Ch 3.3: Even More Linear Regression

Lecture 8 - CMSE 381

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Announcements

Last time:

- 3.2 Multiple Linear Regression

Announcements:

- Dr. Munch back next week!

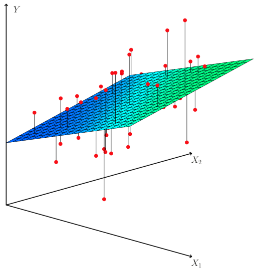
Covered in this lecture

- Extending the linear model with interaction terms
- Hierarchy principle
- Polynomial regression

Section 1

Review from last time

Linear Regression with Multiple Variables



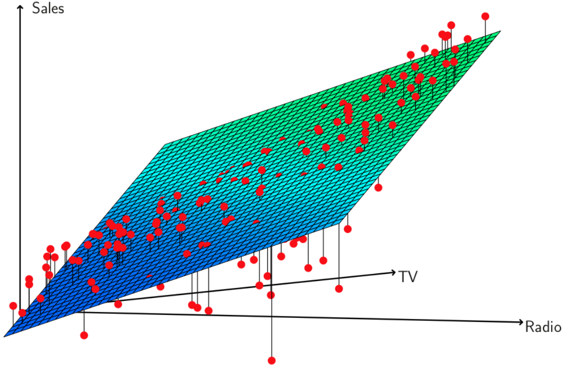
- Predict Y on a multiple variables X

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots \beta_p x_p + \varepsilon$$

- Find good guesses for $\hat{\beta}_0$, $\hat{\beta}_1, \cdots$.
- $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i + \cdots + \hat{\beta}_p x_p$

- $e_i = y_i - \hat{y}_i$ is the ith residual
- RSS $= \sum_i e_i^2$
- RSS is minimized at *least squares coefficient estimates*

If all else fails, look at the data



Section 2

Extending the linear model

Assumptions so far

Back to our Advertising data set

$$\hat{Y}_{sales} = \beta_0 + \beta_1 \cdot X_{TV} + \beta_2 \cdot X_{radio} + \beta_3 \cdot X_{newspaper}$$

Assumed (implicitly) that the effect on sales by increasing one medium is independent of the others.

What if spending money on radio advertising increases the effectiveness of TV advertising? How do we model it?

Interaction Term

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \varepsilon$$

$$\begin{aligned} Y_{sales} &= \beta_0 + \beta_1 X_{TV} + \beta_2 X_{radio} + \beta_3 X_{radio} X_{TV} + \varepsilon \\ &= \beta_0 + (\beta_1 + \beta_3 X_{radio}) X_{TV} + \beta_2 X_{radio} + \varepsilon \\ &= \beta_0 + \tilde{\beta}_1 X_{TV} + \beta_2 X_{radio} + \varepsilon \end{aligned}$$

Interaction term

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \varepsilon$$

	Coefficient	Std. error	t-statistic	p-value
`Intercept`	6.7502	0.248	27.23	< 0.0001
`TV`	0.0191	0.002	12.70	< 0.0001
`radio`	0.0289	0.009	3.24	0.0014
`TV`×`radio`	0.0011	0.000	20.73	< 0.0001

$$\begin{aligned} Y_{sales} &= \beta_0 + \beta_1 X_{TV} + \beta_2 X_{radio} + \beta_3 X_{radio} X_{TV} + \varepsilon \\ &= \beta_0 + (\beta_1 + \beta_3 X_{radio}) X_{TV} + \beta_2 X_{radio} + \varepsilon \end{aligned}$$

Interpretation

	Coefficient	Std. error	t-statistic	p-value
`Intercept`	6.7502	0.248	27.23	< 0.0001
`TV`	0.0191	0.002	12.70	< 0.0001
`radio`	0.0289	0.009	3.24	0.0014
`TV`×`radio`	0.0011	0.000	20.73	< 0.0001

Do the section on "Interaction Terms"

Hierarchy principle

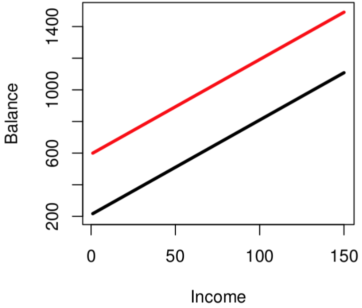
Sometimes p-value for interaction term is very small, but associated main effects are not.

The hierarchy principle:

For credit data set:
Predict balance using income (quantitative) and student (qualitative)

$$\texttt{balance}_i \approx \beta_0 + \beta_1 \cdot \texttt{income}_i + \begin{cases} \beta_2 & \text{if student} \\ 0 & \text{if not} \end{cases}$$

$$\approx \beta_1 \cdot \texttt{income}_i + \begin{cases} \beta_0 + \beta_2 & \text{if student} \\ \beta_0 & \text{if not} \end{cases}$$



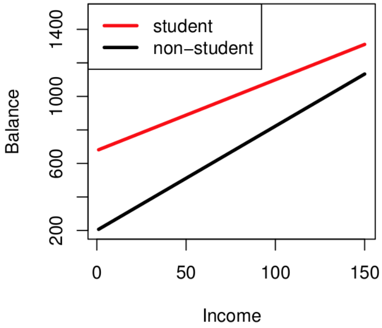
Interaction term for qualitative variables

With interaction term

For credit data set:
Predict balance using income (quantitative) and student (qualitative)

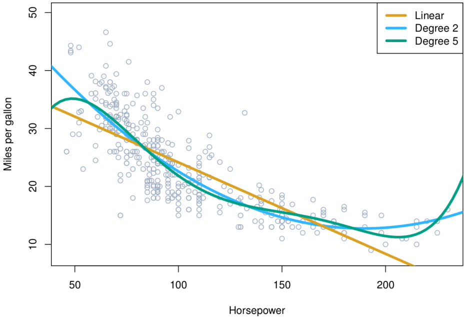
$$\texttt{balance}_i \approx \beta_0 + \beta_1 \cdot \texttt{income}_i + \begin{cases} \beta_2 + \beta_3 \cdot \texttt{income}_i & \text{if student} \\ 0 & \text{if not} \end{cases}$$

$$\approx \begin{cases} (\beta_0 + \beta_2) + (\beta_1 + \beta_3) \cdot \texttt{income}_i & \text{if student} \\ \beta_0 + \beta_1 \cdot \texttt{income}_i & \text{if not} \end{cases}$$



Nonlinear relationships

$$\texttt{mpg} = \beta_0 + \beta_1 \cdot \texttt{horsepower} + \beta_2 \cdot \texttt{horsepower}^2 + \varepsilon$$



	Coefficient	Std. error	t-statistic	p-value
`Intercept`	56.9001	1.8004	31.6	< 0.0001
`horsepower`	-0.4662	0.0311	-15.0	< 0.0001
$\texttt{horsepower}^2$	0.0012	0.0001	10.1	< 0.0001

Practice and Question Time!

For the remaining part of class, work on the jupyter notebook in groups. If you have any lingering questions or confusion, this is a great time to call me over as well!

Next time

Lec #	Date			Reading
1	Mon	Aug 28	Intro / First day stuff / Python Review Pt 1	1
2	Wed	Aug 30	What is statistical learning?	2.1
	Fri	Sep 1	Assessing Model Accuracy	2.2.1, 2.2.2
3	Mon	Sep 4	No class - Labor day	
4	Wed	Sep 6	Linear Regression	3.1
5	Fri	Sep 8	More Linear Regression	3.1/3.2
6	Mon	Sep 11	Even more linear regression	3.2.2
7	Wed	Sep 13	Probably more linear regression	3.3
8	Fri	Sep 15	Intro to classification, Logisitic Regression	2.2.3, 4.1, 4.2, 4.3
9	Mon	Sep 18	More logistic regression	
10	Wed	Sep 20	Multiple Logistic Regression / Multinomial Logistic Regression	
11	Fri	Sep 22	Overflow/Project day?	
	Mon	Sep 25	***Review***	
	Wed	Sep 27	**Midterm #1**	
	Fri	Sep 29	No class - Dr Munch out of town	
12	Mon	Oct 2	Leave one out CV	5.1.1, 5.1.2
13	Wed	Oct 4	k-fold CV	5.1.3
14	Fri	Oct 6	More k-fold CV,	5.1.4-5
15	Mon	Oct 9	k-fold CV for classification	5.1.5
16	Wed	Oct 11	Resampling methods: Bootstrap	5.2
17	Fri	Oct 13	Subset selection	6.1