

## Exercise 3

First. we derive the convex conjugates F\* and Gr\*, and subdifferentials

$$F^*(u,v) = \sup_{(\omega,\chi)} \left[ \begin{bmatrix} \omega \\ \chi \end{bmatrix}, \begin{bmatrix} u \end{bmatrix} \right] - F(\omega,\chi)$$

$$= \sup_{(\omega, x)} \left[ \omega^T x^T \right] \left[ u \right] - F(\omega, x)$$

$$= \sup_{(\omega, x)} \omega^{\mathsf{T}} u + x^{\mathsf{T}} \mathsf{V} - \mathsf{F}(\omega, x)$$

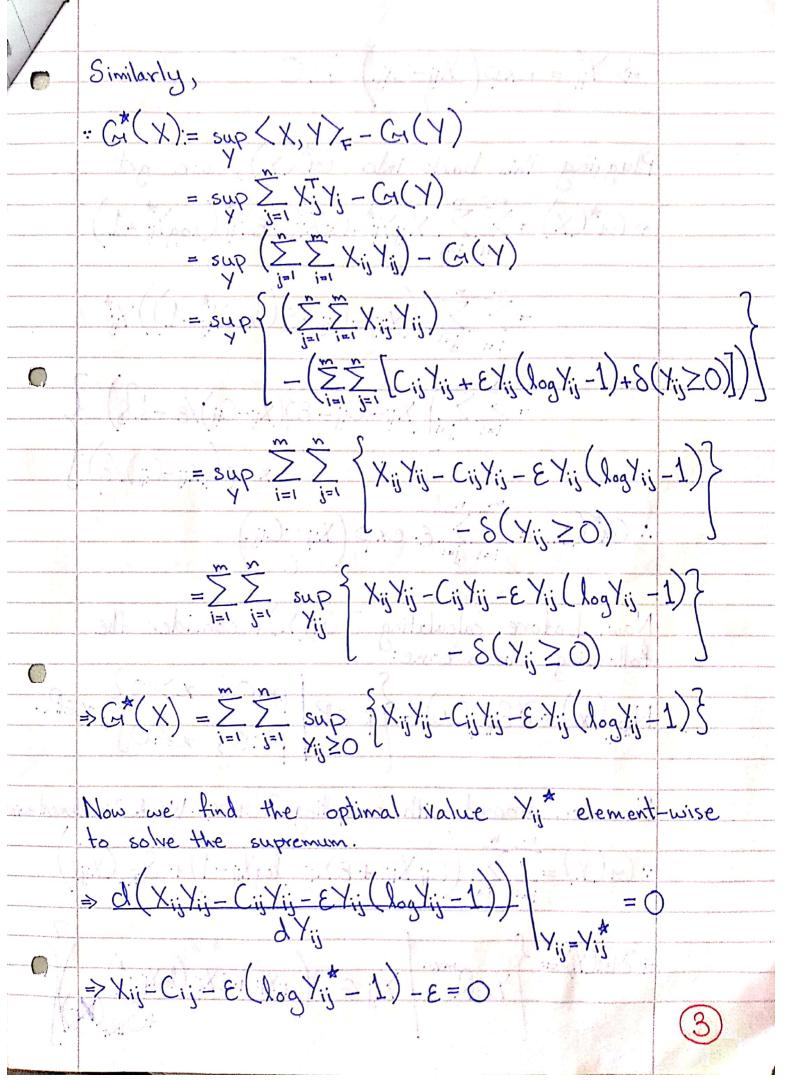
$$= \sup_{(\omega, x)} \omega^{\mathsf{T}} u + x^{\mathsf{T}} v - \delta_{\mathsf{T}}^{\mathsf{T}} (\omega, x) = (\mu, \nu)^{\mathsf{T}}$$

$$= sup \quad \omega^{T}U + \chi^{T}V$$

$$(\omega, \chi) = (\mu, \chi)$$

$$F^*(u,v) = \mu^T u + \nu^T v = \left\langle \begin{bmatrix} \mu \\ \nu \end{bmatrix}, \begin{bmatrix} u \end{bmatrix} \right\rangle$$

$$\partial F^*(u,v) = \{ \nabla F^*(u,v) \} = [\mu^T \nu^T]$$



Plugging this back into 
$$C_i^*(X)$$
, we get

$$C_i^*(X) = \sum_{i=1}^{m} \sum_{j=1}^{n} X_{ij} Y_{ij}^* - C_{ij} X_{ij}^* - E Y_{ij}^* (\log Y_{ij}^* - 1)$$

$$= \sum_{i=1}^{m} \sum_{j=1}^{n} (X_{ij} - C_{ij} - E(\log Y_{ij}^* - 1)) Y_{ij}^*$$

$$= \sum_{i=1}^{m} \sum_{j=1}^{n} (X_{ij} - C_{ij} - E(\log Y_{ij}^* - 1)) Y_{ij}^*$$

$$= \sum_{i=1}^{m} \sum_{j=1}^{n} (X_{ij} - C_{ij} - E(X_{ij} - C_{ij}) / E - 1 \frac{1}{2})$$

$$\times \exp((X_{ij} - C_{ij}) / E)$$

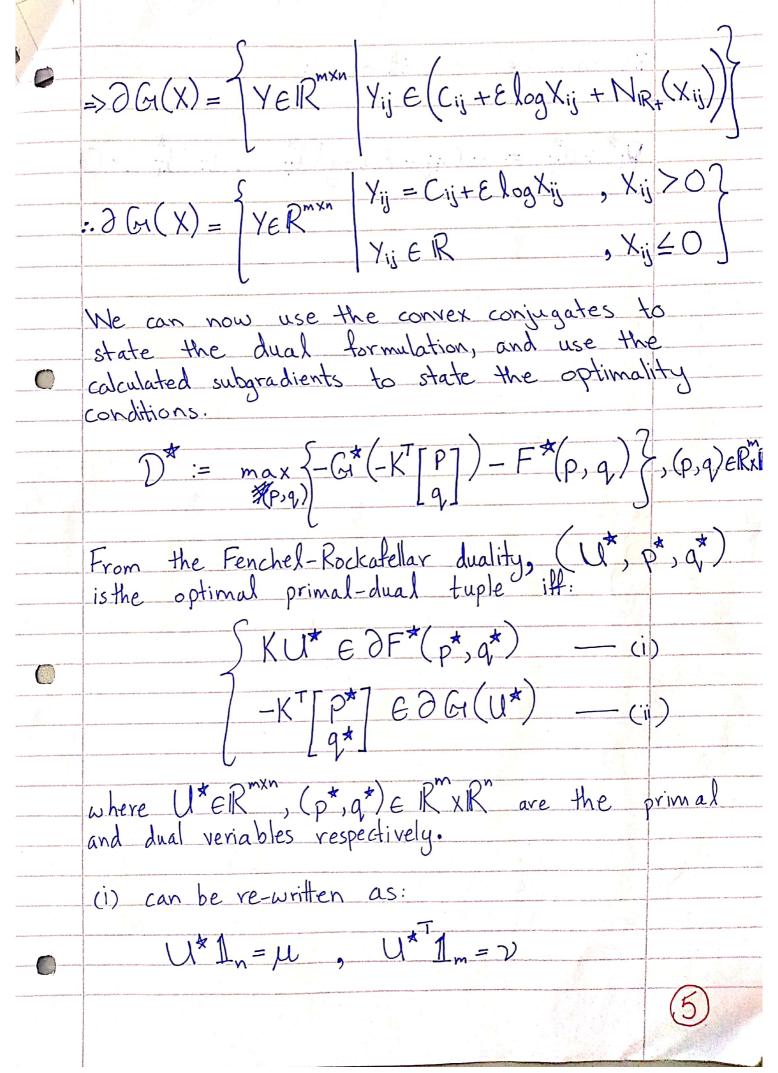
$$\therefore C_i^*(X) = \sum_{i=1}^{m} \sum_{j=1}^{n} E. \exp(X_{ij} - C_{ij}) / E - 1 \frac{1}{2}$$

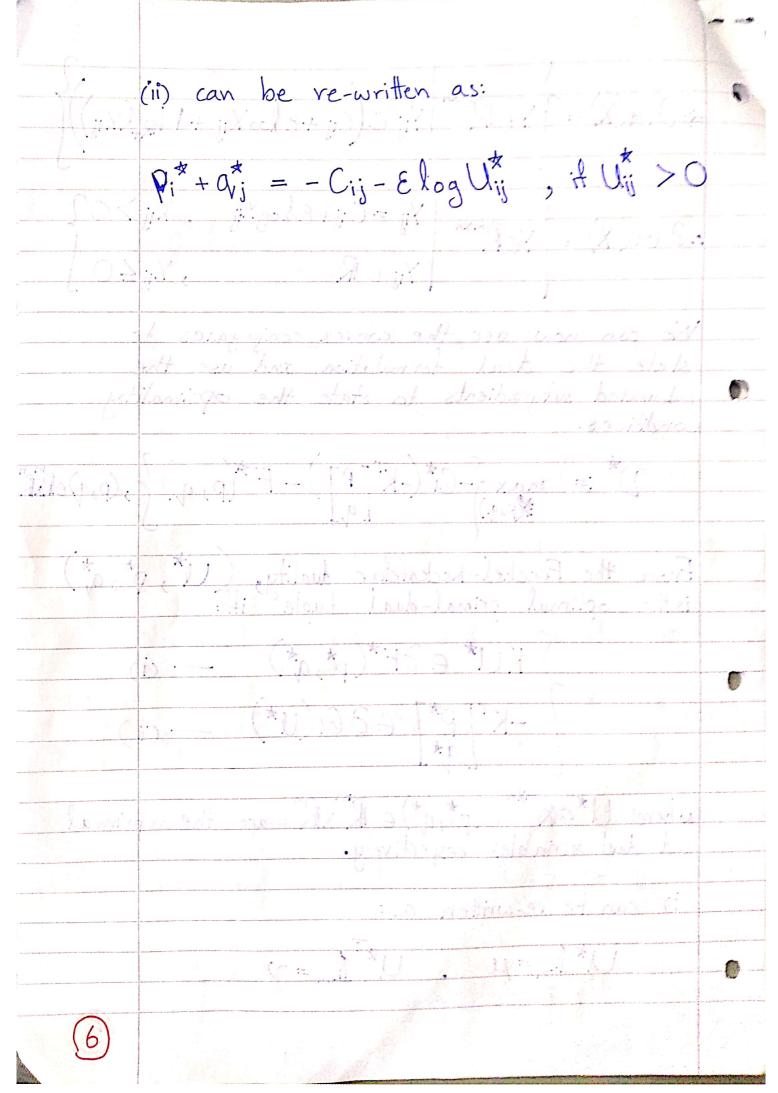
$$\times \exp((X_{ij} - C_{ij}) / E)$$

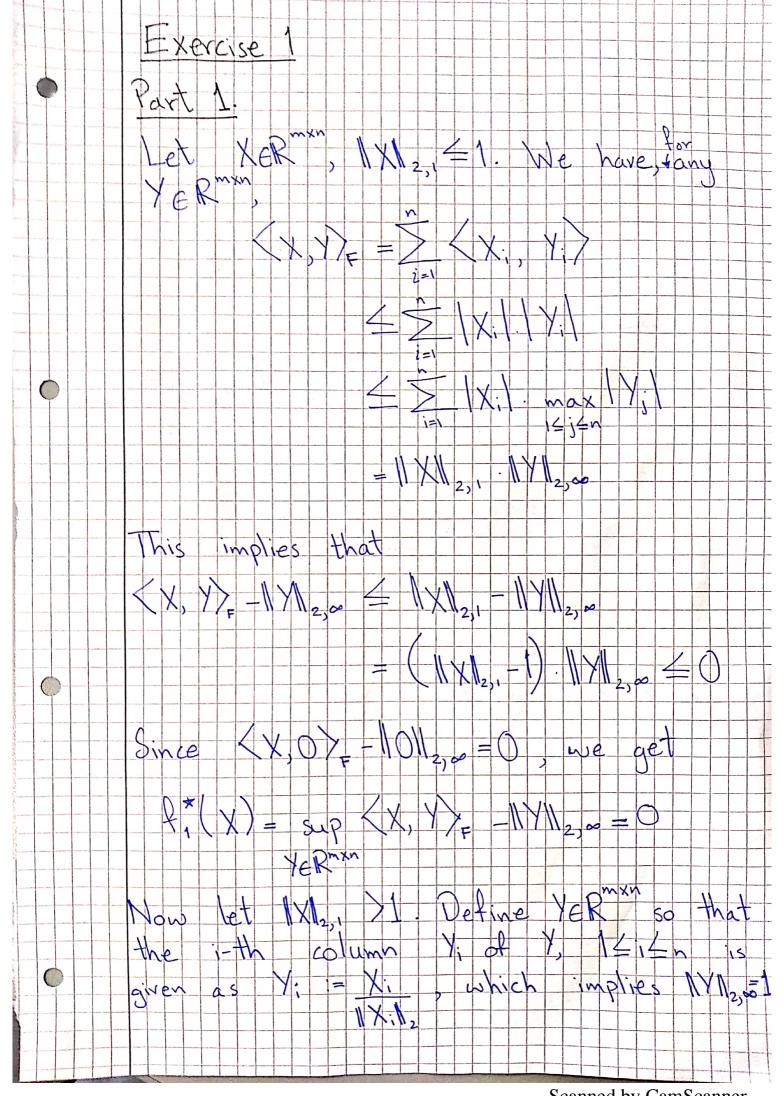
$$Now, before calculating  $\partial C_i(X)$ , consider the following norm-cone:
$$N_{R_i}(X) = \partial S_{R_i}(X) = \int P \in R \left| P = O, X > O, X \in R \right|$$

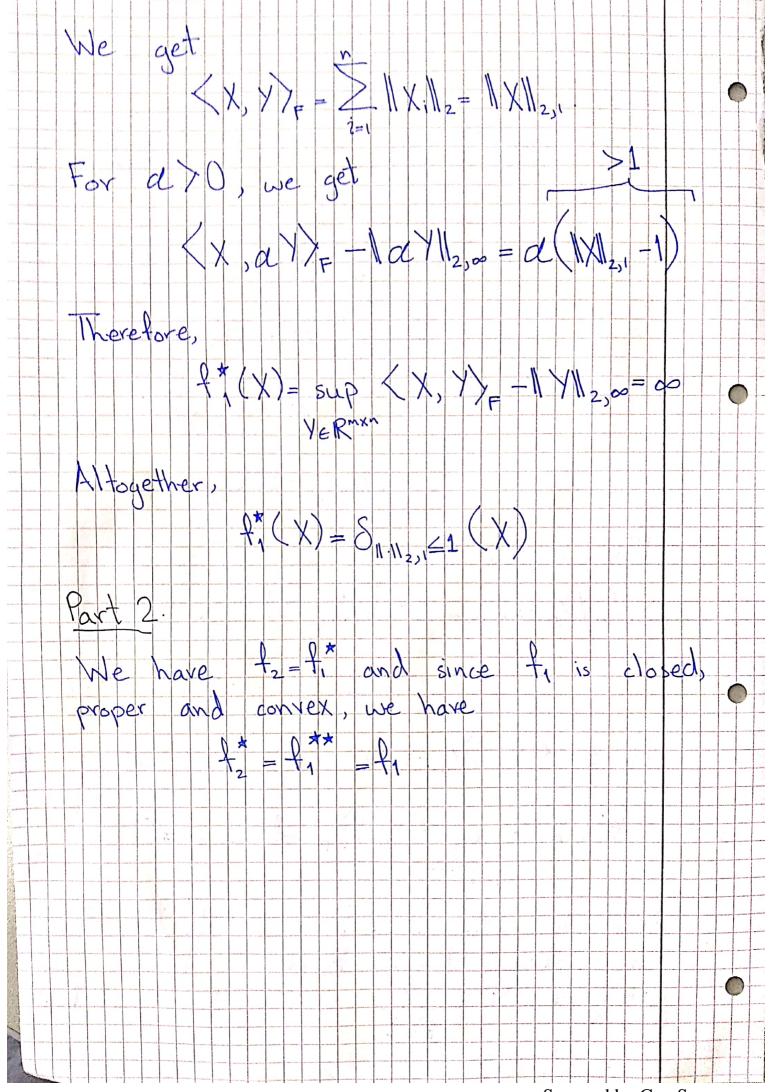
$$Now we proceed with computing  $\partial C_i(X)$ . We do this element-us:
$$C_i(X) = \sum_{i=1}^{m} \sum_{j=1}^{n} (C_{ij} X_{ij} + E X_{ij} (\log X_{ij} - 1) + S_{R_i}(X_{ij})$$

$$\Rightarrow \partial C_i(X) = \begin{cases} Y \in R^{m \times n} \middle Y_{ij} \in \partial(C_{ij} X_{ij} + E X_{ij} (\log X_{ij} - 1) + S_{R_i}(X_{ij}) \end{cases}$$$$$$









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