Homework 3B: Current Sensor Fault Report

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The homework exercise is based on a simulation of a DC motor. There are two models, one for healthy state and the other for faulty state. Two sensors are modelled: rotation speed and current. It is assumed that current sensor is not woking from time to time, and the task is to detect the faulty regions.

Problem 1: Design the bank of state-space models to model healthy and faulty behaviour.

Problem Statement:

```
% Design healthy (m=1) state-space model
% Design faulty  (m=2) state-space model
% Find controller gain K_p to keep current limited to 200 A and no
% overshoot
% Analyze observability of m1
% Analyze observability of m2
```

Solution:

Healthy:

$$\begin{bmatrix} \dot{\omega} \\ i \end{bmatrix} = \begin{bmatrix} 0 & \frac{K_m}{J} \\ -\frac{K_p}{L} & -\frac{R+K_m}{L} \end{bmatrix} \begin{bmatrix} \omega \\ i \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{J} \\ \frac{K_p}{L} & 0 \end{bmatrix} \begin{bmatrix} \omega_{ref} \\ T_d \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \omega \\ i \end{bmatrix} + 0$$

Faulty:

$$\begin{bmatrix} \dot{\omega} \\ i \end{bmatrix} = \begin{bmatrix} 0 & \frac{K_m}{J} \\ -\frac{K_p}{L} & -\frac{R+K_m}{L} \end{bmatrix} \begin{bmatrix} \omega \\ i \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{J} \\ \frac{K_p}{L} & 0 \end{bmatrix} \begin{bmatrix} \omega_{ref} \\ T_d \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \omega \\ i \end{bmatrix} + 0$$

```
% Motor parameters
R = 0.1;                 % Ohms
L = 0.5;                 % Henrys
Km = 0.5;                % motor constant
J = 10;                  % kg.m^2/s^2
Kp = 2;
```

Disturbance (Load torque) has been modelled as pseudo-binary signal from -40 to 60 Nm, therefore depicting changing torque values with mean 50 and deviation +- 10 Nm.

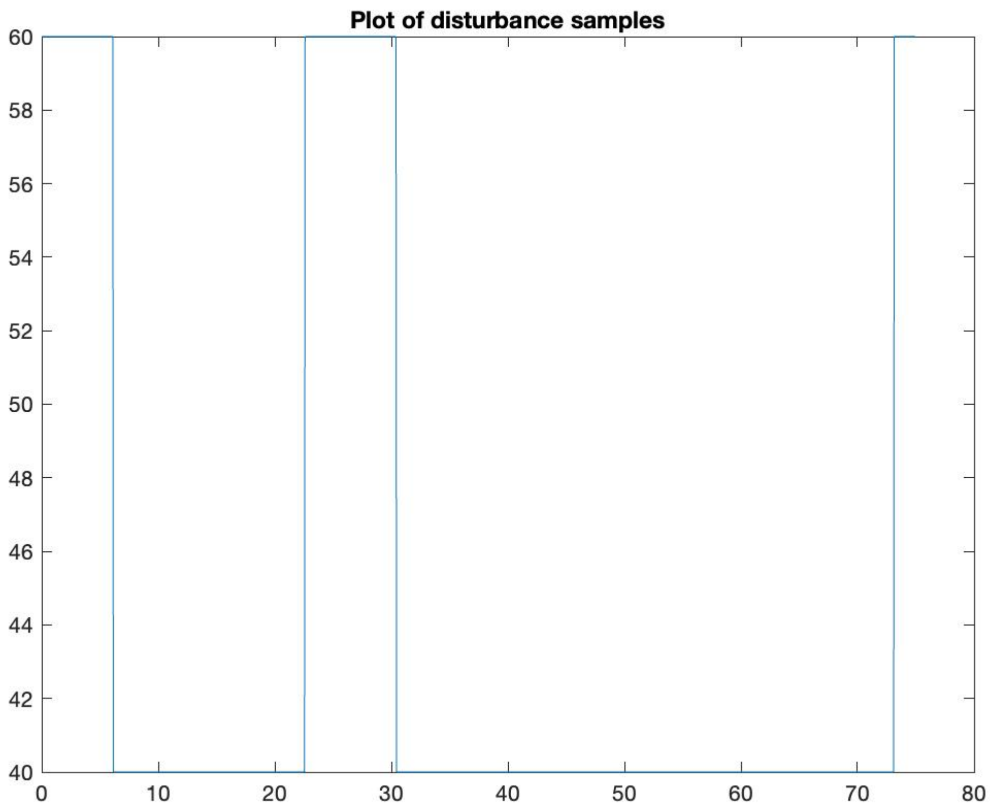

Fig.1 Disturbance (Nm) vs time (s)

Reference rotation speed was selected as a rising signal from 0 to 100 rad/s.

The number of samples is selected to be equal to 1500

It is possible to observe that even with simple P controller it is possible to achieve reasonable behaviour. Previously, it was thought that some more complicated PID controller could be beneficial. Nevertheless, the controller tracks the reference:

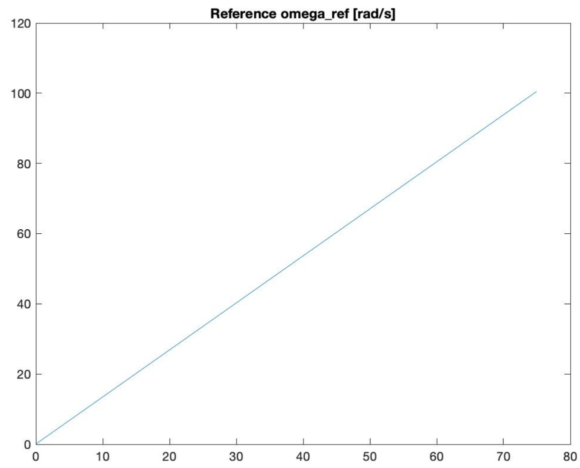

Fig.2 Reference rotation speed (rad/s) vs time (s)

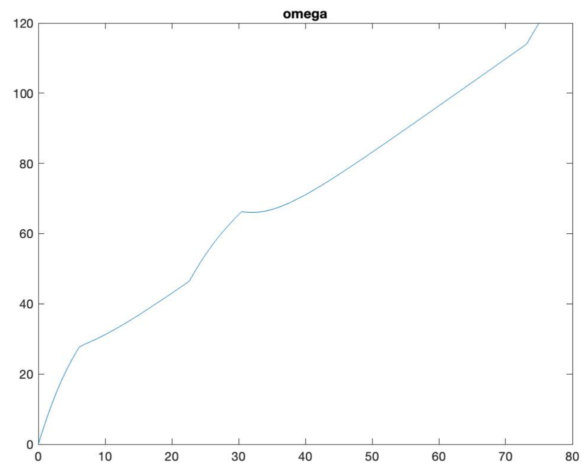

Fig. 3 Response of healthy system(rotation speed) vs. time (s)

And the current is within bounds of +- 200 A:

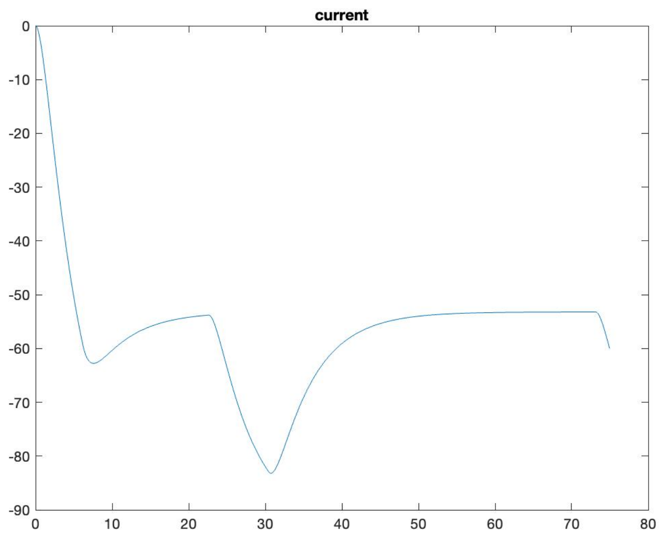

Fig.4 Response of healthy system(current A) vs. time (s)

Observability of the two systems have been identified with number of unobservable modes being equal to 0 for both cases.

Problem 2: Process-noise properties. Input-output data generation.

Problem Statement:

```
% Select and justify process noise properties
% Find input-output data with multiple sensor faults
```

Solution

For solving this task, the equations for noise properties of sampling were obtained. (Slide 108 in the course's lecture slides)

$\epsilon = \frac{T_s - T_c}{T_s}$
$A_{async} = e^{A_c T_s}$
$B_{async} = \int_0^{T_s} e^{A_c \nu} d\nu B_c$
$C_{async} = C_c e^{A_c \epsilon Ts}$
$D_{async} = C_c \int_0^{\epsilon T_s} e^{A_c \nu} d\nu B_c$
$Q = \int_0^{T_s} e^{A_c \nu} Q_c e^{A_c^T \nu} d\nu$
$S = \int_0^{\epsilon T_s} e^{A_c \nu} Q_c e^{A_c^T \nu} C_c^T d\nu$
$R = \int_0^{\epsilon T_s} C_c e^{A_c \nu} Q_c e^{A_c^T \nu} C_c^T d\nu + R_c$

Equations from slides, asynchronous schema

After integration:

$B_{async} = A^{-1}(e^{A_c Ts} - e^{A_c * 0})B_c$
$D_{async} = C_c A_c^{-1}(e^{A_c \epsilon Ts} - e^{A_c * 0})B$

$Q = Q_c(A + A^T)^{-1}(e^{(A+A^T)Ts} - e^{(A+A^T)*0})$
$S = Q_c(A + A^T)^{-1}(e^{(A+A^T)\epsilon Ts} - e^{(A+A^T)*0})C^T$
$R = R_c + C_cQ_c(A + A^T)^{-1}(e^{(A+A^T)\epsilon Ts} - e^{(A+A^T)*0})C^T$

Equations after integration, asynchronous schema

The controller time Tc is selected as 0, thus leading to synchronous sampling. Sampling Ts time is given and is equal to 0.05 s.

With the selection of corresponding matrices A and A_f, B and B_f, C and C_f, D and D_f, the models of each of the two mode-dependent Kalman Filters were constructed.

Qc is selected to be equal to 0.4 and 0.1 for healthy and faulty state-space models respectively. Rc is given is equal to 0.1 .

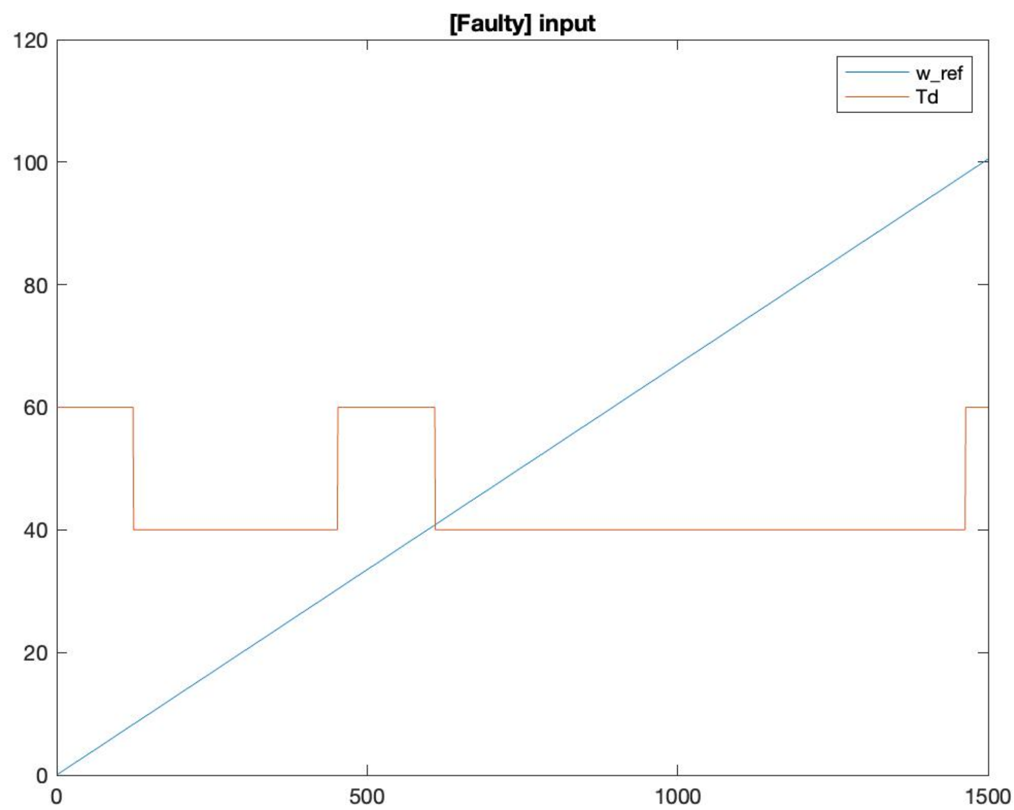

Fig5. Inputs to the system (w_ref [rad/s] Ts[Nm]) vs samples(N = 1500)

The faulty signal is obtained by calculating state from faulty models at certain periods of the whole time of simulation. The graph that depicts the situation is following:

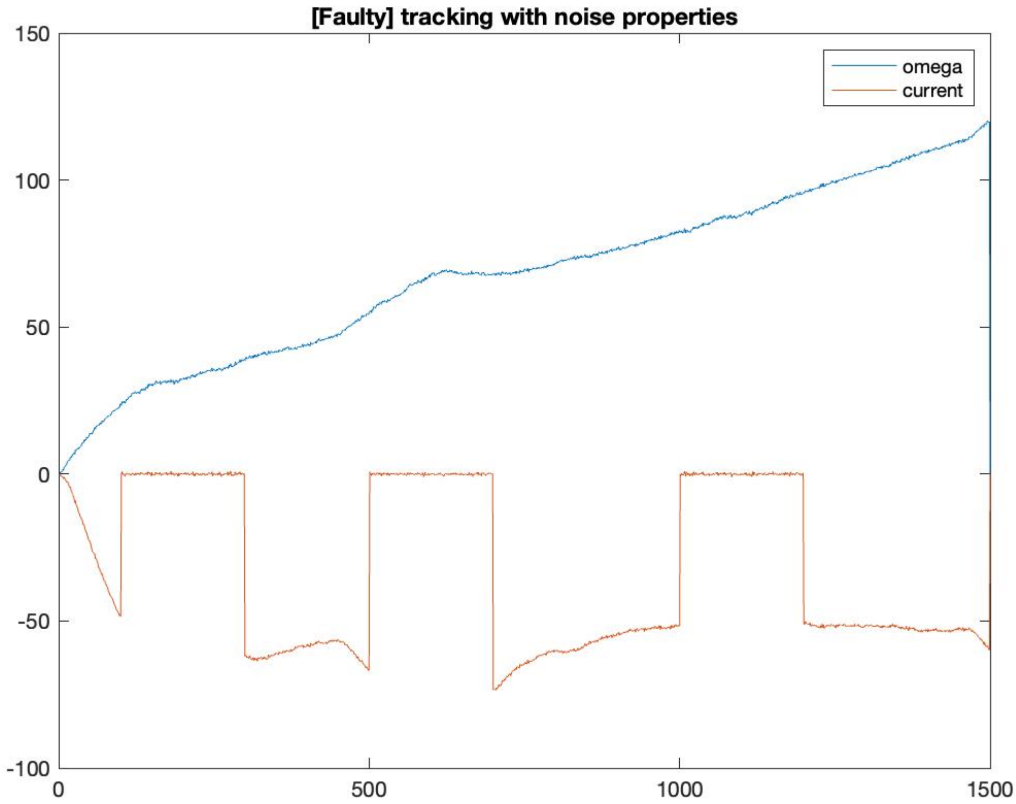

Fig. 6 System response with multiple current sensor faults (3) : omega[rad/s] current[A] vs samples [N=1500]

It can be observed that there are three occasions where sensor does not work and its signal is equal to zero.

Problem 3: IMM design. Faults detection. Estimation of load.

Problem Statement:

```
% Design IMM algorithm
% Evaluate unmeasurable load
% Detect faults of current sensor
% p12 = 0.001 transition to fault
% p21 = 0.005 recovery
```

Solution:

In order to implement Iteractive-Multiple Models filter that would track two modes of operation of the current sensor on DC motor, two Kalman filters that run at the same time were constructed. Unlike included in Matlab function "kalman", the algorithm had to run each step of the KF in parallel. This was achieved by taking each step of KF twice: one of mode "healthy" and one for mode “faulty".

At the input, the algorithm takes previously estimated states from the two filters and mixes them accordingly to transition probabilities. Depending on the previous mode probabilities, it selects some portions of the first input and some portions of the second input. Data update and time update steps are similar to a regular KF. The main part of the algorithm are the lines where the probabilities of each of the signals are updated. There were found many examples with usage of mvnpdf, which is multi-variable normal probability density function. However, due to lack of comprehension of its usage, the transition probabilities decreased to 0 for both states, which resulted into division by zero problem. Finally, it was possible to think through and find another way of how to update probabilities with usage of co-variance information of each of the filters.

By taking size(determinant) of the estimation co-variance as the objective factor, it is possible to design whether the signal should be taken more from first filter or the second.

Additionally it is possible to improve the fault detection by adding residual co-variances to estimation co-variances and taking the size(determinant) of the two.

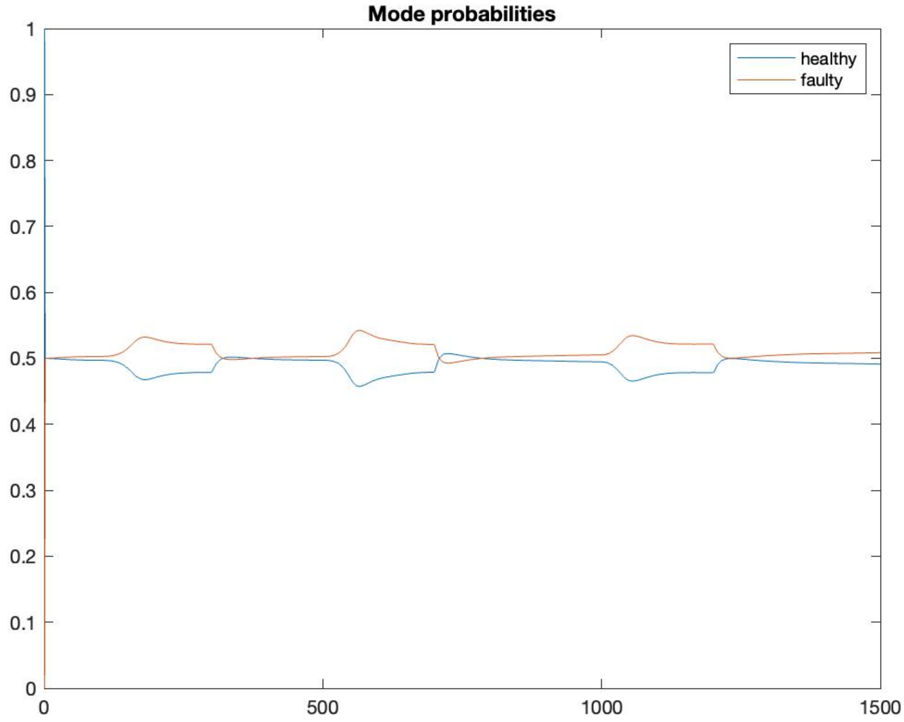

Fig. 7 Probabilities of both modes over samples (N=1500)

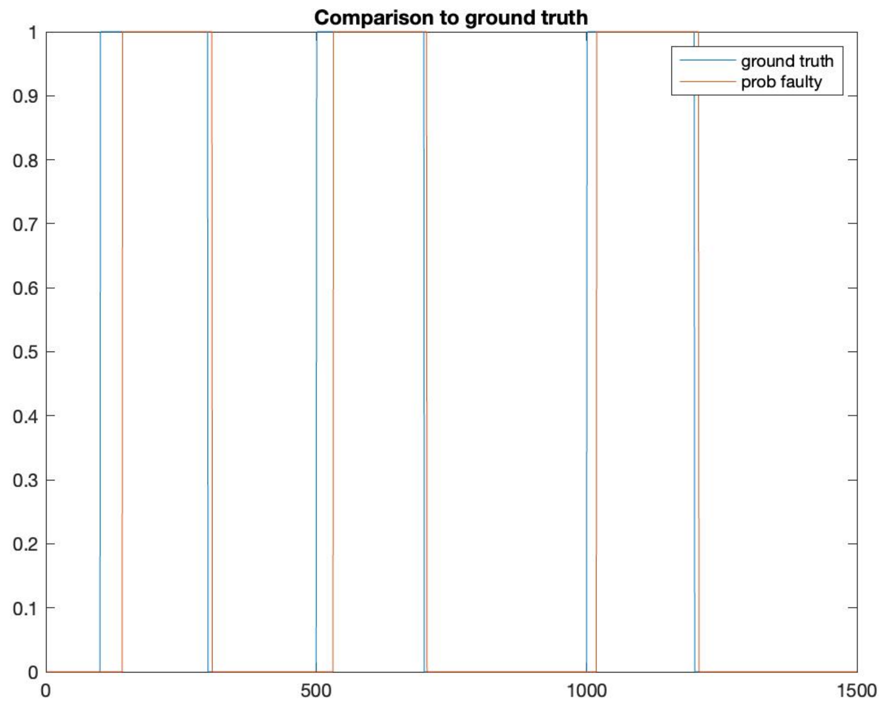

Fig. 8 Comparison of fault detection with ground truth information

Unmeasurable torque is computed from the first state space equation that

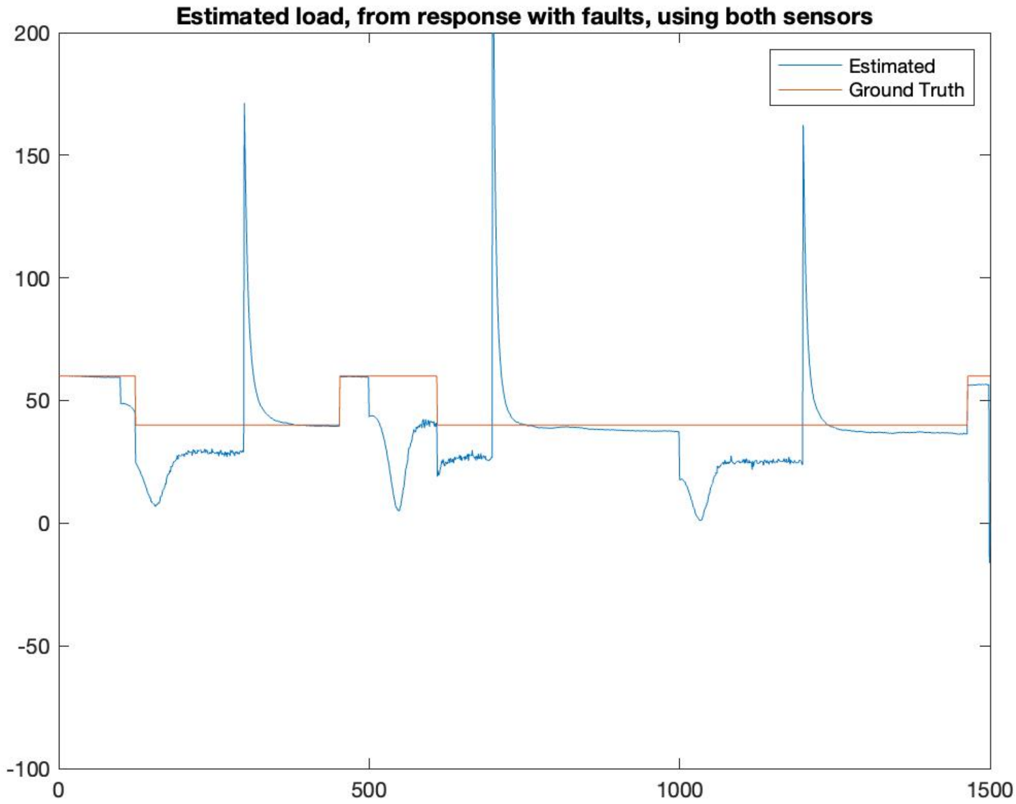

Fig. 9 Estimated load and ground truth load [Nm] vs samples (N=1500)

defines rot speed. The calculation involving second state space equation did not bring any reasonable results.

It can be seen that there are spike of estimated signal, exactly at points of recovery from fault. Corrupted signal results in corrupted estimation of load. There can be seen three similar regions that correspond to sensor faults (just before the recovery spike).

Problem 4: Analysis of performance, comparison to single rotation speed Kalman Filter, justification of usage of current sensor

Problem Statement:

```
% Analyse the performance of fault detection
% Compare IMM to KF using only rotation speed sensor
% Is it possible to justify use of current sensor?
```

Solution:

Although there are not high probability differences between mode 1 and mode 2, this configuration of the algorithm is probably the most accurate so far, because, it identifies the faults in exact time, or with a delay of approximately 50 samples, that is 50*0.05 = 2.5 seconds.

The error of fault detection with respect to ground truth is approximately: 0.28.

Error of load estimation with respect to ground truth is approximately: 19.50 Nm.

Interestingly, it is possible to estimate the load in a most accurate way, by using single Kalman Filter with no current sensor (faulty model).

The error in this case drops down to 1.56 Nm

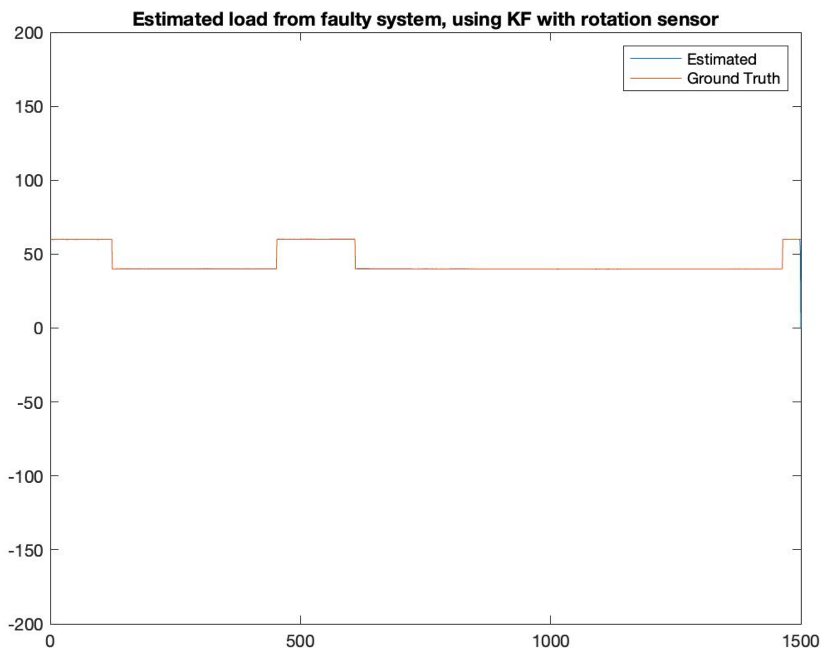

Fig. 10 - Estimation of load using only rotation KF

Due to outstanding result of load estimation of single Kalman Filter, I cannot justify usage of current sensor. This, probably, is the reason why the current sensors are not so predominant in a typical DC motor.

Problem 5: Lessons learned

1) IMM is best suited for fault detection

2) KF using only rotation speed sensor is enough for estimation of load

3) KF is better for estimation of load than IMM