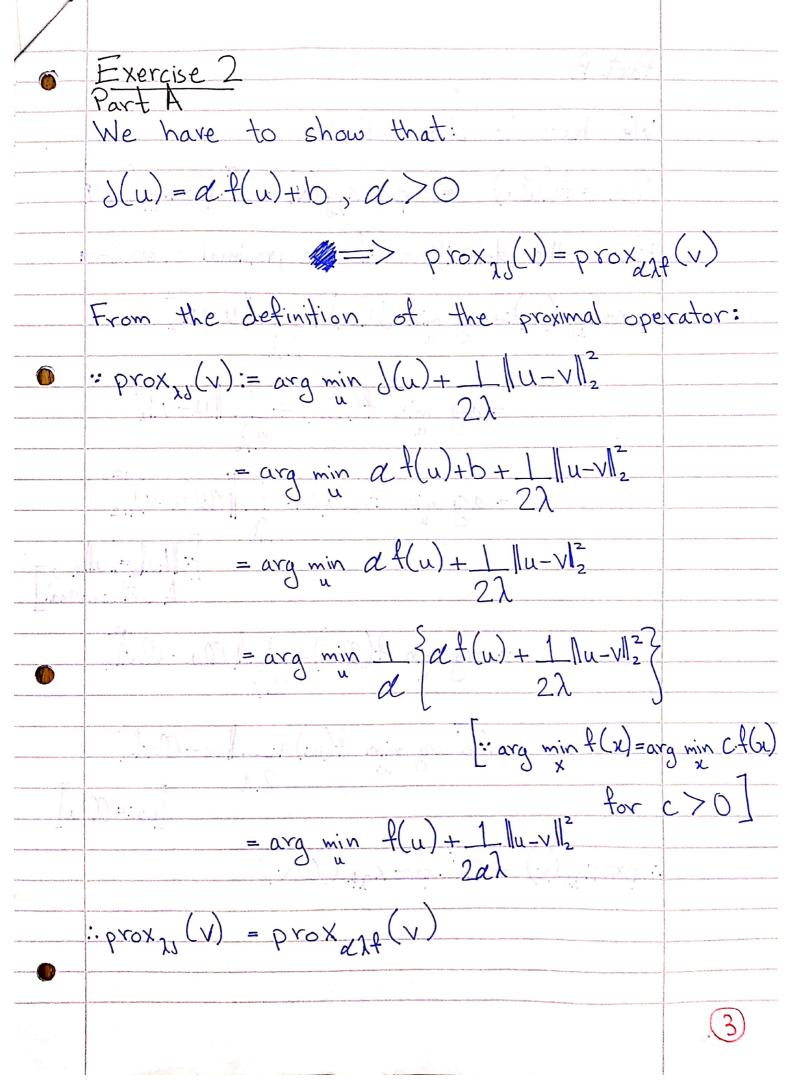
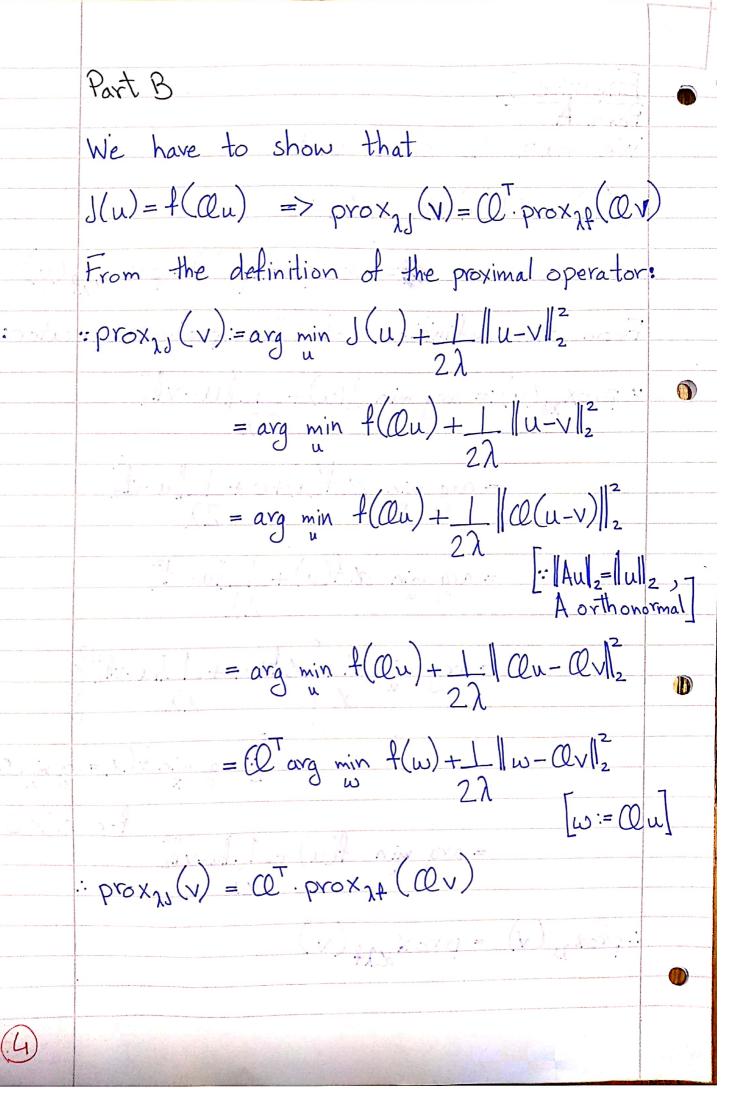
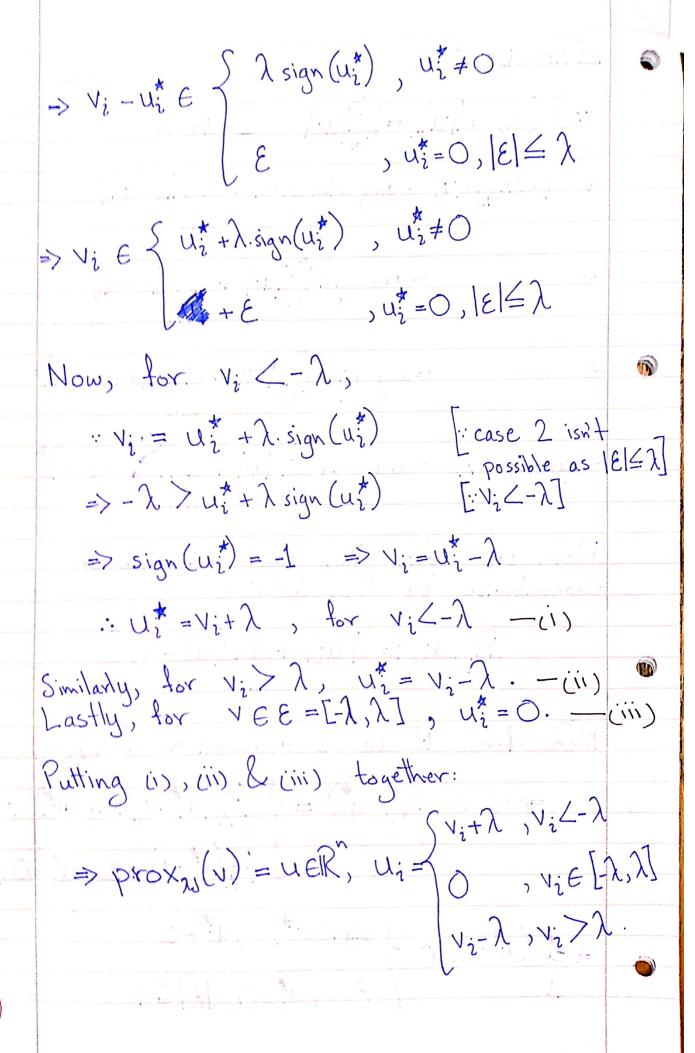


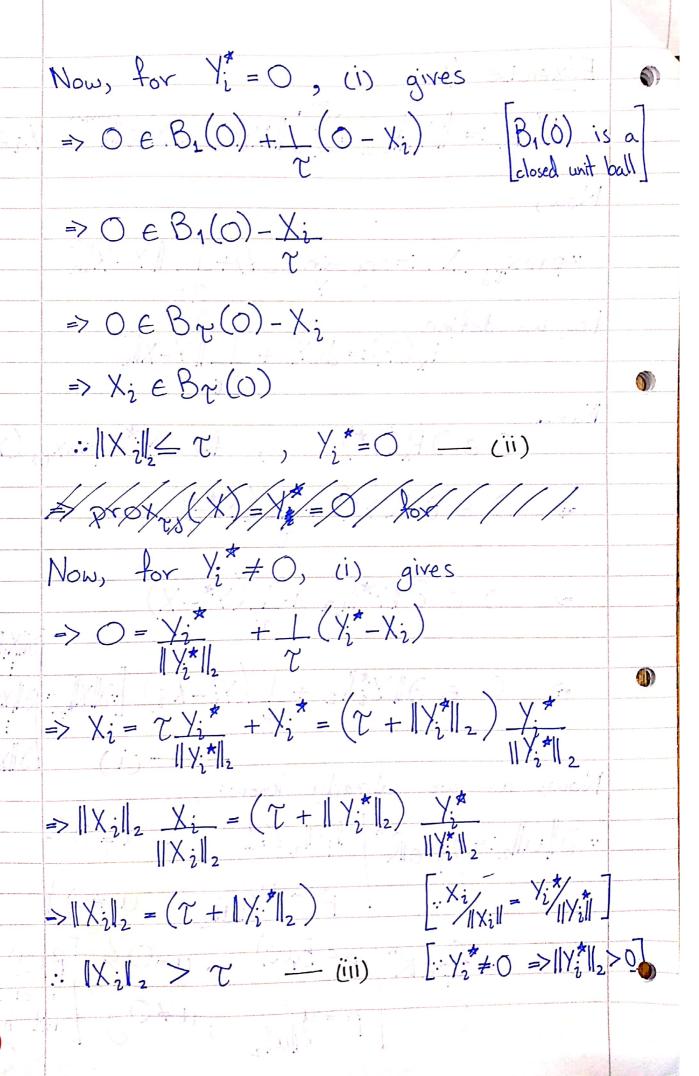
=> min Pu\*(v) = Pu\*(u\*) [tor Pu(v) = J(v) + I |v-u|] => arg min Pux (V) = U\* : prox (u\*) = u\* [Definition of proxy(v)] Assume U\* = prox, (u\*) Now, define Pu(v):= J(v)+ Ilv-ul Then,  $u^* = arg min Pu^*(v)$ => 0 E 2 Pa\* (u\*) => 0 e 2 / J(v) + 1 || u\* - u\*||^2 } => 0 E d { J(u\*) }  $0.00 \in 33(u^{*})$ 







Exercise 4  $J(X) := \|X\|_{1,2} = \sum_{i=1}^{m} \|X_{i}\|_{2}, X \in \mathbb{R}^{m \times n}, X_{i} \in \mathbb{R}^{n}$ Now, "prox<sub>2</sub>(X) := arg min d(Y)+1 ||Y-X||<sub>2</sub> let us define  $P_{x}(Y) := J(Y) + I ||Y - X||_{2}^{2}$ 0 e d Px (y\*), for y = proxy (x) 0 E 3 1 /\* 11,2 + 1 (Y\*-X)  $0 \in \partial \|Y_{i}^{*}\|_{2} + \frac{1}{2} (Y_{i}^{*} - X_{i}) [\partial \|X\|_{1,2} = \partial \|X_{i}\|_{2}]$ Now, as we already know:  $\frac{1}{2} \frac{1}{2} = \frac{1}{2} \frac{$  $= \left\{ p \in \mathbb{R}^n \middle| p \in \mathcal{B}_1(0) \right\}, \quad u = 0 \right\}, \quad \left\{ \begin{array}{c} \mathcal{B}(0) \text{ is } \\ \mathcal{A} \text{ closed} \\ \mathcal{A} \text{ out ball.} \end{array} \right\}$ 



	Also, for $V_i^* \neq 0$ ,	
	$\Rightarrow \bigcirc = \frac{\chi_i^*}{\ \chi_i^*\ _2} + \underline{\downarrow} (\chi_i^* - \chi_i)$	
	$-> 0 = \underbrace{\times_{i}}_{1} + \underbrace{\times_{i}}_{1} \times \underbrace{\times_{i}}_{1}$	Yi*
	Xill2 T	ea betone.
	$\Rightarrow Y_i^* = X_i - 7 \underline{X_i} - (iV)$ $\ X_i\ _2$	
	Therefore, from (ii) & (iii) + (iv),	
	$\Rightarrow p_{X_{i}}(X) = Y^* = \begin{cases} y \in \mathbb{R}^{m \times n} & \text{so}, \ X_{i}\ _{1} \\ y_{i} = \begin{cases} x_{i} - \frac{x_{i}}{\ X_{i}\ _{2}} \end{cases}$	2 <del>-</del>
	$X_{i}-X_{i}$ , $X_{i}$	$ X_i  _2 > \tau$
		,
		9
9		