Ch 7.1-7.2: Polynomial regression and Step Functions

Lecture 22 - CMSE 381

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Announcements

Last time:

- PLS
- High dimensions

This lecture:

- 7.1 Polynomial regression
- 7.2 Step functions

Announcements:

- HW #6 due Monday

Section 1

Last time

High-Dimensional Data

Low-Dimensions

$n \gg p$

- Low here means p is low, or at least small relative to n
- Can do all the stuff we've talked about so far

High-Dimensions

$n \ll p$

- Issues show up even if p is close to or slightly smaller than n
- Classical approaches not appropriate since lots of overfitting

What to do about it?

Be less flexible....

Key points

- regularization or shrinkage plays a key role in high-dimensional problems,
- appropriate tuning parameter selection is crucial for good predictive performance, and
- the test error tends to increase as the dimensionality of the problem increases, unless the additional features are truly associated with the response.

- Curse of dimensionality
- Report results on an independent test set, or cross-validation errors.

Section 2

Polynomial Regression

Polynomial regression

Replace linear model

$$y_i = \beta_0 + \beta_1 x_1 + \varepsilon_i$$

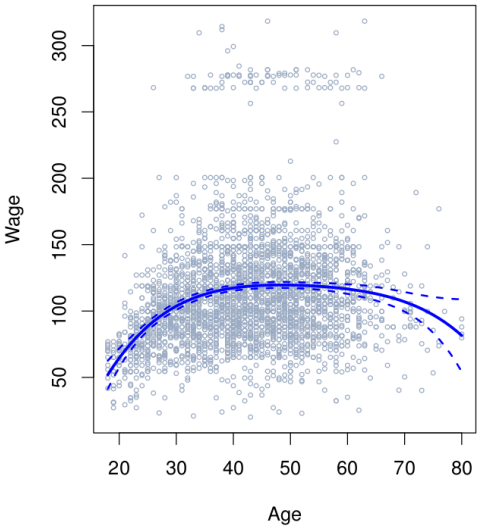
with

$$y_i = \beta_0 + \beta_1 x_1 + \beta_2 x_i^2 + \cdots + \beta_d x_i^d + \varepsilon_i$$

Coding bit

$$\texttt{wage} = \beta_0 + \beta_1 \texttt{age} + \beta_2 \texttt{age}^2 + \cdots + \beta_p \texttt{age}^p + \varepsilon.$$

Example with wage data



Section 3

Step function

Step functions

$$\mathrm{I}(X < c) \qquad \mathrm{I}(c_1 \leq X < c_2) \qquad \mathrm{I}(c \leq X)$$

More on step function setup

$$
\begin{aligned}
C_0(X) &= I(X < c_1), \\
C_1(X) &= I(c_1 \leq X < c_2), \\
C_2(X) &= I(c_2 \leq X < c_3), \\
&\vdots \\
C_{K-1}(X) &= I(c_{K-1} \leq X < c_K), \\
C_K(X) &= I(c_K \leq X),
\end{aligned}
$$

Example

Given knots $c_1 = 3$, $c_2 = 5$, $c_3 = 7$, determine the entries in the columns for $C_i(X)$ in the below matrix.

X	$C_0(X)$	$C_1(X)$	$C_2(X)$	$C_3(X)$
1				
2				
3				
4				
5				

X	$C_0(X)$	$C_1(X)$	$C_2(X)$	$C_3(X)$
6				
7				
8				
9				
10				

Draw the function

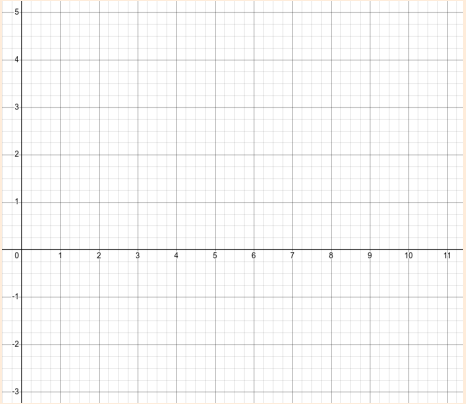
My code doing regression on the step function input returned the function.

$$f(X) = -1 + 3C_1(X) + 4C_2(X) - 2C_3(X).$$

Fill in the table of values, then draw this function below.

X	$F(X)$
1	
2	
3	
4	
5	

X	$F(X)$
6	
7	
8	
9	
10	



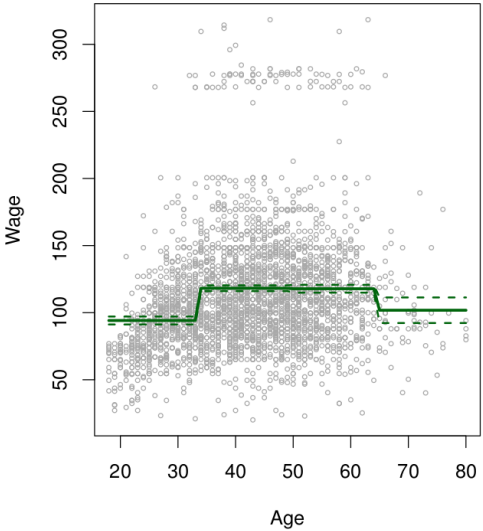
Step function: Learned model

$$y_i = \beta_0 + \beta_1 C_1(x_i) + \beta_2 C_2(x_i) + \cdots + \beta_K C_K(x_i) + \varepsilon_i$$

Coding bit

Back to the `wage` data set

Step function example



Next time

Lec #	Date			Reading	Homeworks
20	Fri	Oct 27	Dimension Reduction	6.3	
21	Mon	Oct 30	More dimension reduction; High dimensions	6.4	
22	Wed	Nov 1	Polynomial & Step Functions	7.1,7.2	
23	Fri	Nov 3	Step Functions	7.2	HW #6 Due
24	Mon	Nov 6	Basis functions, Regression Splines	7.3,7.4	
25	Wed	Nov 8	Decision Trees	8.1	
26	Fri	Nov 10	Random Forests	8.2.1, 8.2.2	
27	Mon	Nov 13	Maximal Margin Classifier	9.1	
28	Wed	Nov 15	SVC	9.2	
29	Fri	Nov 17	SVM	9.3, 9.4	
30	Mon	Nov 20	Single layer NN	10.1	
31	Wed	Nov 22	Overflow/project day?		
	Fri	Nov 24	No class - Thanksgiving		
	Mon	Nov 27	**Review**		
	Wed	Nov 29	**Midterm #3**		
32	Fri	Dec 1	Multi Layer NN	10.2	
33	Mon	Dec 4	CNN	10.3	
34	Wed	Dec 6	Unsupervised Learning & Clustering	12.1, 12.4	
35	Fri	Dec 8	Overflow/Project day?		Project due