Mathematical models

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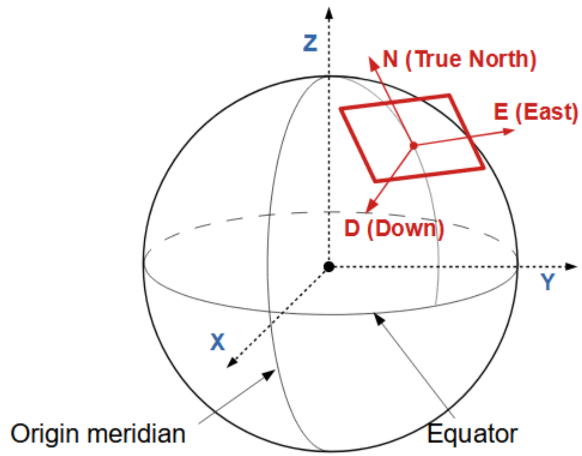
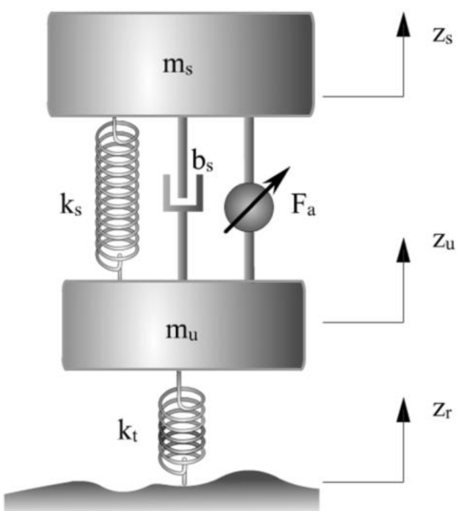
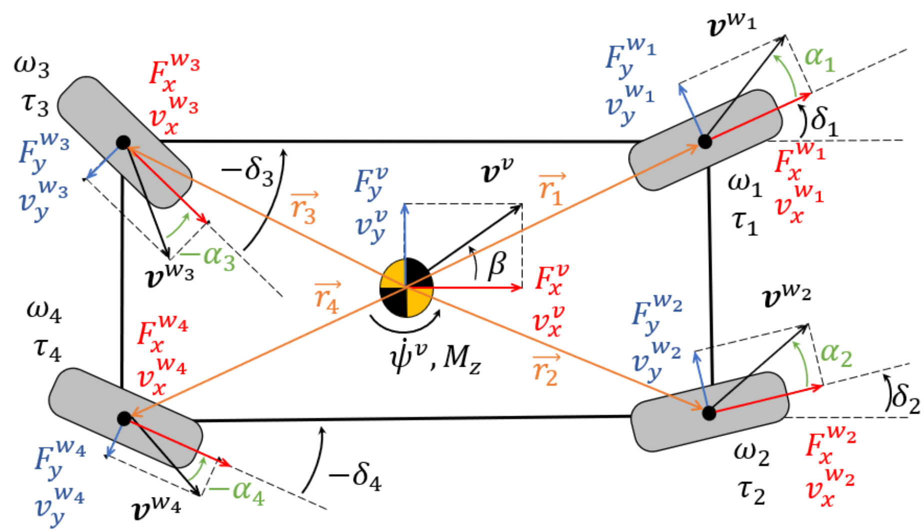
Coordinate systems

Important points

- **Inertial frame – NED**
- **Vehicle coordinate system**
  - **Body – Center of Gravity**
  - **Body – Wheel**
- **Wheel**
  - **Contact patch**
  - **Sprung/Unsprung mass**

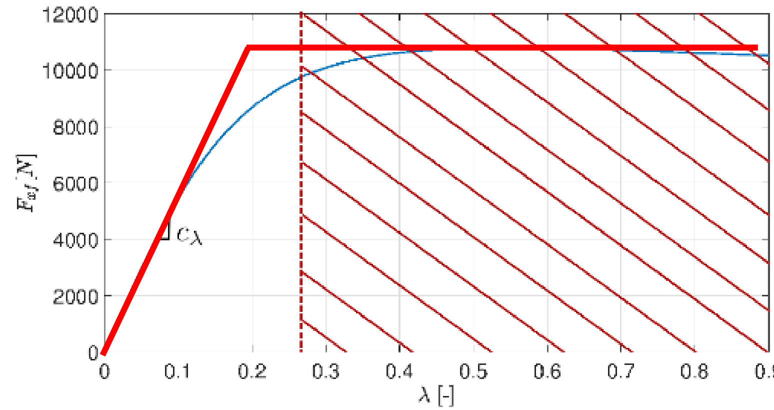


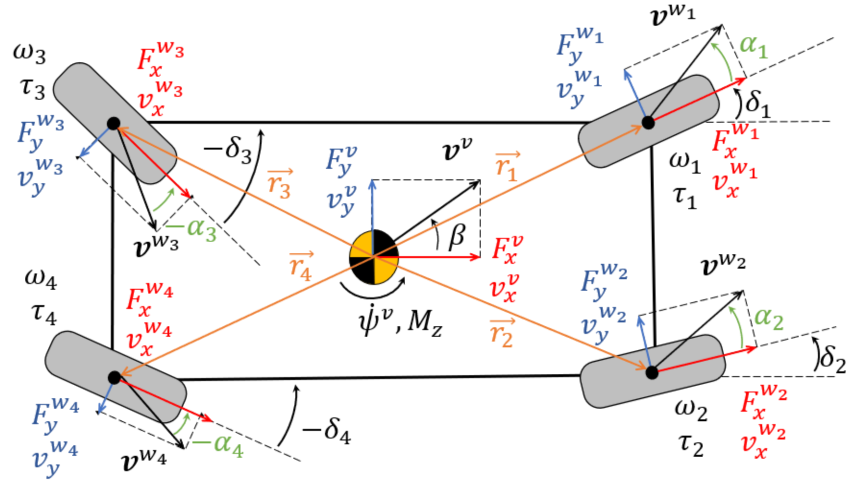


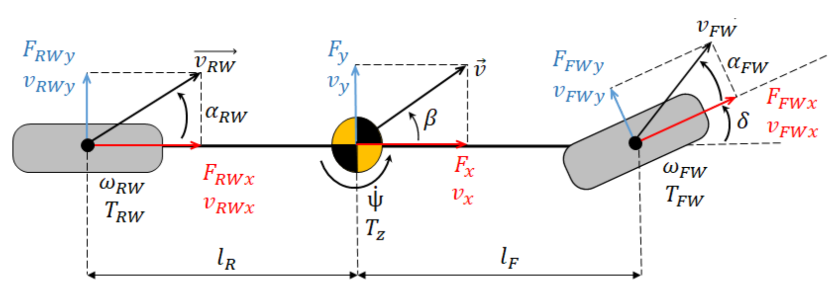


Vehicle models

$$\vec{v} = f(F_x, F_y)$$







Are the forces considered?

- × **Kinematic model**
- ✓ **Dynamic model**

Is body motion considered?

- × **Single-track model**
- ✓ **Twin-track model**

Steering model

- × **Parallel steering**
- ✓ **Ackerman steering**
- ✓ **Steering allocation**

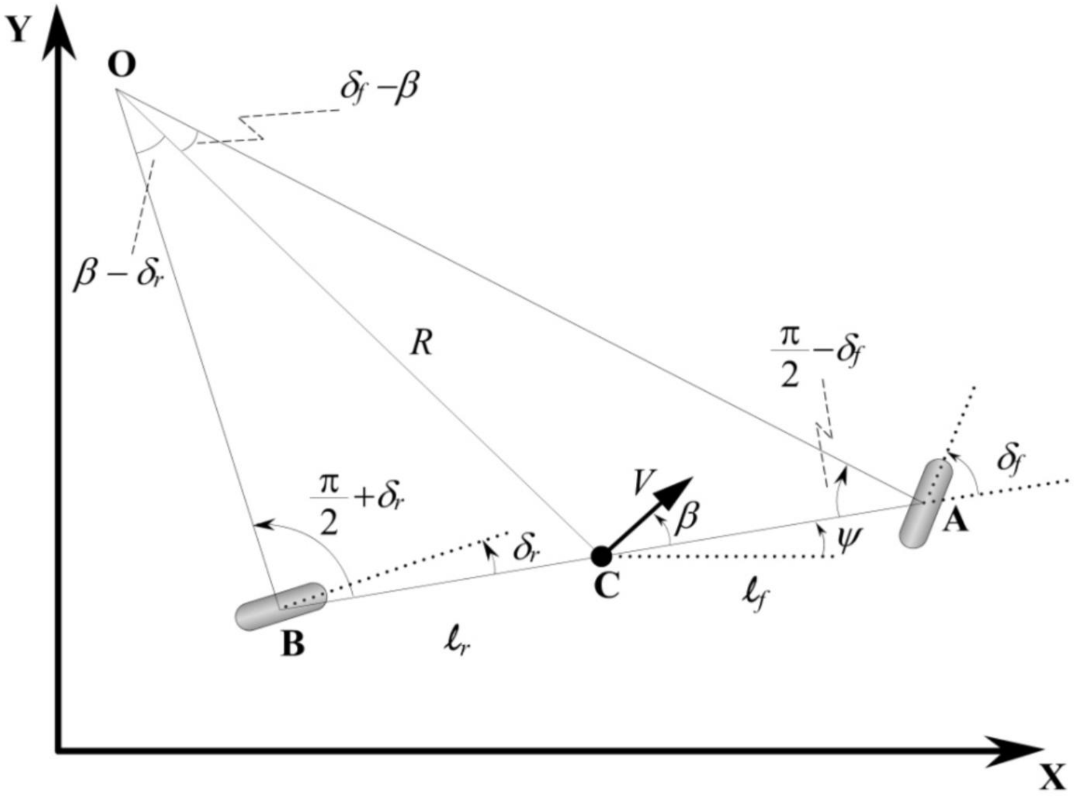
Suspension model

- × **Kinematic, Single/Twin-track**
- ✓ **Quarter/Half/Full model**

Kinematic model

Assumptions

- **Wheel velocity vectors are in direction of wheels (slow motion, v<5km/h)**



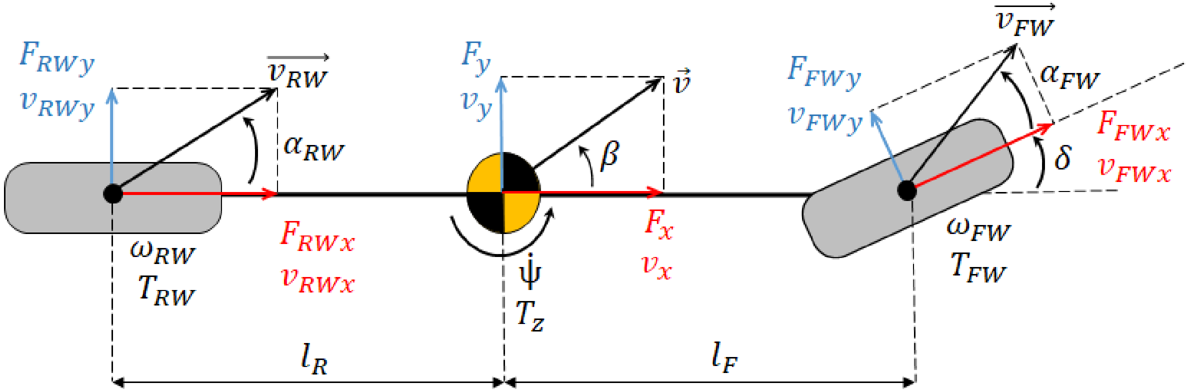
$$\dot{X} = V\cos(\psi + \beta)$$

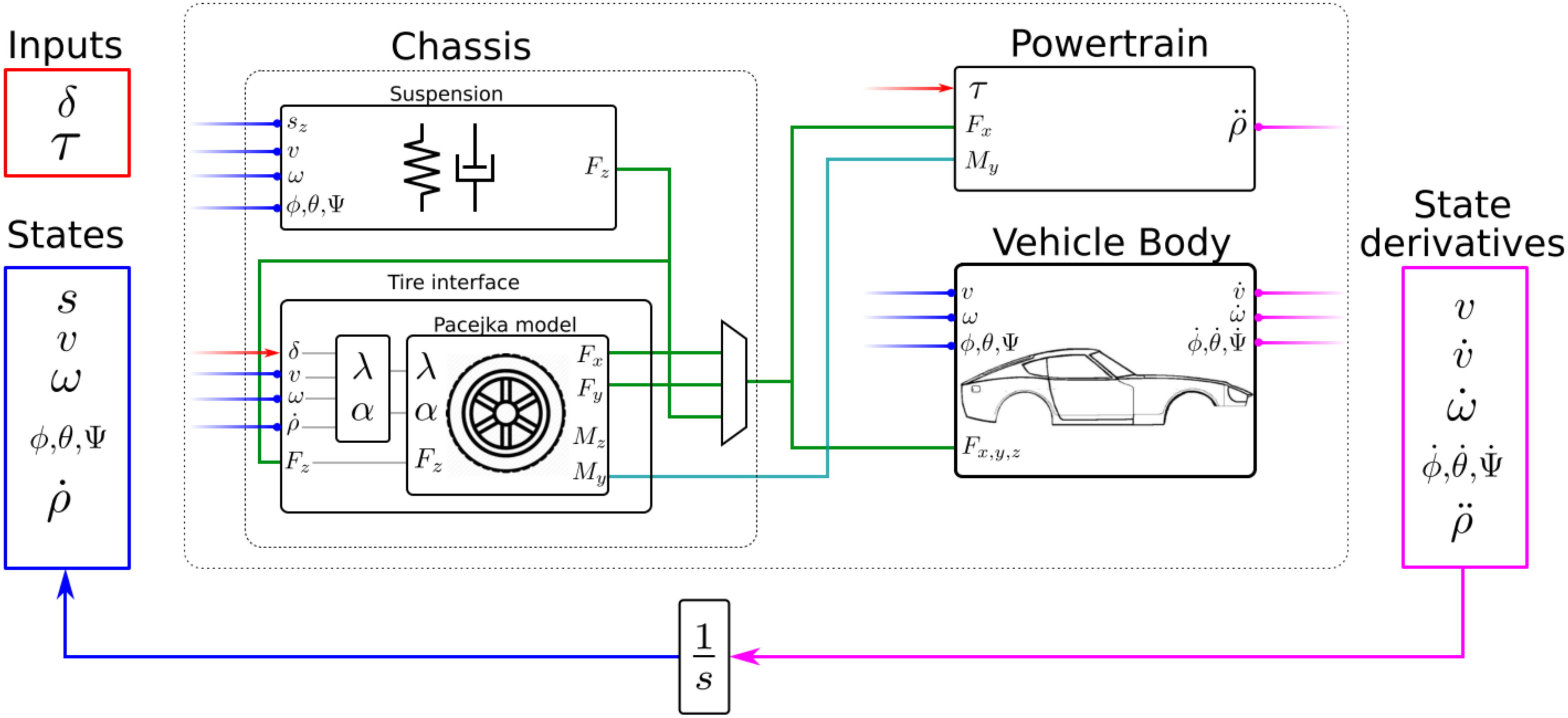
$$\dot{Y} = V\sin(\psi + \beta)$$

$$\dot{\psi} = \frac{V\cos(\beta)}{l_f + l_r}\left(\tan\delta_f - \tan\delta_r\right)$$

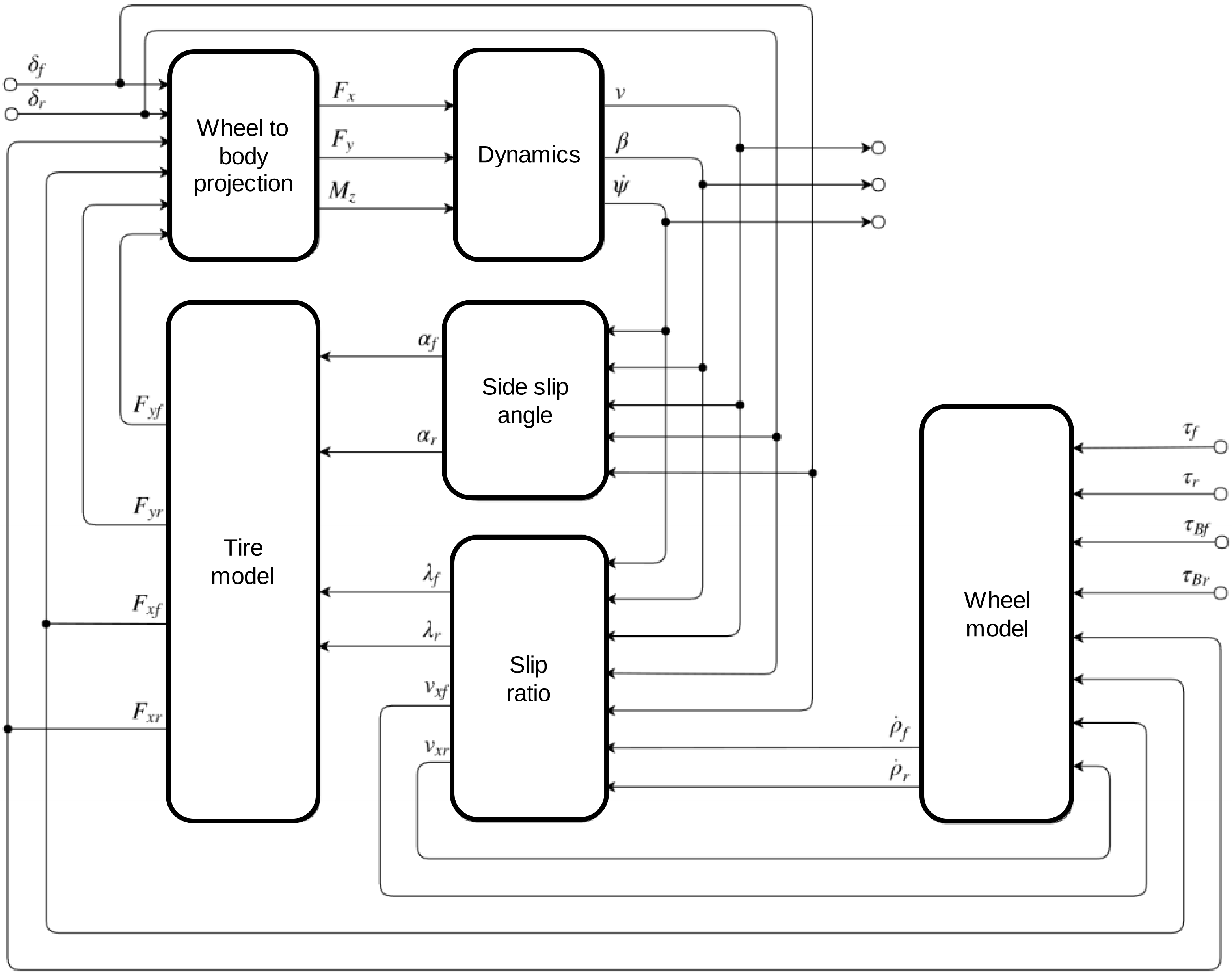
$$\beta = \tan^{-1}\left(\frac{l_r \tan\delta_f + l_f \tan\delta_r}{l_f + l_r}\right)$$

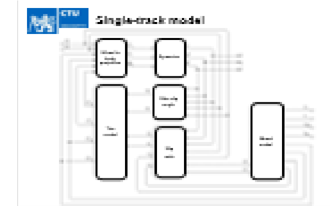
Single-track model



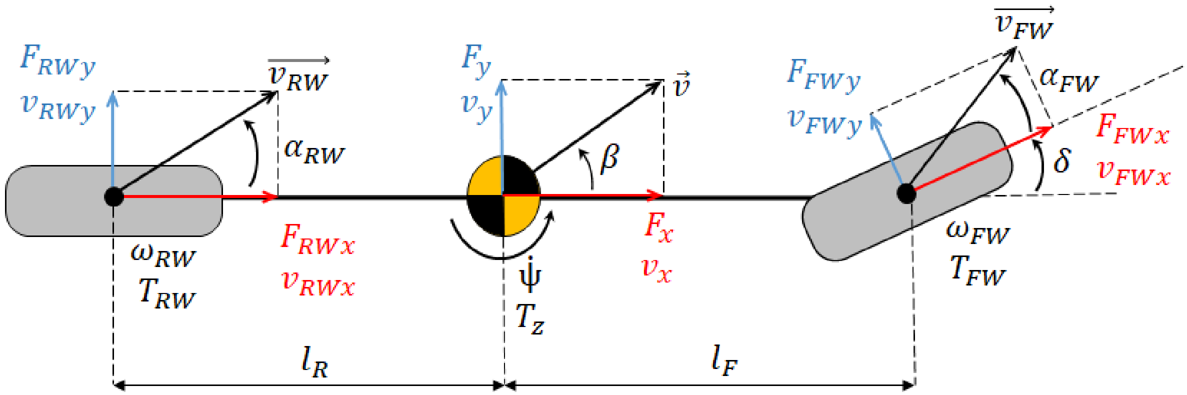


Single-track model





ST model – dynamics



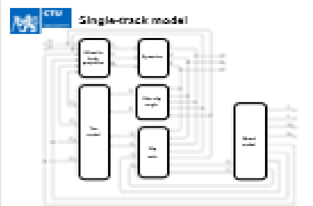
$$\vec{v}$$

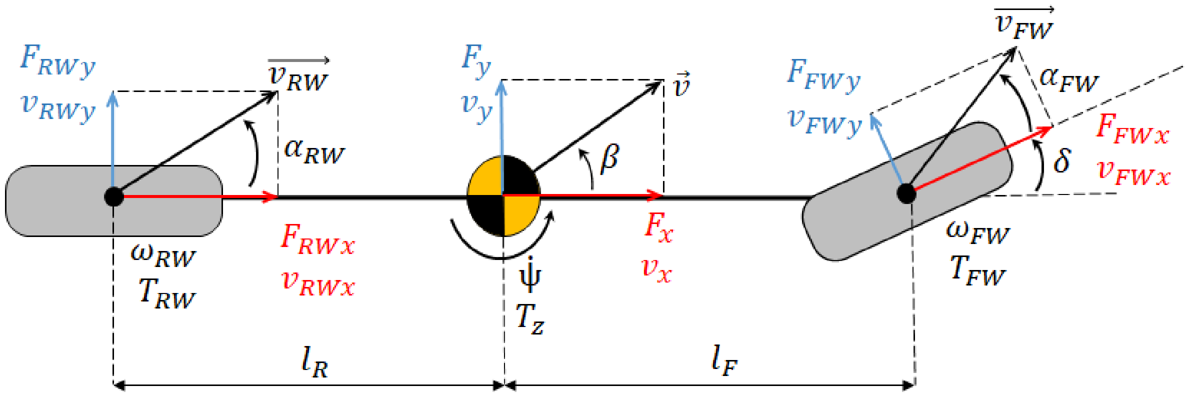
$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} \leftrightarrow \begin{bmatrix} \beta \\ |\vec{v}| \end{bmatrix}$$

$$m\dot{v}_x = F_x$$
$$m(\dot{v}_y + \dot{\psi}v_x) = F_y$$
$$I_z\ddot{\psi} = T_z$$

$$-m|\vec{v}|(\dot{\beta} + \dot{\psi})\sin\beta + m|\dot{\vec{v}}|\cos\beta = F_x$$
$$m|\vec{v}|(\dot{\beta} + \dot{\psi})\cos\beta + m|\dot{\vec{v}}|\sin\beta = F_y$$
$$I_z\ddot{\psi} = T_z$$

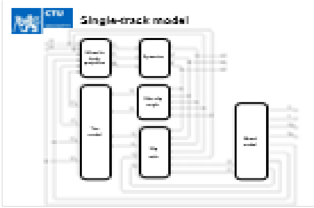
ST – Wheel to body projection

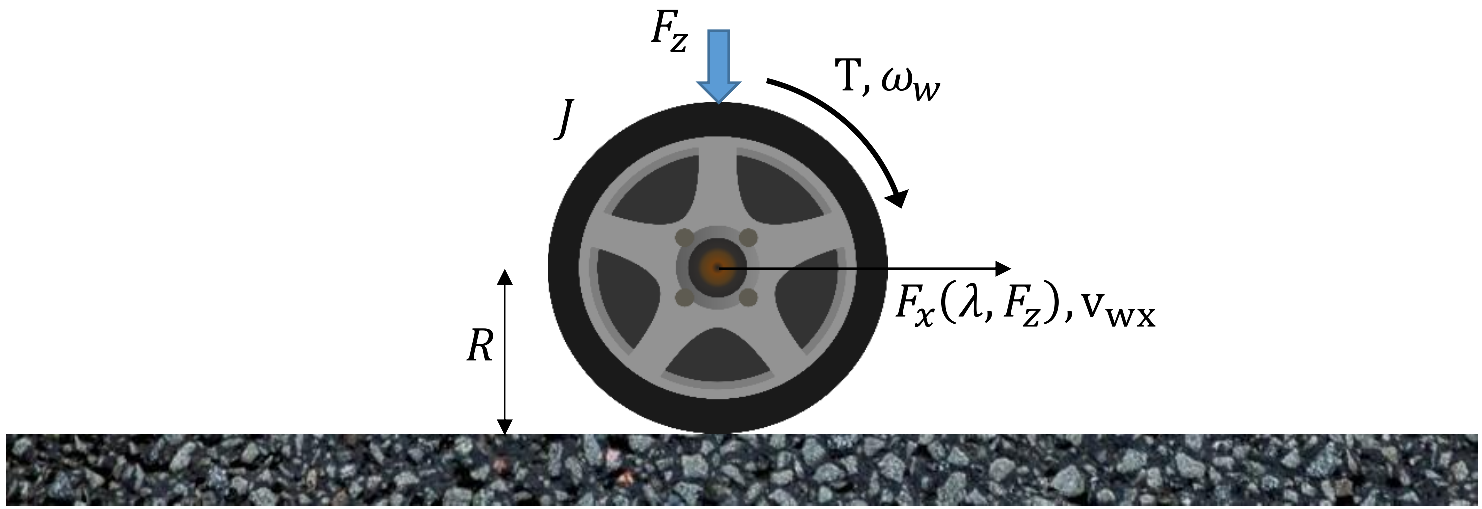




$$F_{xf}\cos\delta_f + F_{xr}\cos\delta_r - F_{yf}\sin\delta_f - F_{yr}\sin\delta_r = F_x$$
$$F_{yf}\cos\delta_f + F_{yr}\cos\delta_r + F_{xf}\sin\delta_f + F_{xr}\sin\delta_r = F_y$$
$$l_f F_{yf}\cos\delta_f - l_r F_{yr}\cos\delta_r + l_f F_{xf}\sin\delta_f - l_r F_{xr}\sin\delta_r = T_z$$

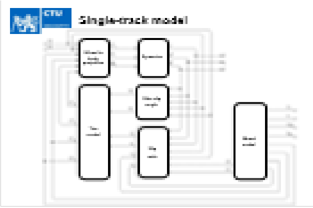
ST – Wheel model

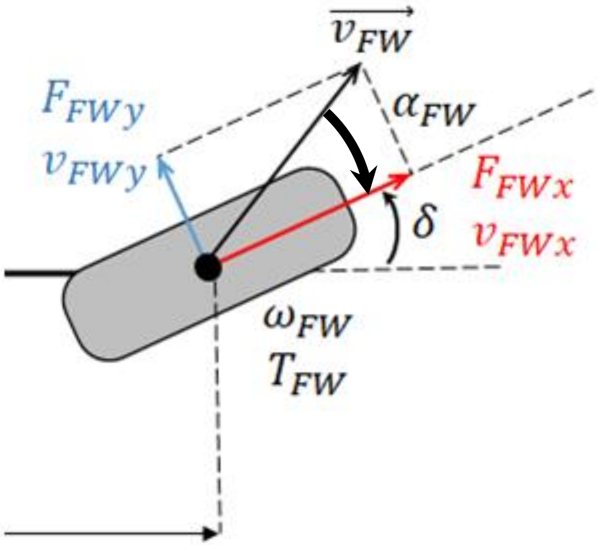




$$J\dot{\omega}_w = \mathrm{T} - F_{wx}(\lambda, F_z)R - sign(\omega_w)\mathrm{T_b} - \mathrm{kv_{wx}}$$

ST – Side slip angle

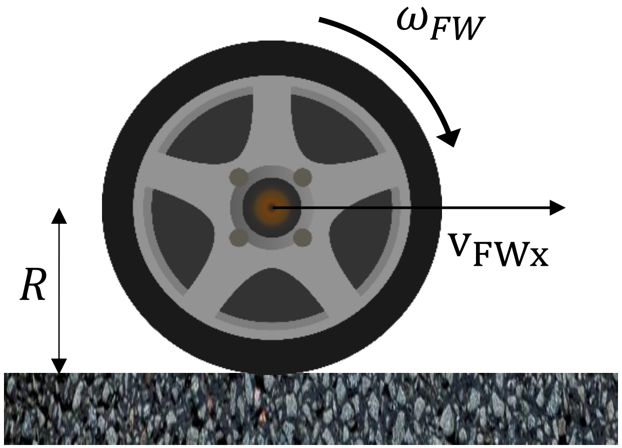




$$\alpha_{FW} = -\tan^{-1}\left(\frac{v_{FWy}}{v_{FWx}}\right)$$

$$\alpha_{FW} = -\tan^{-1}\left(\frac{\left(v_y + l_f\dot{\psi}\right)\cos\delta_f - v_x\sin\delta_f}{\left(v_y + l_f\dot{\psi}\right)\sin\delta_f + v_x\cos\delta_f}\right)$$

ST – Slip ratio

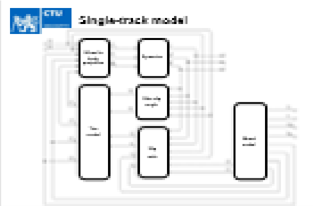


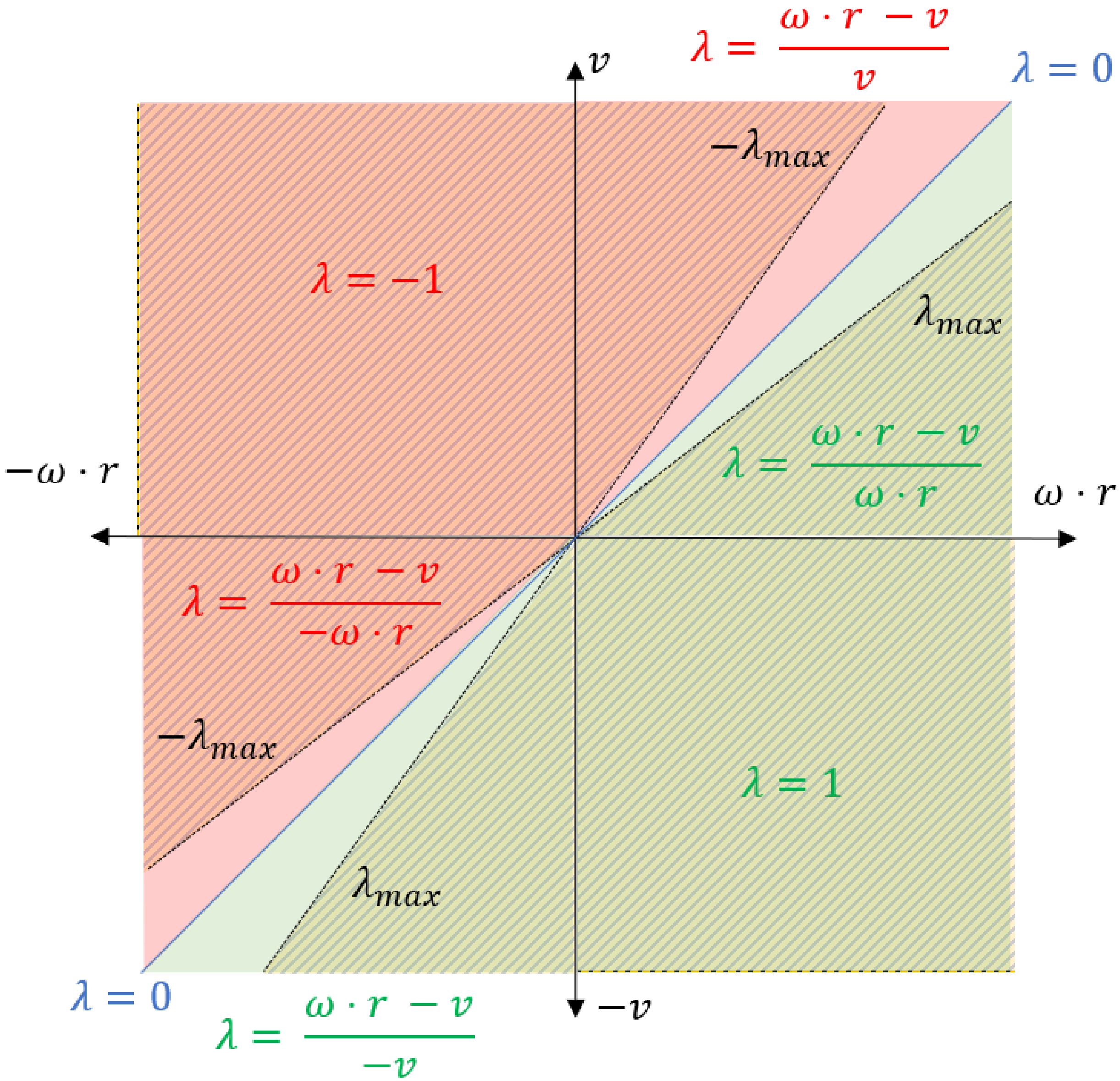
$$v_{FWci} = \omega_{FW} \cdot R$$

$$v_{FWx} = \left(v_y + l_f \dot{\psi}\right) \sin \delta_f + v_x \cos \delta_f$$

$$\lambda_{FW} = \frac{v_{FWci} - v_{FWx}}{\max(|v_{FWx}|, |v_{FWci}|)}$$

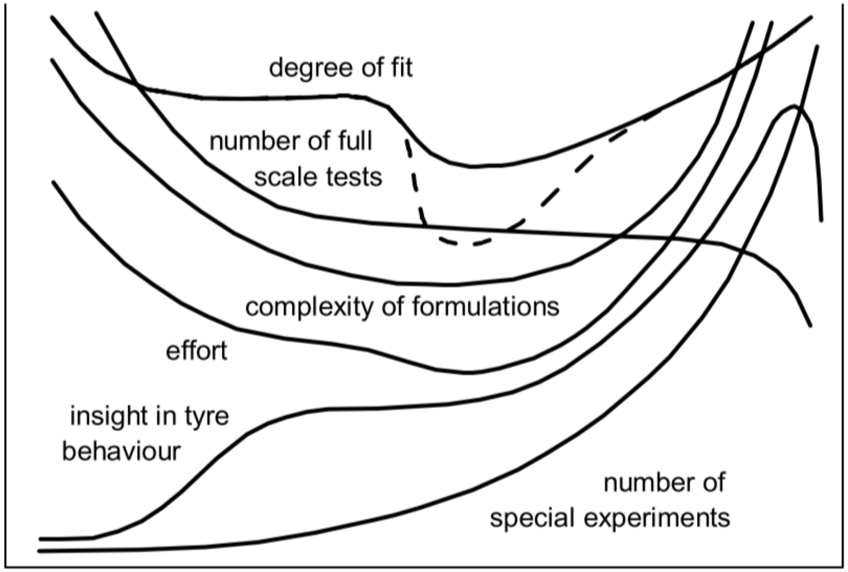
ST – Slip ratio





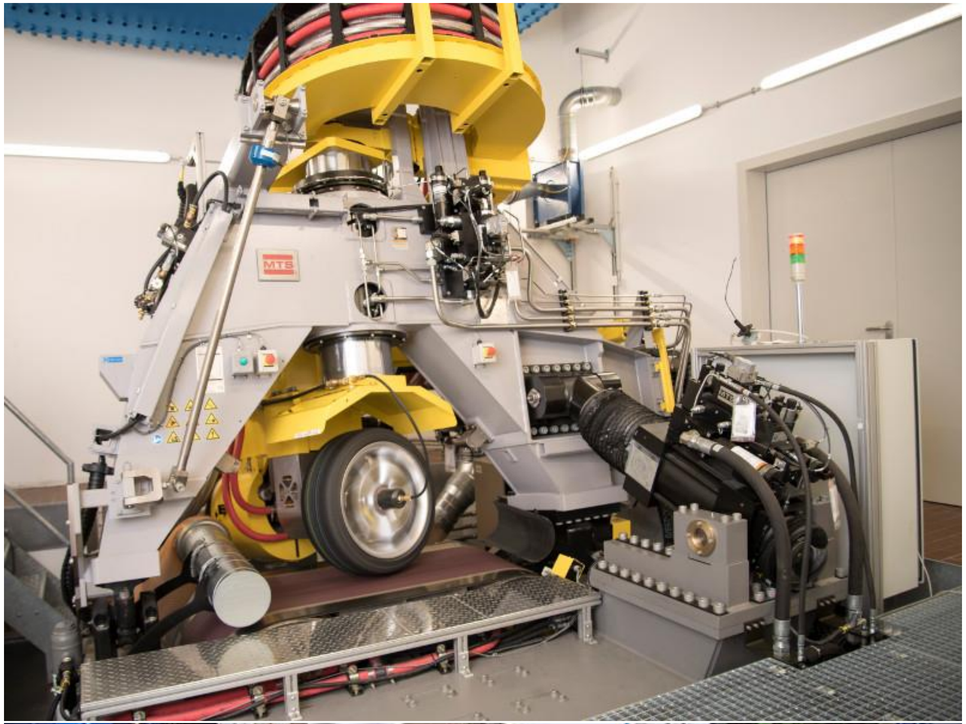
ST – Tire model (1)

from experimental data only	using similarity method	through simple physical model	through complex physical model
fitting full scale tyre test data by regression techniques	distorting, rescaling and combining basic characteristics	using simple mechanical representation, possibly closed form solution	describing tyre in greater detail, computer simulation, finite element method

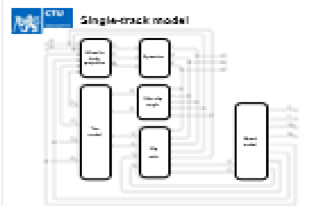


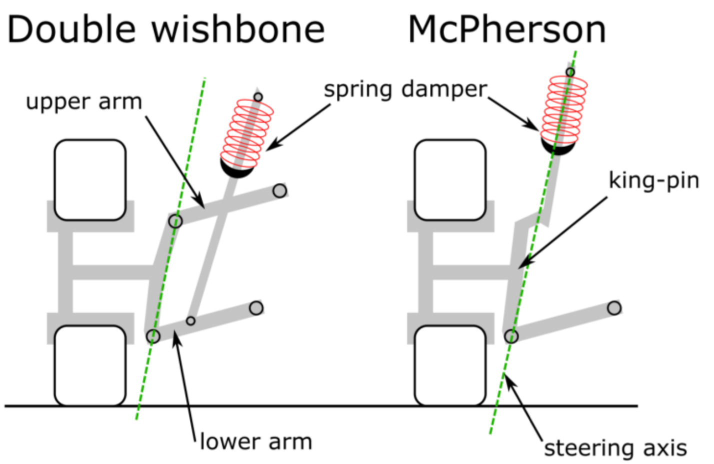
approach more

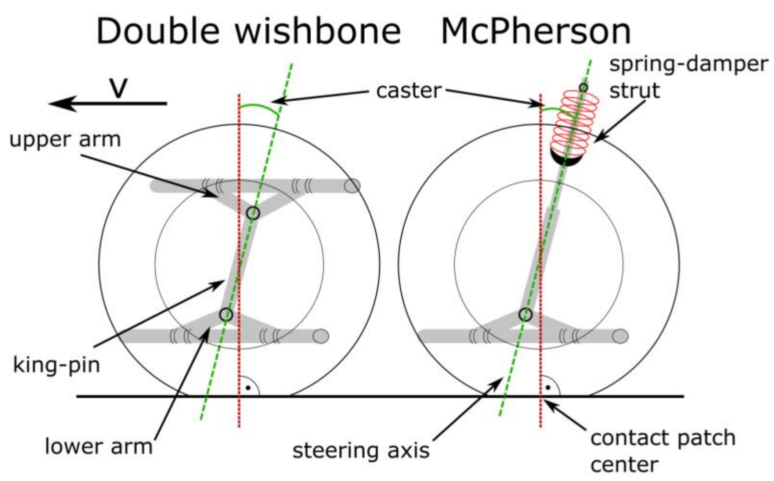
← empirical theoretical →

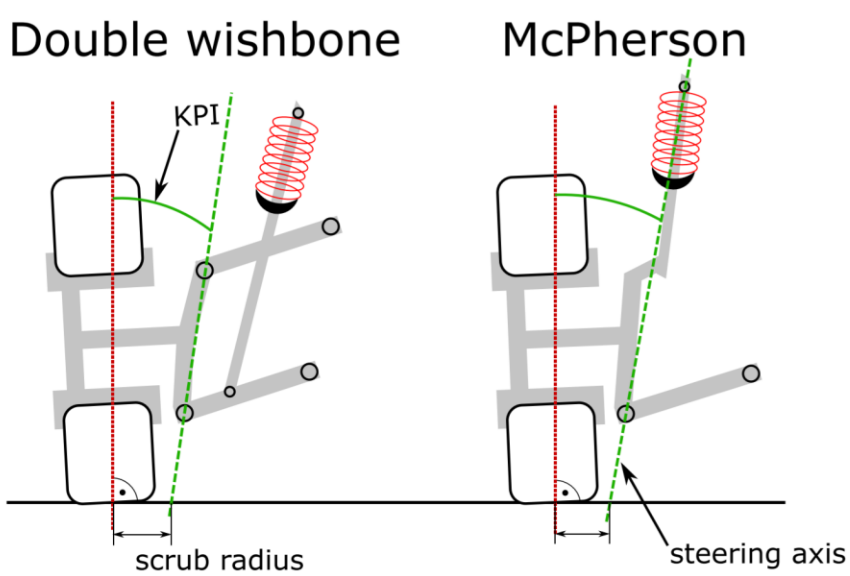


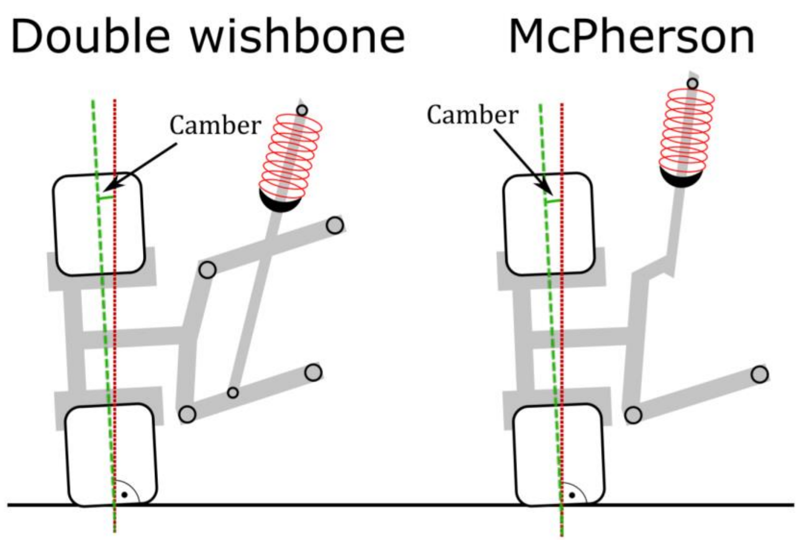
ST – Tire model (2)



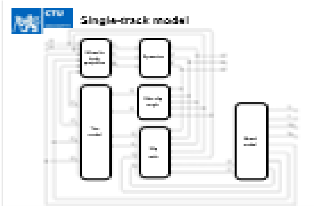


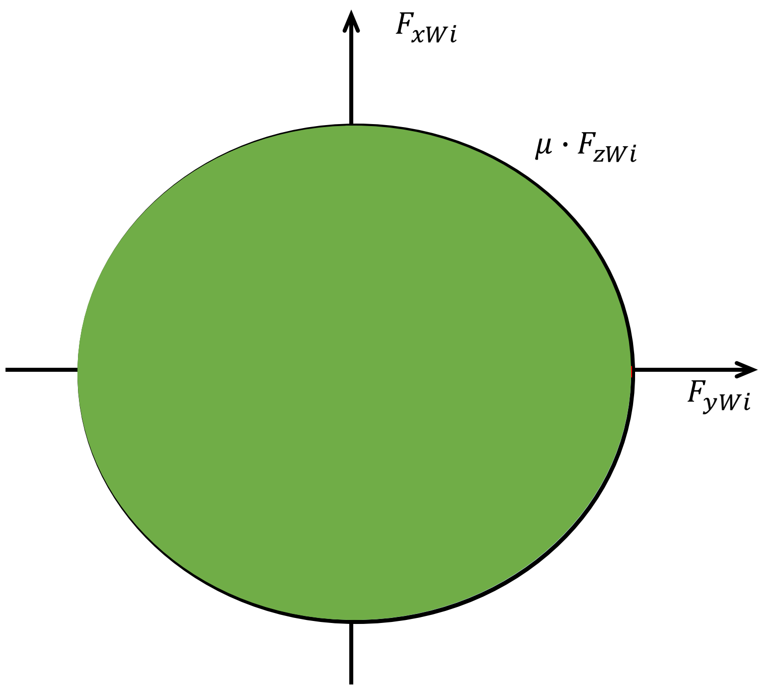


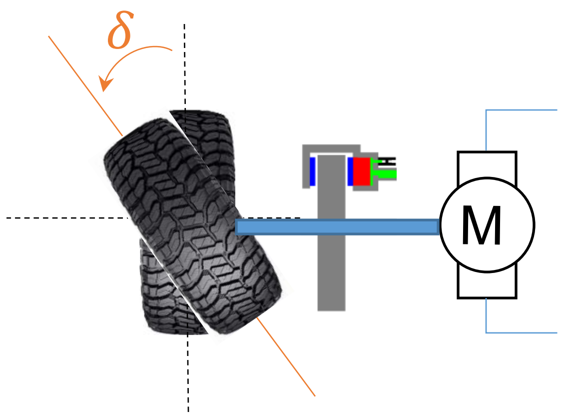


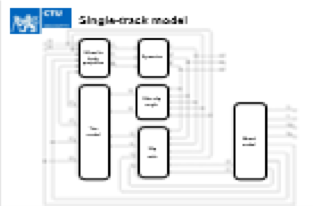


ST – Tire model (3)

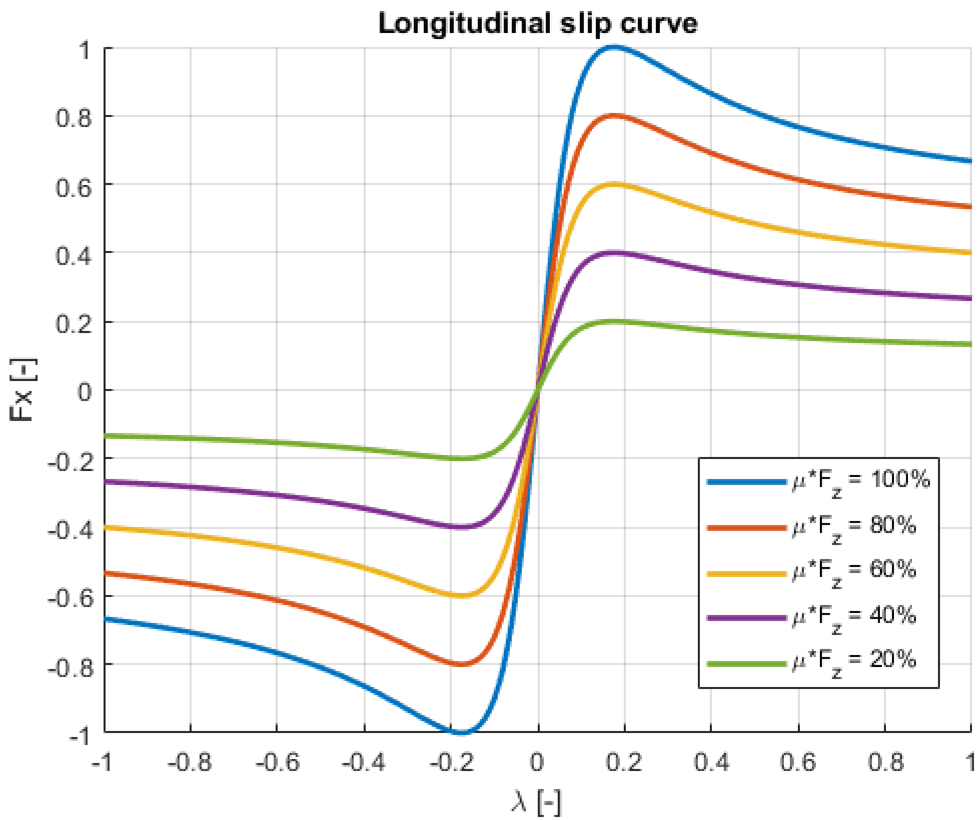








ST – Tire model (4)



$$F_{FWx} = D\mu F_{FWz}\sin\left(C\tan^{-1}\left(B\lambda_{FW} - E\left(B\lambda_{FW} - \tan^{-1}(B\lambda_{FW})\right)\right)\right)$$