy, your method can return any such seam.

- **Finding a horizontal seam.** The behavior of `findHorizontalSeam()` is analogous to that of `findVerticalSeam()` except that it returns an array of length W such that entry x is the row number of the pixel to be removed from column x of the image. For the 6-by-5 image, the method `findHorizontalSeam()` should return the array $\{2,2,1,2,1,2\}$ because the pixels in the minimum energy horizontal seam are $(0,2)$, $(1,2)$, $(2,1)$, $(3,2)$, $(4,1)$, and $(5,2)$.

energy of seam = 111.10 + 138.69 + 151.02 + 133.07 + 107.89 + 143.75 = 785.53

240.18	225.59	302.27	159.43	181.81	192.99
124.18	237.35	**151.02**	234.09	**107.89**	159.67
111.10	**138.69**	228.10	**133.07**	211.51	**143.75**
130.67	153.88	174.01	284.01	194.50	213.53
179.82	175.49	70.06	270.80	201.53	191.20

the minimum energy horizontal seam in a 6-by-5 image

- **Performance requirements.** The `width()`, `height()`, and `energy()` methods should take constant time in the worst case. All other methods should run in time proportional to $W \times H$ (or better) in the worst case.

- Submit `SeamCarver.java`, and any other files needed by your program to LMS. You may not call any library functions other than those in `java.lang`, `java.util`, and `java.awt.Color`.
- Finally, submit a `readme.txt` file describing concisely your algorithm to compute the horizontal and vertical seam. Also give the running times to remove one row and one column from a W-by-H image as a function of W, and H.