Introduction

Public transport is the backbone of cities which aspire at being accessible to all its residents and beyond. A well-constructed public transport network allows the community to have better access to most of the elements which drive economic growth such as employment, education, and health sectors. According to the United States Department of Transportation, over 90% of the public transport users do not own any form of transportation[1] such as cars, and therefore heavily rely on the public transport network. And in alignment with the federation of Canadian Communities idea that "Better transit means less congestion, faster commutes, more convenience, higher productivity, and lower emissions. Simply put: public transit builds better lives[2]" we had the idea of rethinking a service our team member use on a daily basis – Montreal bus network. For the purpose of this project, we are aiming to optimize the operational cost of the bus network in downtown Montreal.

The Montreal Bus Route Problem is an example of vehicle routing problem, an optimization method widely used to address public transport planning. The vehicle routing problem (VRP) is a generalization of the Travelling Salesman Problem (TSP). The TSP optimization problem investigates the optimal route a salesman should take given a list of locations to visit and the distances between location pairs. The Montreal Bus Route Problem is a multiple-vehicle routing problem as multiple routes buses can take was included in our problem formulation.

This optimization problem is a minimization of the operational cost of Montreal bus fleet and the Gurobi package was used to solve the problem in Python. The data on the latitude and longitude of bus stop location used for this optimization problem was retrieved from the Société de Transport de Montreal (STM) and processed by overlying it with road network data using ArcGIS.

This report is organized in the following fashion: I) the problem description and formulation, II) Numerical Implementation and Solution III) Challenges Encountered & Problem Extensions.

[1] U.S Department of Transportation Federal Highway Administration, (2002), The Importance of Public Transportation, https://www.fhwa.dot.gov/policy/2002cpr/pdf/ch14.pdf

[2] Federation of Canadian Municipalities, (2021), https://fcm.ca/en/focus-areas/public-transit/building-better-public-transit

Problem Description and Formulation

The Montreal Bus Network Problem aims to find the solutions in order to minimize the total number of buses k required to serve all the bus stops in the Montreal downtown area. The identified problem is an example of the Vehicle Routing problem, which is a widely use method for optimizing public transport planning. The Vehicle Routing Problem aims at finding the optimal set of routes for a given fleet of vehicles. This type of problem finds its roots in the widely known Travelling Salesman problem. The problem explored in this report is a multiple vehicle routing problem. To take a more realistic approach to problem solving, this problem includes a fleet of up to 15 buses and 200 bus stops.

The Montreal Bus Network problem can be thought of as a graph with a set of V bus stops with A routes connecting stops (i, j) for a total of n number of stops. c_{ij} is the distance matrix with each bus stops (i,j) $\in$ A. Let L be the maximum number of stops visited by a single bus and K be the minimum number of stops each bus can visit. We can have at most m_max number of buses and at least m_min number of buses that we can operate in our road network.

Parameters:

i,j: Bus stops

c_{ij}: Travel Distance for route between stops (i,j)

L: Maximum Number of Stops Visited by a single bus

K: Minimum Number of Stops Visited by a single bus

V: Set of bus stop combination

A: Set of routes between stops (i,j)

n: Number of stops

m_max: Maximum number of buses we can operate

m_min: Minimum number of buses we can operate

q = Cost of travelling unit distance

b = Cost of procuring a bus

r = Times we traverse the network

Decision Variables:

x_{ij} = Route to be taken visited from stop i to stop j, 0 if we don't take this route otherwise 1

u_i = Dummy variable that allows elimination of sub-tours for a vehicle

Objective Function: In order to minimize the cost of serving all the bus stops in the Montreal downtown area, we developed the following function which includes two summation terms, respectively representing the cost of visit all the stops and the cost of procurement the required number of buses.

$$\text{Minimize} \sum_{(i,j)\in A}^{n} c_{ij}x_{ij}qr + \sum_{j\in A}^{n} bx_{1j}$$

Constraints:

(1) At least m_min buses should start and end their route at the bus terminal – Bonaventure Station.

$$\sum_{j=2}^{n} x_{1j} >= m\_min$$

$$\sum_{j=2}^{n} x_{j1} >= m\_min$$

(2) At most m_max buses should end their route at the bus terminal – Bonaventure Station.

$$\sum_{j=2}^{n} x_{1j} <= m\_max$$

$$\sum_{j=2}^{n} x_{j1} <= m\_max$$

(3) Each stop should be entered only once

$\sum_{i=1}^{n} x_{ij} = 1;$ j = 2 . . . n;

(4) Each stop should be exited only once

$\sum_{j=1}^{n} x_{ij} = 1;$ i = 2 . . . n;

(5) Maximum number of stops a bus can visit – this constraint sets the upper bounds for the number of bus stops visited by each bus*

$u_i + (L-2)x_{1i} - x_{i1} <= L-1; \qquad i = 2 \ldots n;$

(6) Minimum number of stops a bus can visit - this constraint sets the lower bounds for the number of bus stops visited by each bus*

$u_i + x_{1i} + (2-K)x_{i1} >= 2; \qquad i = 2 \ldots n;$

(7) To prevent a bus from coming back to the starting depot after visiting one node*

$x_{1i} + x_{i1} \leq 1;$

(8) Sub-tour elimination constraint – this constraint prevents the formation of subtour between bus stops, by ensuring that $u_j = u_i + 1$ happens only if $x_{ij} = 1$.

$u_i - u_j + Lx_{ij} + (L-2)x_{ji} <= L-1; \qquad 2 \leq i \neq j \leq n$

(9) Stops can either be visited (value: 1) or not (value: 0)

$x_{ij} \in \{0,1\}; \qquad \forall(i,j) \in A$

(10) One of the buses should visit the stops along the old port.

$x_{i(i+1)} = 1; \qquad i \in \mathrm{Index}[60868, 60720, 60719, 60726, 60728, 60717]$

(11) A single route must cover the direct route from St Antoine/ de la Cathédrale (53696) as it is the bus stop the nearest to the suburbs trains terminal to Decelles/ Queen Mary (51278) as it is near multiple large universities including HEC Montreal and Université de Montreal.

$x_{ij} = 1; \qquad i \in Index[53696], \ j \in \mathrm{Index}[51278]$

* Constraint (7) doesn't allow $x_{1i} = x_{i1} = 1$. We have the following 3 considerations:

1. If $x_{1i} = x_{i1} = 0$, constraints (5) and (6) impose $2 \leq u_i \leq L-1$. This is the condition for non-visited stops from the depot.
2. If $x_{1i} = 1$ and $x_{i1}=0$, we have $u_i = 1$ as constraints (5) and (6) impose $1 \geq u_i$ and $1 \leq u_i$. This is for all cases where the bus leaves the depot and doesn't come back to the depot from the stop.

3. If $x_{1i} = 0$ and $x_{i1}=1$, constraints (5) and (6) impose $K \leq u_i \leq L$. This means that a bus can visit any number of total stops u_i between K and L for all cases where the bus doesn't return to the depot from the first stop that it visited.

*Constraint (7) becomes redundant for K>=4 as (5) and (6) already serve this purpose.

*Constraints (5) and (6) are valid for $2 <= K <= \frac{n-1}{m\_max}$

Numerical Implementation, Challenges, and Solution:

Cost of Travel Data. The Montreal Bus Network Problem requires data collection of geographic points to accurately locate the bus stops' localization and distances between bus stops in the Montreal bus network. The localization of each bus stops is captured using the latitude and longitude of each stop and are available for public use on the Société de Transport de Montreal's website. From this website geographic localization for 8500 bus stops in Montreal were retrieved. To accurately capture the measurement of the distance between bus stops given real life road constraints such as road speed limit and one-way streets, we obtained the mapping of the Montreal road network from McGill library. We then used the ArcGIS software to overlap the road network with the longitude and latitudes of the data points. The data were loaded on ArcGIS which overlaid the bus stops on the map and used the software to obtain many-to-many mapping of the distances and time it would take to travel from one stop to another. ArcGIS allowed us to have a clear picture of the road network and bus stops' locations, to subsequently feed this processed information to our model on Gurobi to calculate the optimal distance it would take from bus stops based on road speed limit. Only part of the data collected was used in the Montreal Bus Network problem as the local machine used to run it could not support such large data along with the numerous constraints addressed above. In the interest of coherence with constraints 10 and 11, we narrowed data input to Montreal downtown, which includes 200 of bus stops. Additionally, we determined the hours of operations per days, the number of hours per route, and the total number of days. The hours of operation were determined based on STM's available information whereby the bus network operates from 5 a.m. to 12a.m. every day – which means a total of 19 hours of operation each day[3] and the number of minutes per routes were estimated to be 15 mins. We have formulated the

[3] Note, the hours of operations are excluding all days where bus network is disrupted due to public holidays or other unexpected cause(s).

problem to minimize cost for a run-time of 6 months.

Cost of Bus Procurement Data. The cost of bus procurement includes the cost of buying each bus along with the operational cost – excluding fuel cost. The values were estimated based on the available information in Magder's article[4]. The buses will actually cost the Société de transport de Montreal CA$ 1.13 million per bus based on the minutes of the June meeting of the STM's board of directors. In addition, the STM is buying 530 replacements and will pay CA$ 941.2 million for the 830 buses in total. The buses were ordered as part of a group purchase with eight other Quebec cities, and the total contract for 1,525 buses will cost CA$ 2.02 billion.

Problem Formulation: There are many formulations of Travelling Salesman Problem and Vehicle Routing Problem, we therefore chose to formulate our problem by including serving all the stops present in Montreal while minimizing the cost of operating the buses. Since we do not have information about the usage of each stop, we are going to model this as a multiple TSP problem with the cost of introducing a new bus route to be added to the road network. In order to narrow the focus on this problem, we are assuming there is a single depot from which all the buses start from and must return to which is located at Bonaventure station. Our model is based on the Single Depot mTSP formulation[5] and the aforementioned formulation makes use of all the available salesmen (m) to traverse the routes as shown in the constraint below:

$$\sum_{j=2}^{n} x_{1j} = m$$

Model and constraints were inspired by Euchi & Mraihi's scientific article[6].

To address our needs, we replaced the above constraint with two constraints (1) and (2) (see constraints 1 and 2 under section I on page 5). This new formulation gives flexibility by having a minimum and maximum number of buses we want to operate in addition to the flexibility of having minimum and maximum number of stops a single bus can visit. Additionally, in order to have an

[4] Magder, J. (2018). Plante's pledged new hybrid buses to cost more than $1 million each, https://montrealgazette.com/news/local-news/plantes-pledged-new-hybrid-buses-to-cost-more-than-1-million-each

[5] Imdat Kara, Tolga Bektas, Integer linear programming formulations of multiple salesman problems and its variations, https://doi.org/10.1016/j.ejor.2005.03.008.

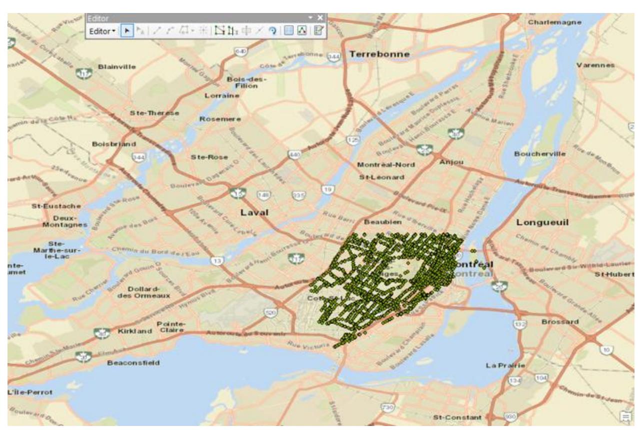
[6] Euchi, J., & Mraihi, R. (2012). The urban bus routing problem in the Tunisian case by the hybrid artificial ant colony algorithm. *Swarm and Evolutionary Computation*, *2*, 15–24. https://doi.org/10.1016/j.swevo.2011.10.002

optimization problem which captures real life constraints we added additional constraints. For example, we added constraint (10) which forces a bus route to make stops at the old port and constraint (9) allows for a route to service students travelling from the suburb station (Bonaventure Station) by taking them to the proximity of various universities like HEC Montreal and Université de Montréal. We have considered the cost to traverse the route for 6 months from 5a.m. till 12 a.m. every day.

Challenges:

Data: Obtaining a matrix of distances between the bus stops was a challenge as there was no already available dataset online for such data. To obtain accurate information, we overlaid the bus stops data collected on STM on the Montreal's road network provided by McGill library. We used OD Cost Matrix functionality in ArcMap for finding the shortest road distance between stops. The access to the latest road network data for Montreal was provided by McGill library. Since these were 8500 stops, this mapping generation took extensive time and generated a large .csv file of over 72 million records containing both time and distance mappings between stops. Processing this large dataset of 4.5 GB was an additional challenge and required dask arrays to process the data into a matrix of distance mappings. In order to obtain a solution, considering the available computational power, we reduced the number of stops to those close to downtown (200 stops):

Map with all bus stops



Map with downtown bus stops only

Obtaining a Solution: The travelling salesman problem is an NP-hard problem. For the purpose of this problem, we selected 200 stops and this would add 40,000 binary variables and 200 additional dummy integer variables. This was a challenge to run on the local machine. MIP problems are solved by using the branch and bound method which essentially relaxes the integrality constraint to obtain solutions. We used the following parameters in Gurobi to speed up the process and obtain a solution[7]:

1. PreSolve: Before running the branch and bound method, we apply transformations to reduce the size of the problem. PreSolve=2 was used to apply problem reduction.
2. Cuts: Cutting planes are used to remove undesirable fractional solutions during the solution process, tightening the formulation of the problem. This was set applied by setting Cuts=3.
3. MIPFocus: We noticed that the best objective bound (lower-bound) remained stagnant for the initial iterations that we ran the problem for so we set the parameter MIPFocus=3 so that Gurobi can focus on changing the bound.
4. MIPGap: This controls when we conclude our search for a better solution. This was set to 1% considering the time it took to get a subsequently better solution.

Additional Parameters. Given our computational capacity and the challenges described above, we narrowed down the scope of the problem and used the following parameters.

- **Number of Stops (n): 200** – Number of bus stops in selected area of Montreal.
- **Maximum number of stops visited by a single bus (L): 79** – based on the maximum number of bus stops per bus according to STM website.
- **Minimum number of stops a bus can visit (K): 4** – arbitrary number of buses assigned.
- **Maximum number of buses we can operate (m_max): 15** - Arbitrary number of buses. We can specify as many as we want, given that it does not violate the constraint 2<= K <= (n-1)/m_max.
- **Minimum number of buses we can operate (m_min): 5** - This value can be any number >= 2. We selected 5 as the lower bound of minimum buses we can operate to make sure we get a solution when running the problem, on our machine. Additionally, having higher gap of m_max and m_min made the solution very slow on Gurobi.

Optimization Results: The following results were found for 200 bus stops:

Bus*	Total Stops	Distance (km)
1	35	19.27

7 Gurobi Optimization, (2021), Mixed-Integer Programming: A Primer on the Basics https://www.gurobi.com/resource/mip-basics/

2	6	5.00
3	4	7.80
4	79	47.85
5	75	30.48

*For route details and each stops traversed refer to Appendix 1.

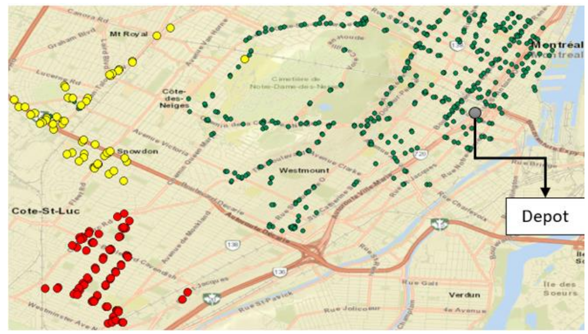
Total length of all bus routes: **124.67 km**

Total cost of operation for 6 months: **CA$ 7,080,716**

Fixed cost of purchasing 5 buses: **CA$ 5,650,000**

Cost of traversing the route for 6 months: **CA$ 1,430,716**

We notice that the stops on the two-longest bus routes are the farthest away from our depot. Perhaps dedicated routes for faraway stops are the best way to minimize travel costs.



Red and Yellow stops are the longest routes.

Problem Extensions

Problem-Specific Extensions. Based on the above-described challenges faced when collecting data, developing the model, and when running it on our local machine, below are the avenues for extensions on the selected topic.

1) Maintenance costs: Based on the available data for academic purposes, we found limited information on the value for maintenance cost associated with the hybrid buses which are currently operating by STM. Inclusion of this cost perhaps as a function of the time/routes traversed or the distance travelled by the bus could give a better estimate of the total cost of operating a bus based on its operational life.
2) Bus operational life: Buses on average have a life of 200,000 miles or 7 years[8]. This information could be incorporated as a constraint.
3) Multiple depots: Buses can have multiple depots in the city instead of starting at a single point[9].

[8] US Department of Transportation Federal Transit Administration. (2007). Useful Life of Transit Buses and Vans. https://www.transitwiki.org/TransitWiki/images/6/64/Useful_Life_of_Buses.pdf

[9] Computer and Operations Research. (2014). The multi-depot vehicle routing problem with heterogenous vehicle fleet: Formulation and a variable neighborhood search implementation. https://www.sciencedirect.com/science/article/abs/pii/S0305054813001408

4) Carrying Capacity & Passenger Demand: We can use the passenger demand: how many travellers use a particular bus stop to reformulate the problem. We can also add constraints to on how many passengers a bus can carry.

Alternative Solvers. It is very difficult to solve combinatorial optimization problems especially as the number of the decision variables & complexity increases. Gurobi is a derivative-based solver and uses branch and bound method to estimate the integer solutions, which takes a long time to compute a solution. Derivative-free methods can also be used to obtain solutions to the problems. Some of the commonly suggested alternatives to get faster solutions are: 1) Any Colony Optimization Algorithms, and 2) Genetic Algorithms[10].

Public Transport Industry Extensions. To reduce greenhouse gas emissions from the transportation industry, many cities are aiming to electrify their bus fleets to provide a greener public transportation system and improve air quality in these cities. For this reason, the Société de transport de Montreal has begun pilot projects to test the use of 100% electric buses in its fleet with the goal of eventually converting the entire fleet. Thus, starting in 2025, the STM plans to acquire only 100% electric buses to serve its network[11]. However, it is not possible to simply replace gasoline buses with electric buses and hope that the network continues to be served to the high standards of the customers. Indeed, because of the constraints that the batteries of these vehicles bring such as the limited autonomy and the long recharging time, it is necessary to rethink the whole planning of the initially formulated problem. The main concern for these types of routing problems is the energy consumption of electric buses. Thus, when formulating such problems, parameters affecting energy consumption such as the topography of the route and the profile of the driver must be considered. It is also necessary to consider the traffic during the day because electric vehicles consume more energy when they are driving faster and recover energy when they are stopped[12]. Once the factors influencing the energy consumption of electric buses have been taken into consideration, it is necessary to plan the optimal route that plans, when necessary, the stops for recharging and the time required to do so in order to minimize the time required to complete a

[10] Alhanjouri. M, Alfarrah. B, Ant Colony versus Genetic Algorithm based on Travelling Salesman Problem. https://tarjomefa.com/wp-content/uploads/2016/09/5281-English.pdf

[11] https://www.stm.info/fr/a-propos/grands-projets/grands-projets-bus/electrification-du-reseau-de-surface/bus-electriques

[12] Basso, R., Kulcsár, B., Egardt, B., Lindroth, P., & Sanchez-Diaz, I. (2019). Energy consumption estimation integrated into the Electric Vehicle Routing Problem. *Transportation Research Part D: Transport and Environment*, *69*, 141–167. https://doi.org/10.1016/j.trd.2019.01.006

route[13] and it is also important to minimize the number of stops required for recharging[14]. It is essential to ensure that buses never run out of power while completing a route, which requires planning as close to reality as possible.

Until recently, researchers did not consider the environmental aspect when formulating these problems. Since a few years, they start to integrate in the environmental constraints such as the minimization of greenhouse gases generated by vehicles[15]. As mentioned above, the optimization problem must be as close to reality as possible to avoid buses running out of energy to complete their path. To this end, Basso and al. developed a bus route planning model adapted for electric buses. First, they developed a linear model with coefficients for factors influencing the energy consumption of electric vehicles. Then, they developed a model that determines the optimal route that buses should take considering stops and charging times while minimizing the energy consumption[16]. To ensure that their models were as close to reality as possible, the researchers collected actual route data from electric buses in the city of Gothenburg, Sweden, to compare the energy consumption data of the routes using simulation[17].

Despite not being included in the research, we must consider other factors that are very important in terms of electric vehicles and apply them to the city like Montreal. First, while fast charging enables lower downtime times for buses, it also has a negative impact on the vehicle's battery. One should minimize the use of fast chargers to prevent battery degradation. To incorporate this constraint in the optimization model, one should also consider the cost of the loss in battery life during fast charging[18]. Another important factor to consider is the impact of the weather on the range of the electric buses, especially in a city like Montreal. Indeed, cold weather impacts directly on the battery capacity, which can reduce the range of electric vehicles

13 Basso, R., Kulcsár, B., Egardt, B., Lindroth, P., & Sanchez-Diaz, I. (2019). Energy consumption estimation integrated into the Electric Vehicle Routing Problem. *Transportation Research Part D: Transport and Environment*, *69*, 141–167. https://doi.org/10.1016/j.trd.2019.01.006

14 Widuch, J. (2019). Current and emerging formulations and models of real-life rich vehicle routing problems. In *Smart Delivery Systems: Solving Complex Vehicle Routing Problems* (pp. 1–35). Elsevier. https://doi.org/10.1016/B978-0-12-815715-2.00006-3

15 Basso, R., Kulcsár, B., & Sanchez-Diaz, I. (2021). Electric vehicle routing problem with machine learning for energy prediction. *Transportation Research Part B: Methodological*, *145*, 24–55. https://doi.org/10.1016/j.trb.2020.12.007

16 Basso, R., Kulcsár, B., Egardt, B., Lindroth, P., & Sanchez-Diaz, I. (2019). Energy consumption estimation integrated into the Electric Vehicle Routing Problem. *Transportation Research Part D: Transport and Environment*, *69*, 141–167. https://doi.org/10.1016/j.trd.2019.01.006

17 *Idem*

18 Widuch, J. (2019). Current and emerging formulations and models of real-life rich vehicle routing problems. In *Smart Delivery Systems: Solving Complex Vehicle Routing Problems* (pp. 1–35). Elsevier. https://doi.org/10.1016/B978-0-12-815715-2.00006-3

considerably. Also, in those temperatures, the usage of heating increases, causing even more energy consumption. Combining those two factors, we can impact the range of electric vehicles up to 40% of their initial range. Researchers should therefore take this information into consideration when designing their optimization problems to either review their optimal routes for winter or to have routes that work no matter the outside temperature.

Conclusion

This report addresses the path used to optimize the operational cost of the bus network in downtown Montreal. Our approach suggests the optimal bus routes and the number of buses needed to traverse all the existing bus stops in Montreal. In the interest of accurately capture the parameters which impact the cost associated, we collected data using the following sources: Société de Transport de Montreal, McGill Library, and Magder's *Plante's pledged new hybrid buses to cost more than $1 million each* article. To obtain a reasonable solution in a practical amount of time, we have used a subset of the entire Montreal stop data that comprises areas close to Downtown Montreal along with setting additional specific constraints. Our solution suggests 5 bus routes that serve all the bus stops. This approach can be scaled to include all the bus stops currently being used by the STM.

The main limitation for us in the project were the compute power and time required to solve this complex combination problem. To obtain a reasonable solution, we had to set the MIP Gap threshold to 1%. For real-world applications, there is usually a trade-off between a better solution and the time or processing power it takes to reach a better solution. We must solve the problem in a reasonable time while having a reasonably good optimal solution. It would require domain expertise judgment to suggest this threshold. The other challenge was to obtain an accurate estimate of the distances between the various stops in Montreal. While the locations of the bus stops were readily available on the STM website, free-to-use options to calculate the road distances were limited. Google Maps provides such as API. For the purpose of this project, we explored free GIS0based resources. The road network data was provided by McGill library, and this was fed to Geospatial Analysis Software to obtain a mapping of distances between all the Montreal bus stops.

In order to obtain and devise the plan for a better road network, this problem can be modelled as a multiple Vehicle Routing Problem where we consider the passenger carrying capacities as well. For practical reasons, not all the buses should start from a single starting point. Instead, we could explore a formulation which has multiple depots. For accurate cost estimates, we can incorporate operational costs, maintenance costs, costs of operating depots and cost to employ drivers.

Furthermore, as complexity of Mixed Integer Problems increases, branch and bound provide a very slow solution as they employ derivative-based approach. We can employ heuristic algorithms like ant-colony algorithms, genetic algorithms or K-Nearest Neighbours to obtain faster solutions while not necessarily reaching the best solution-as obtaining the most optimal solution is not a reasonable endeavour in practical use cases.

In the future, to further save energy and reduce pollution, electric buses will stand up to provide a greener public transportation system and improve air quality in the cities. However, replacing traditional gas buses with 100% electric buses based on the current network won't be realistic or be able to meet the high standards of the customers. Long recharging time and other energy consumption of the electric battery need to be taken care of. Cold temperature and potential harm from fast recharging can also negatively impact battery capacity and longevity.

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Appendix 1

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Routes Traversed:

Bus 1 (35 Stops, 19.27 km):
53696 > 51278 > 50789 > 50810 > 50807 > 50753 > 50721 > 50690 > 50654 > 50580 > 50582 > 50587 > 50583 > 50655 > 50691
> 50683 > 50663 > 50680 > 50629 > 50630 > 50631 > 50701 > 50722 > 50745 > 50744 > 50778 > 50777 > 50697 > 50738 > 507
37 > 50736 > 50818 > 50817 > 50819 > 50820 > 50791 > 53696

Bus 2 (6 Stops, 5.00 km):
53696 > 60868 > 60720 > 60719 > 60726 > 60728 > 60717 > 53696

Bus 3 (4 Stops, 7.80 km):
53696 > 50111 > 50112 > 50115 > 50123 > 53696

Bus 4 (79 Stops, 47.85 km):
53696 > 50114 > 50116 > 50104 > 50821 > 50822 > 50735 > 50746 > 50692 > 50694 > 50693 > 50608 > 50609 > 50581 > 50547
> 50548 > 50534 > 50533 > 50560 > 50476 > 50500 > 50532 > 50509 > 50531 > 50508 > 50599 > 50607 > 50598 > 50550 > 505
44 > 50584 > 50642 > 50641 > 50643 > 50714 > 50795 > 50728 > 50658 > 50660 > 50659 > 50703 > 50684 > 50702 > 50806 >
50790 > 50700 > 50628 > 50591 > 50510 > 50475 > 50435 > 50477 > 50478 > 50479 > 50397 > 50398 > 50436 > 50410 > 50377
> 50561 > 50633 > 50699 > 50780 > 50782 > 50781 > 50809 > 50808 > 50779 > 50698 > 50632 > 50573 > 50579 > 50682 > 507
48 > 50739 > 50676 > 50588 > 50675 > 50715 > 50552 > 53696

Bus 5 (75 Stops, 30.48 km):
53696 > 50796 > 50799 > 50798 > 50626 > 50604 > 50605 > 50603 > 50638 > 50637 > 50636 > 50618 > 50569 > 50570 > 50606
> 50627 > 50677 > 50752 > 50776 > 50827 > 50797 > 50762 > 50761 > 50725 > 50727 > 50724 > 50726 > 50688 > 50689 > 506
61 > 50662 > 50749 > 50783 > 50823 > 50826 > 50824 > 50825 > 50784 > 50750 > 50730 > 50729 > 50696 > 50695 > 50623 >
50622 > 50613 > 50651 > 50674 > 50673 > 50597 > 50614 > 50596 > 50577 > 50578 > 50525 > 50489 > 50526 > 50576 > 50650
> 50649 > 50648 > 50717 > 50716 > 50766 > 50764 > 50763 > 50747 > 50741 > 50723 > 50110 > 50743 > 50765 > 50801 > 508
00 > 50103 > 50102 > 53696
```