Homework 1B: Forgetting methods

Report

Timur Uzakov

The homework exercise is based on simulation model of a continuous-time process, the heat exchanger. The transfer function is following:

$$Y(s)/U(s) = 1/(1 + s\tau)^3$$

Tau is varying from 5,10,20 seconds and the three time intervals are given: 0…1000s,1000…2000s,2000…3000s. Sampling period is chosen to be 1 sec, ARX model structure is assumed. Noise variance is set to 0.01.

Conversion from continuous transfer function to discrete

It is possible to convert a continuous-time model to discrete-time model in two ways: the first is by using c2d function in MATLAB with method selected as 'tustin', which is the best; the second approach is to calculate values of a and b polynomials by inserting

$$s = \frac{z-1}{z+1} * \frac{2}{Ts},$$

which is a result of

$$z = \frac{1 + \frac{s * Ts}{2}}{1 - \frac{s * Ts}{2}}$$

and is known as the best discretisation method.

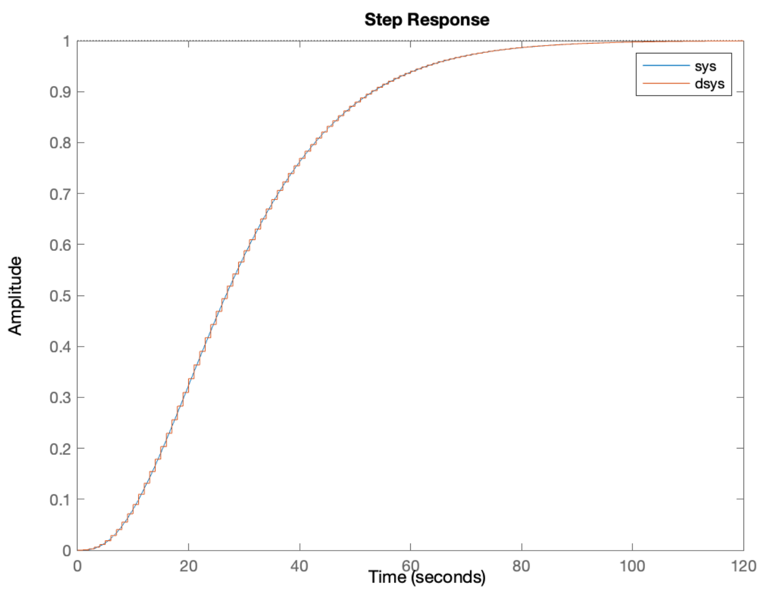


Fig. 1 The step-responses of continuous and discrete time systems

Model derivation and time-profile

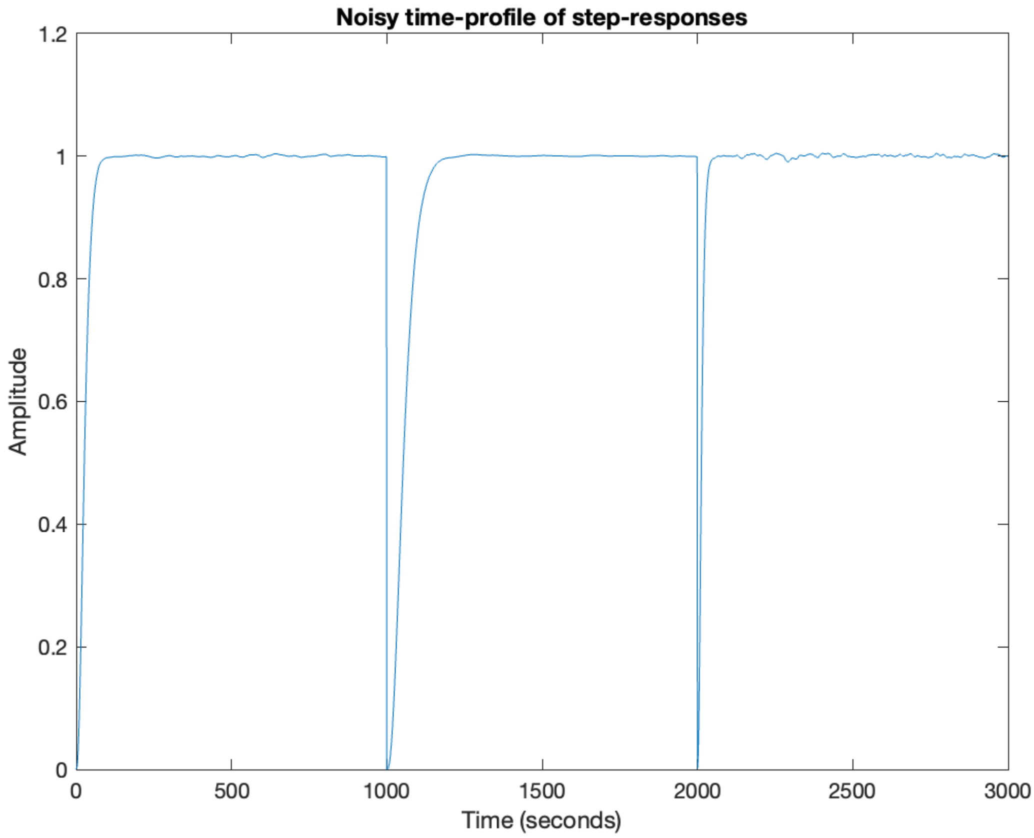


Fig.2 Time-profile of step responses for each of the tau, staring from 10,20 to 5 s

There were obtained three systems that correspond to each of the situations: *tau* being equal to 10,20 and 5 s. It appears that there is no dependancy on *tau* of the *b* polynomial of the ARX model, however when normalising the polynomial *a* by its first term, such that *a(1) = 1*, it is possible to see that *b* is also dependant on *tau*. The structure of the *c* polynomial also changes from 1 to the polynomial dependent on *tau*, because of the normalisation.

By using *syms* and *expand* commands in Matlab, it was possible to derive the terms of the three *a, b* and *c* polynomials. From viewing the time profile of the systems, it is possible to observe how the response time changes for each of the tau, with tau(2) = 20s being the longest.(fig. 2)

The polynomials have the following form:

$$a = \frac{[(8*tau^3+12*tau^2+6*tau+1)\,(-24*tau^3-12*tau^2+6*tau+3)\,(24*tau^3-12*tau^2-6*tau+3)\,(-8*tau^3+12*tau^2-6*tau+1)]}{(8*tau^3+12*tau^2+6*tau+1)}$$

$$b = \frac{[1\;3\;3\;1]}{8*tau^3+12*tau^2+6*tau+1}$$

$$c = \frac{1}{(8*tau^3 + 12*tau^2 + 6*tau + 1)}$$

Table 1 - polynomials of discrete time heat-exchanger system

Input-output data from three pseudo-random binary sequence signals

For simulation purposes three signals were created. All of them are pseudo-random binary sequence signals, that capture probabilistic nature of an input signal. With the threshold set to 0.5 we can obtain a signal with energy spread equally along all frequencies, that is white noise. By raising the factor to 0.9, for example, we can obtain high-frequency PRBS signal. And low frequency signal can be obtained from lower threshold value, for example, 0.01.

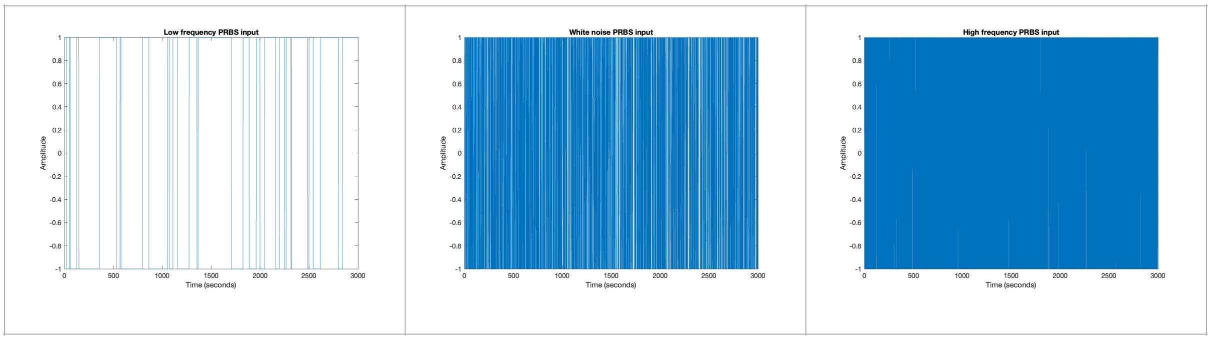


Fig. 3 PRBS signals with threshold set to 0.01, 0.5, 0.9

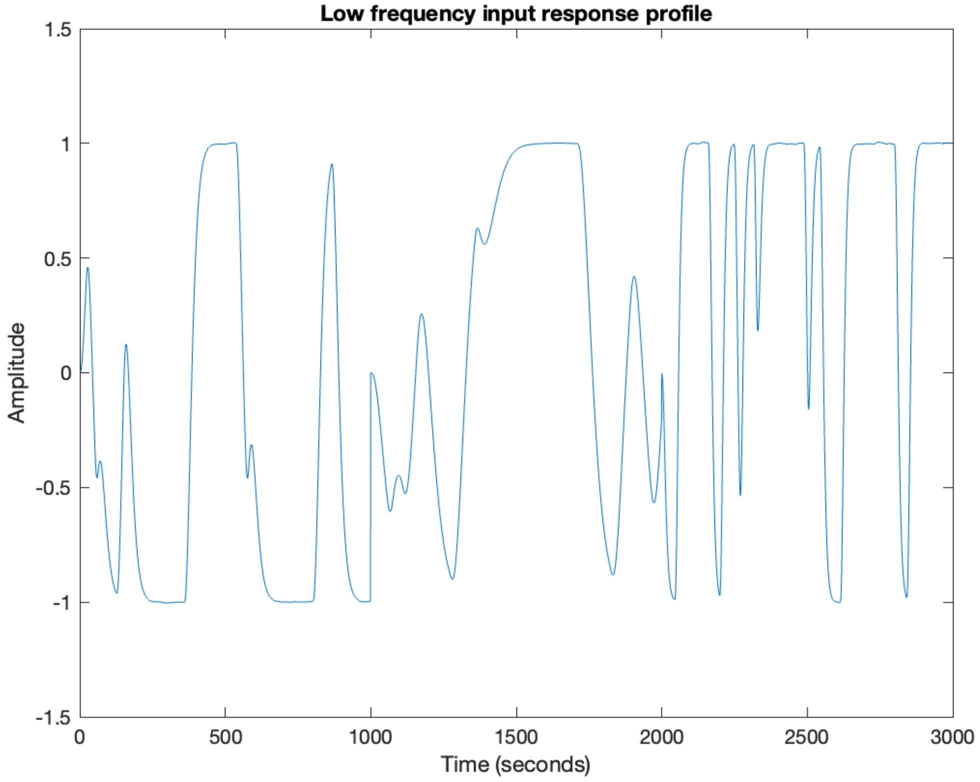
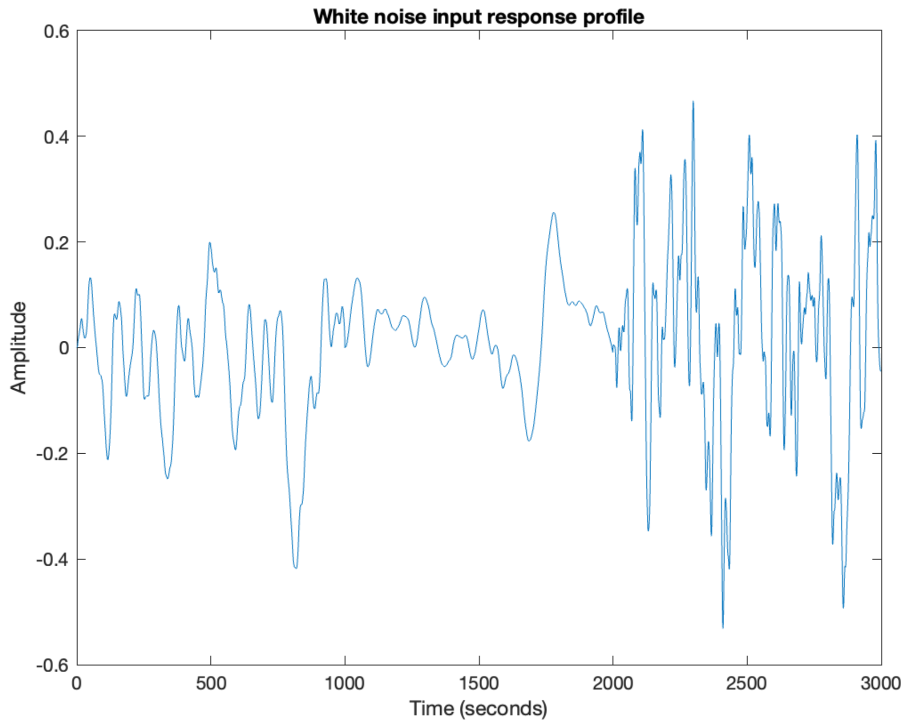


Fig. 4 Low frequency PRBS input response profile

By observing the responses of the three systems to each of the signal, it is possible to conclude that low-frequency PRBS is the best candidate for estimation as it captures the reactions of the system in an observable way.



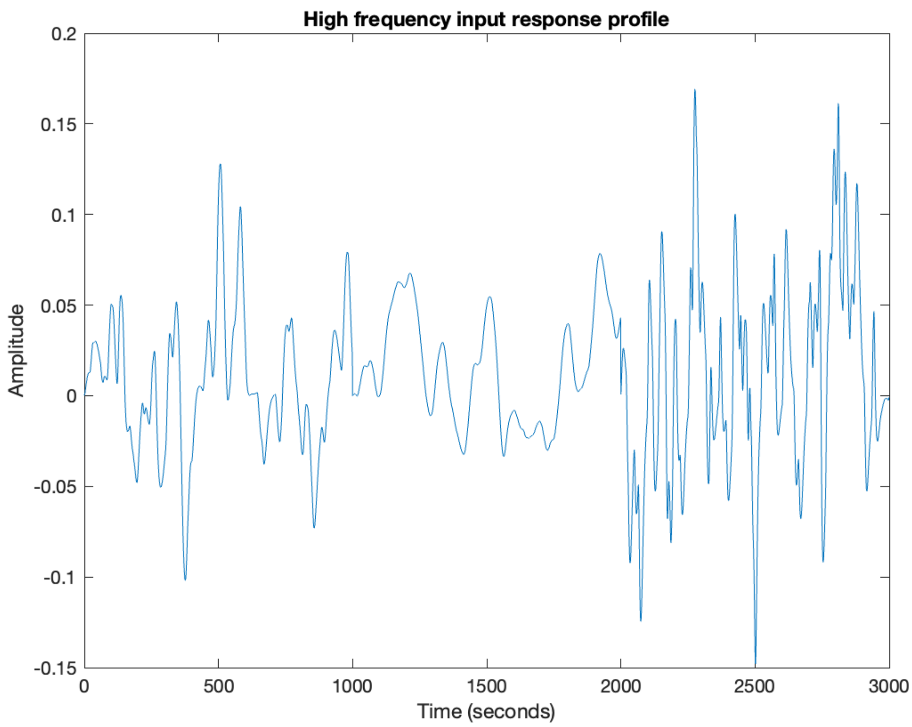


Fig.6 High frequency PRBS input response profile

Forgetting method derivation

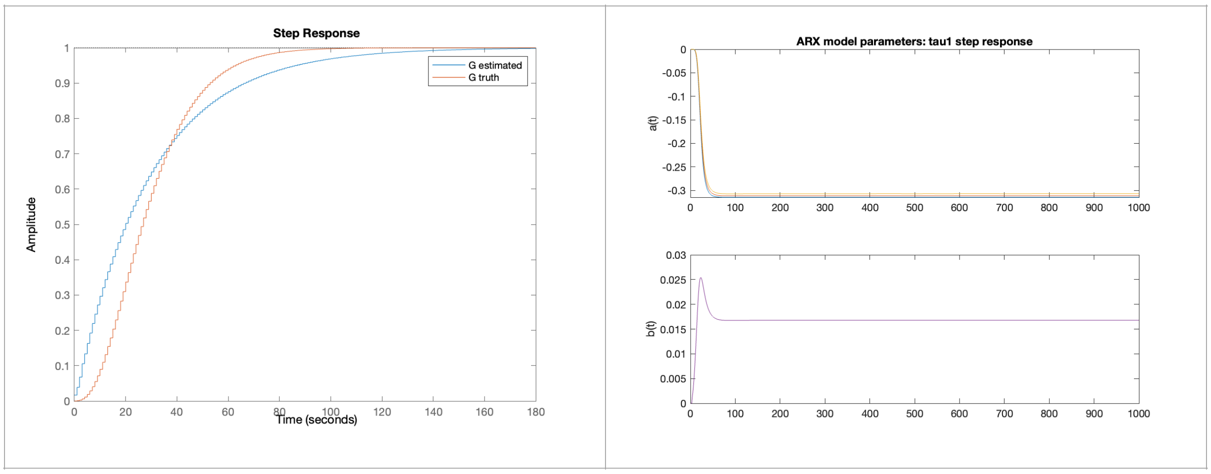


Fig.7 Comparison of estimated and ground truth systems responses(left), parameters convergence(right)

In order to capture how does the system respond to a signal, such as step response, it was, firstly, decided to create an estimation of a single system with tau being equal to 10. (fig.7)

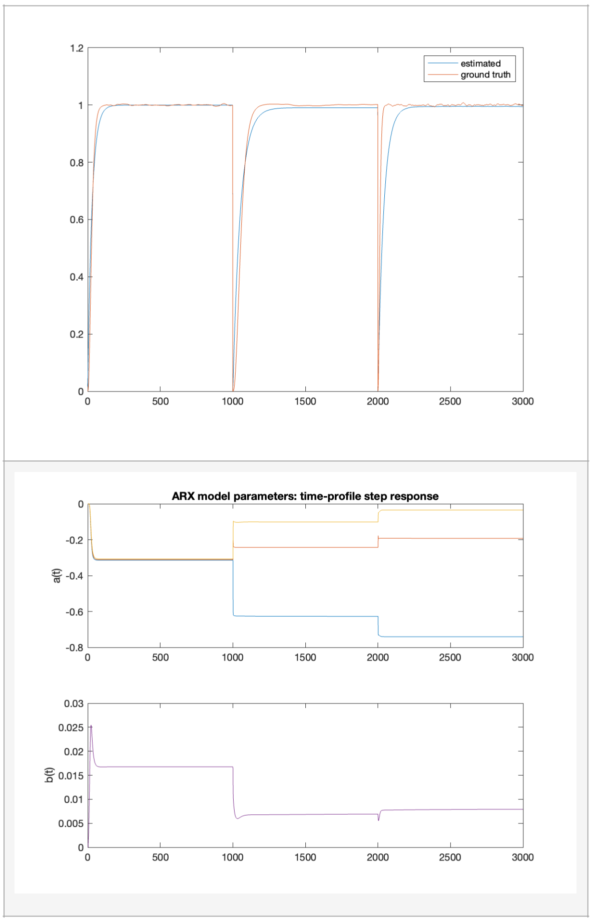


Fig.8 Comparison of estimated and ground truth systems time-profile responses(upper), parameters convergence(lower)

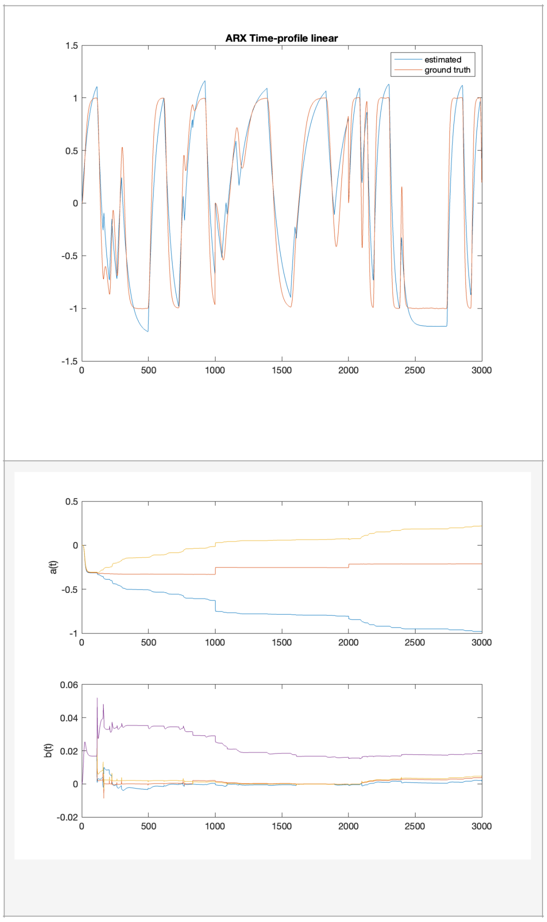


Fig.9 Linear forgetting method

After that, it was necessary to observe how the estimated system reacts to changes of tau and the time-profile was considered. It can be seen that as time passes by, the system struggles to identify correct tau, that is the third tau is the least probable. However, convergence of a and b parameters is stable.(fig. 8)

Forgetting methods comparison

After successful development of a single tau system and time-profiles, it was possible to proceed to using and comparing different forgetting methods.

In this work the methods: linear, exponential, restricted linear, restricted exponential and no forgetting were considered, based on the self-study material.

It can be observed that noisy input produced by PRBS with low frequency has a definite influence on the convergence of parameters. Parameters of *a* polynomial respond to changes in *tau* of the systems and diverge at the end of the time intervals. On the contrast, *b* coefficients tend to converge at the end of time interval, however they are not influenced by changes of time constant *tau*.(fig. 9)

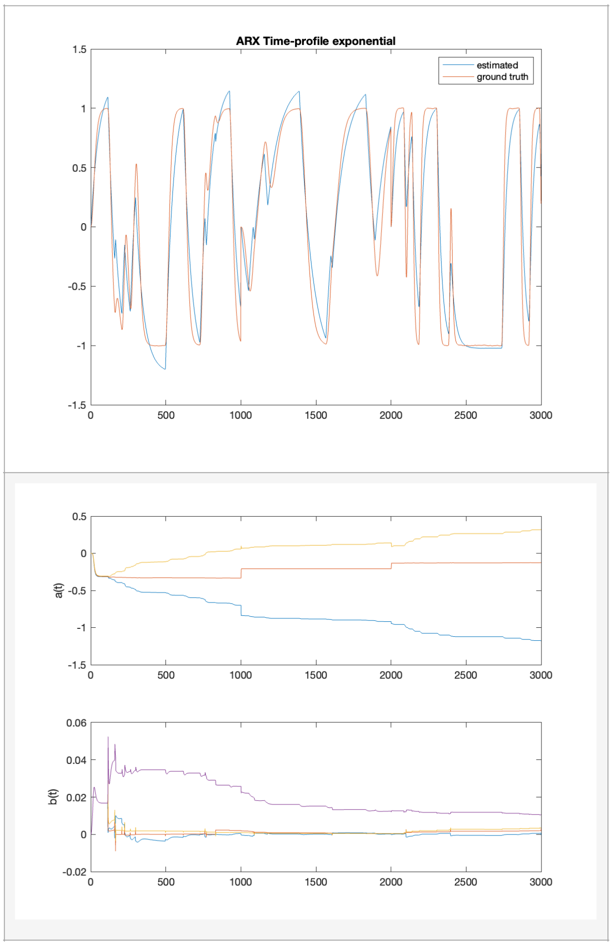


Fig.10 Exponential forgetting method

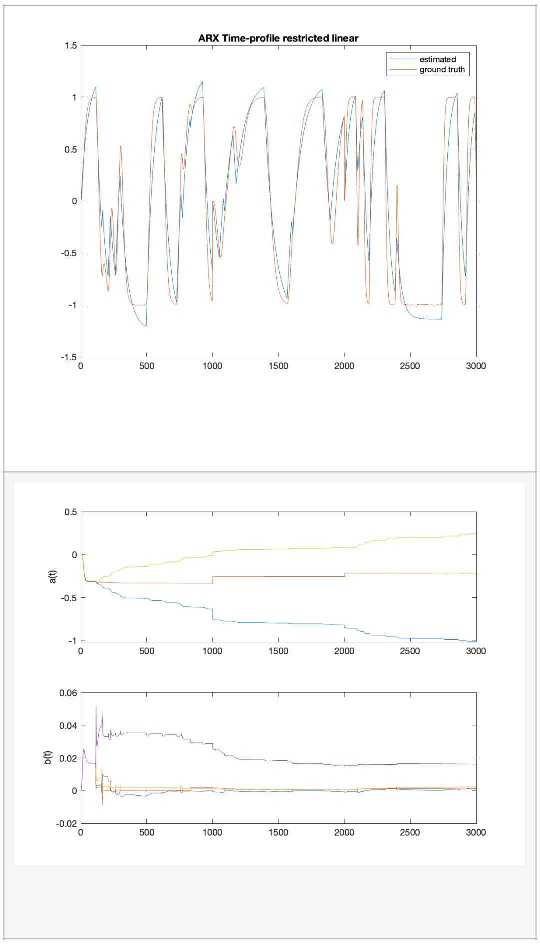
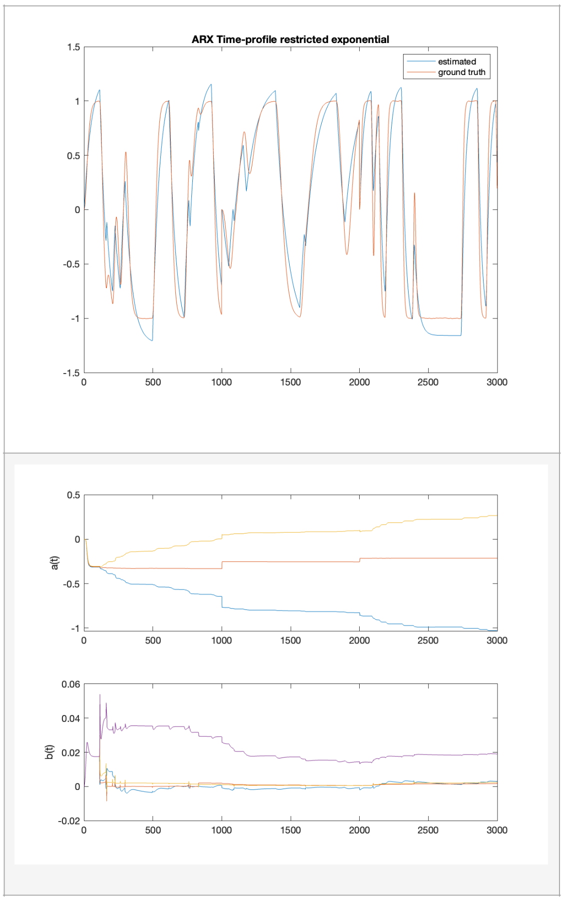


Fig.11 Restricted linear forgetting method



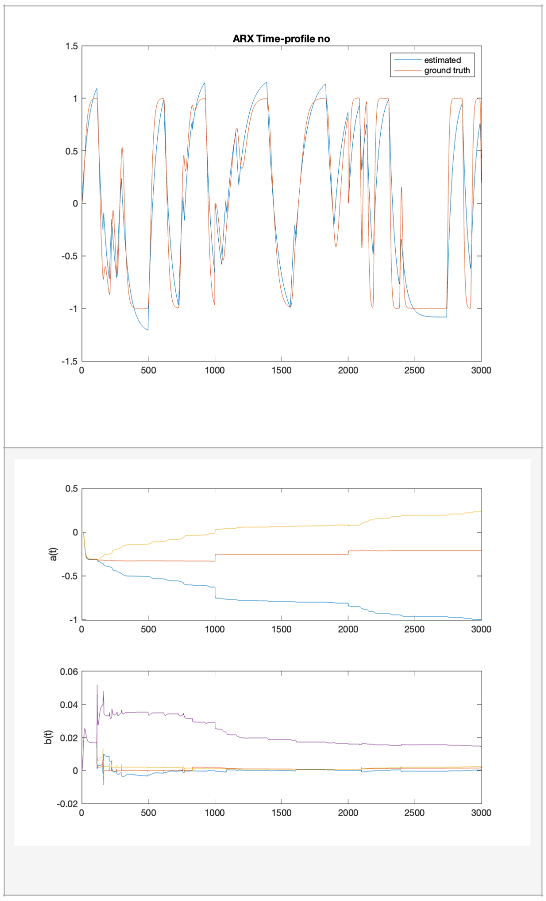


Fig.12 Restricted exponential forgetting method

Fig.13 Without forgetting method

In order to provide some measure of quality of responses of estimated systems using different forgetting approaches, Mean Square Error was taken into consideration.

Linear	Exponential	Restricted Linear	Restricted Exponential	No-forgetting
0.2306	0.23	0.2582	0.2218	0.2774

Table 2 - MSE of different methods

Conclusion: lessons learned

In conclusion, it can be stated that in order to reduce estimation error, it is important to use a forgetting method, because without it MSE is larger. Generally, exponential methods are more accurate than linear, while the best is restricted exponential forgetting method.