Estimation, Filtering and Detection

Homework 2C: Asynchronous Sampling

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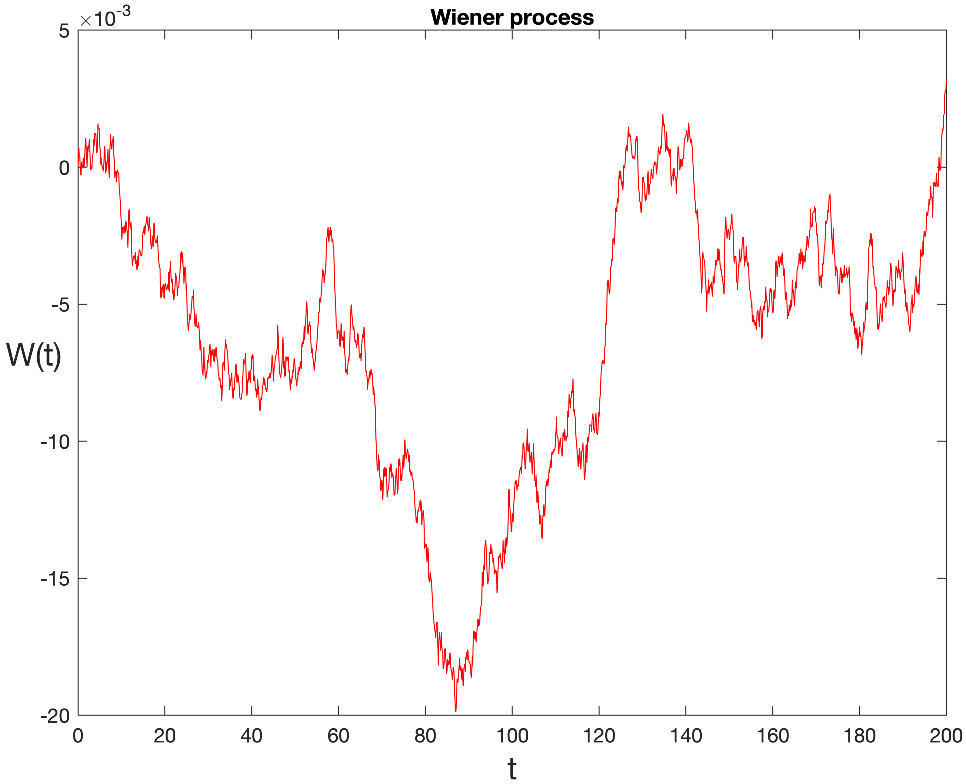
MACROS

```
clear all;
storeFigures = true; % set true if you want to save plots
```

Problem 1:

```
% Y(s) = 1/(1+s*tau)^2 *(U(s)+D(s))
% Find continusous-time stochastic state space model
% Create disturbance as a Wiener process
% Intensity of disturbance Q = 0.001

Qc  = 0.001;
Rc  = 0.01*0.01;
n_seconds = 200;

% Solution
% Continuous-time system

% Wiener Process
N = 2000;
dt = n_seconds/N;

dW = zeros(1,N);
W = zeros(1,N);
dW(1) = Qc*sqrt(dt)*randn();
W(1) = dW(1);
for j = 2:N
    dW(j)= Qc*sqrt(dt)*randn();
    W(j) = W(j-1) + dW(j);
end
figure(1);
plot([0:dt:n_seconds],[0,W],'r-')
xlabel('t','FontSize',16)
ylabel('W(t)','FontSize',16,'Rotation',0)
title("Wiener process");
```



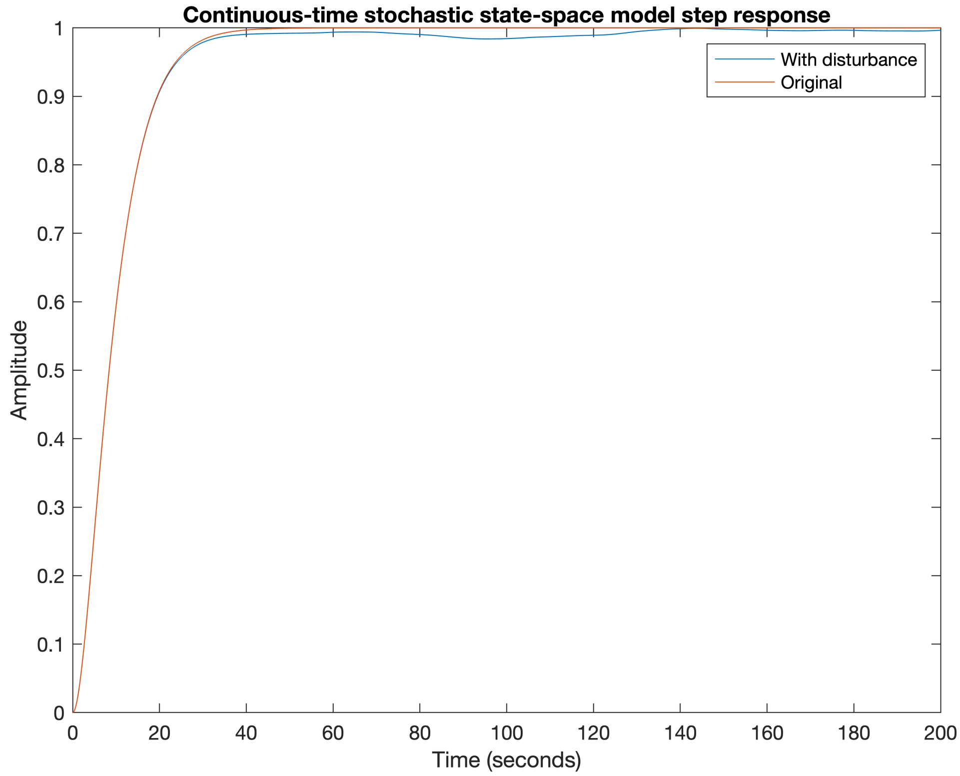

```
% mean and variance
MeanWiener = mean(W)
```

MeanWiener = -0.0062

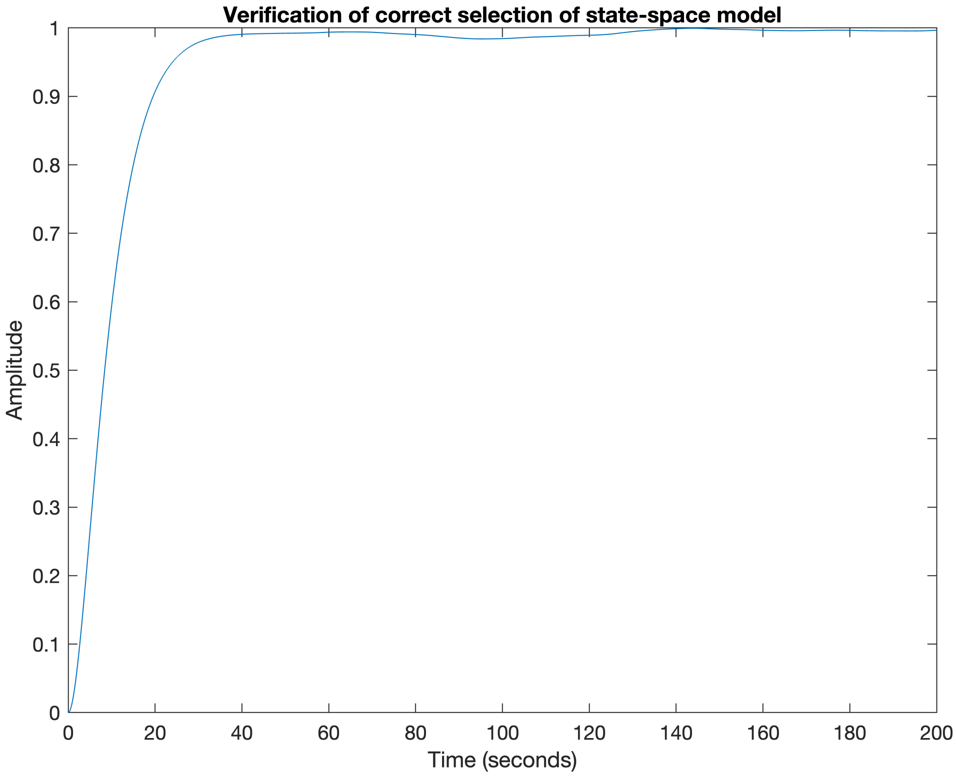
```
VarianceWiener = var(W)
```

VarianceWiener = 2.3800e-05

```
tau = 50*dt; % 50 seconds
sys = tf(1,[tau^2 2*tau 1]);
step_input = ones(1,N+1);
disturbance = [0,W];
response = lsim(sys,step_input+disturbance,[0:dt:n_seconds]);
response_original = lsim(sys,step_input,[0:dt:n_seconds]);

figure(2);
plot([0:dt:n_seconds],response);
ylabel("Amplitude");
xlabel("Time (seconds)");
title("Continuous-time stochastic state-space model step response");
hold on
plot([0:dt:n_seconds],response_original);
legend("With disturbance","Original");
```




```
legend show
fix_ylim = ylim;

% .
% x = Ax + Bu + Gw      {State equation}
% y = Cx + Du +  v      {Measurements}

[A,B,C,D] = tf2ss(1,[tau^2 2*tau 1]);
G = [1;0];
B = [B G];
D = [D 0];
sys = ss(A,B,C,D);
step_input = ones(1,N+1);
disturbance = [0,W];
%% verification of stochastic state space model derivation
response = lsim(sys,[step_input;disturbance],[0:dt:n_seconds]);
figure(3);
plot([0:dt:n_seconds],response);
ylabel("Amplitude");
xlabel("Time (seconds)");
title("Verification of correct selection of state-space model");
```



Problem 2:

```
% Find discrete-time model
% Find Kalman filter
% Ts = 20s
% Evaluate predicted and filtered values
Ts = 20*dt;
dsys = c2d(sys,Ts,'tustin');
Qn = Qc*eye(2);
Rn = Rc*eye(1);
[kest,L,P,M,Z] = kalman(dsys,Qn,Rn);
P % minimal prediction co-variance  P(t|t-1)
```

```
P = 2×2
    0.0043    0.0045
    0.0045    0.0335
```

```
Z % minimal filtering co-variance   P(t|t)
```

```
Z = 2×2
    0.0038    0.0019
    0.0019    0.0212
```

Problem 3:

```
% Find Kalman Filter models
```

```
% Use asynchronous sampling with controller computation time
% Tc = 10/1/0.1/s
% Compare properties of noise models
% Evaluate P (predicted value)

Ts = 20*dt;
Tcs = [10 1 0.1].*dt;
for i = 1:3
    Tc = Tcs(i);
    etta = (Ts-Tc)/Ts;
    funB = @(x) expm(A.*x);
    funD = @(x) expm(A.*x);
    funQ = @(x) Qc*expm(A.*x)*expm(A'.*x);
    funS = @(x) Qc*expm(A.*x)*expm(A'*x)*C';
    funR = @(x) Qc*C*expm(A.*x)*expm(A'.*x)*C';

    Aasync = expm(A.*Ts);
    Casync = C*expm(A.*etta*Ts);
    Basync = integral(funB,0,Ts,'ArrayValued', true);
    Dasync = integral(funD,0,etta*Ts,'ArrayValued', true);
    Q = integral(funQ,0,Ts,'ArrayValued', true);
    S = integral(funS,0,etta*Ts,'ArrayValued', true);
    R = integral(funR,0,etta*Ts,'ArrayValued', true);
    Basync = Basync*B;
    Dasync = C*Dasync*B;
    R = Rc + R;

    disp("Tc")
    disp(Tcs(i)/dt)
    [kest,L,P,M,Z] = kalman(ss(Aasync,Basync,Casync,Dasync,Tcs(i)),Q,R,S);
    P
    Z

end
```

```
Tc
    10
P = 2×2
    0.0114    0.0127
    0.0127    0.0522
Z = 2×2
    0.0082    0.0034
    0.0034    0.0259
Tc
     1
P = 2×2
    0.0090    0.0060
    0.0060    0.0324
Z = 2×2
    0.0069    0.0015
    0.0015    0.0224
Tc
    0.1000
P = 2×2
    0.0086    0.0055
    0.0055    0.0310
Z = 2×2
```

```
    0.0068    0.0014
    0.0014    0.0221
```

Problem 4:

```
% Show impact of neglecting S in case of asynchronous sampling
for i = 1:3
    Tc = Tcs(i);
    etta = (Ts-Tc)/Ts;

    funB = @(x) expm(A.*x);
    funD = @(x) expm(A.*x);
    funQ = @(x) Qc.*expm(A.*x)*expm(A'.*x);
    funS = @(x) Qc.*expm(A.*x)*expm(A'.*x)*C';
    funR = @(x) Qc.*C*expm(A.*x)*expm(A'.*x)*C';

    Aasync = expm(A*Ts);
    Casync = C*expm(A*etta*Ts);
    Basync = integral(funB,0,Ts,'ArrayValued', true);
    Dasync = integral(funD,0,etta*Ts,'ArrayValued', true);
    Q = integral(funQ,0,Ts,'ArrayValued', true);
    S = integral(funS,0,etta*Ts,'ArrayValued', true);
    R = integral(funR,0,etta*Ts,'ArrayValued', true);
    Basync = Basync*B;
    Dasync = C*Dasync*B;
    R = Rc + R;

    disp("Tc")
    disp(Tcs(i)/dt)
    [kest,L,P,M,Z] = kalman(ss(Aasync,Basync,Casync,Dasync,Ts),Q,R,S);
    disp("With S")
    P
    Z
    disp("Value of determinant with S")
    disp(det(P))
    disp("With S=0")
    [kest,L,P,M,Z] = kalman(ss(Aasync,Basync,Casync,Dasync,Ts),Q,R);
    P
    Z
    disp("Value of determinant without S")
    det(P)
end
```

```
Tc
    10
With S
P = 2×2
    0.0114    0.0127
    0.0127    0.0522
Z = 2×2
    0.0082    0.0034
    0.0034    0.0259
Value of determinant with S
   4.3674e-04
With S=0
```

```
P = 2×2
    0.0117    0.0141
    0.0141    0.0559
Z = 2×2
    0.0080    0.0037
    0.0037    0.0263
Value of determinant without S
ans = 4.5568e-04
Tc
     1
With S
P = 2×2
    0.0090    0.0060
    0.0060    0.0324
Z = 2×2
    0.0069    0.0015
    0.0015    0.0224
Value of determinant with S
   2.5477e-04
With S=0
P = 2×2
    0.0102    0.0086
    0.0086    0.0386
Z = 2×2
    0.0072    0.0018
    0.0018    0.0235
Value of determinant without S
ans = 3.2013e-04
Tc
    0.1000
With S
P = 2×2
    0.0086    0.0055
    0.0055    0.0310
Z = 2×2
    0.0068    0.0014
    0.0014    0.0221
Value of determinant with S
   2.3729e-04
With S=0
P = 2×2
    0.0100    0.0081
    0.0081    0.0373
Z = 2×2
    0.0071    0.0017
    0.0017    0.0234
Value of determinant without S
ans = 3.0741e-04
```

```
%% Conclusion: inclusion of S decreases the values of covariances
```

Problem 5:

```
% Evaluate filtered P(t|t)
% Use filter design for system with decorrelated noise
Tc = Tcs(3);
etta = (Ts-Tc)/Ts;

funB = @(x) expm(A.*x);
funD = @(x) expm(A.*x);
funQ = @(x) Qc.*expm(A.*x)*expm(A'.*x);
```

```
funS = @(x) Qc.*expm(A.*x)*expm(A'.*x)*C';
funR = @(x) Qc.*C*expm(A.*x)*expm(A'.*x)*C';

Aasync = expm(A*Ts);
Casync = C*expm(A*etta*Ts);
Basync = integral(funB,0,Ts,'ArrayValued', true);
Dasync = integral(funD,0,etta*Ts,'ArrayValued', true);
Q = integral(funQ,0,Ts,'ArrayValued', true);
S = integral(funS,0,etta*Ts,'ArrayValued', true);
R = integral(funR,0,etta*Ts,'ArrayValued', true);
Basync = Basync*B;
Dasync = C*Dasync*B;
R = Rc + R;

A_decor = Aasync - S*inv(R)*Casync;
B_decor = Basync - S*inv(R)*Dasync;
Q_decor = Q - S*inv(R)*S';

disp("State covariance matrix P(t|t) for system model with decorrelated noise")
```

```
State covariance matrix P(t|t) for system model with decorrelated noise
```

```
[kest,L,P,M,Z] = kalman(ss(A_decor,B_decor,C,D,Ts),Q_decor,R);
Tc
```

```
Tc = 0.0100
```

```
P
```

```
P = 2×2
    0.0112    0.0157
    0.0157    0.0719
```

```
Z
```

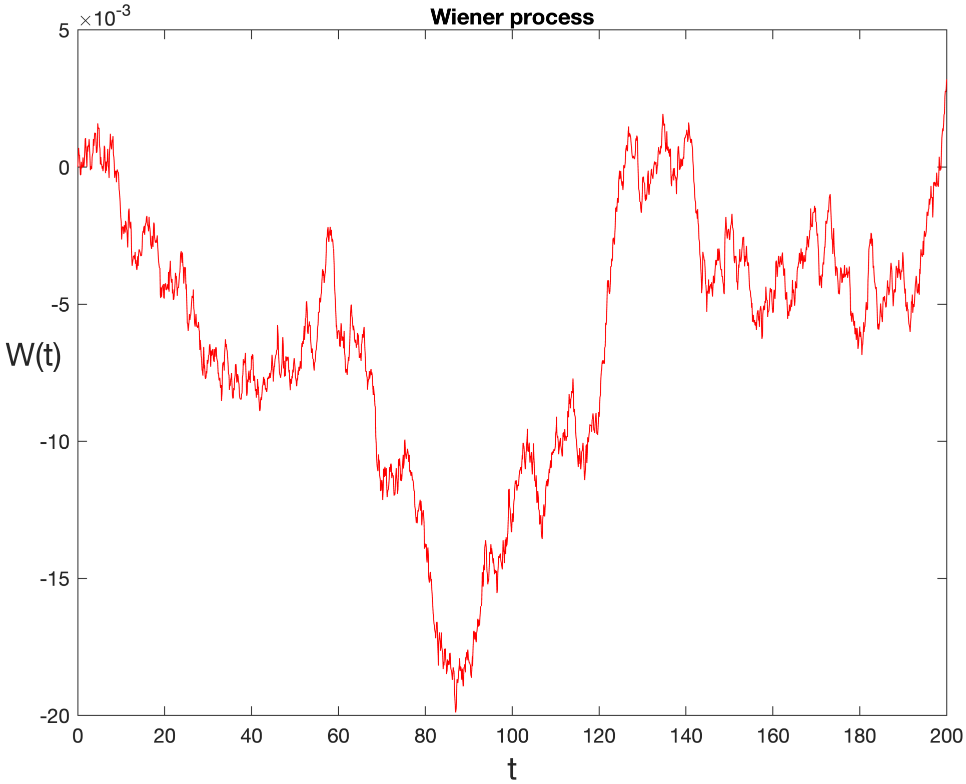
```
Z = 2×2
    0.0094    0.0075
    0.0075    0.0344
```

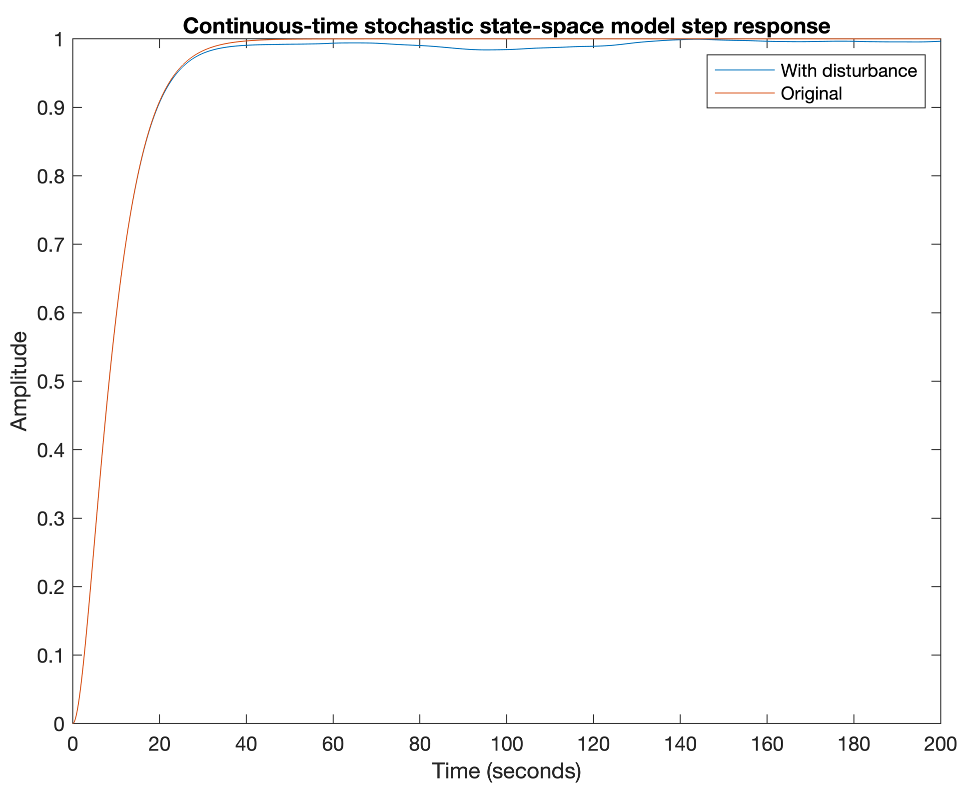
```
det(Z)
```

```
ans = 2.6823e-04
```

Additional tools

```
% Storing figures
if storeFigures
    for i=1:2
        filename = strcat('figure_',num2str(i));
        foldername = './figures/';
        saveas(figure(i),fullfile(foldername,filename),'jpg');
    end
end
```


Wiener process
W(t)
t
×10^-3


Continuous-time stochastic state-space model step response
Amplitude
Time (seconds)
With disturbance
Original