Estimation, Filtering and Detection

Homework 3B: Current Sensor Fault

Timur Uzakov

MACROS

```
clear all;
storeFigures = true; % set true if you want to save plots
```

Problem 1:

```
% Design healthy (m=1) state-space model
% Design faulty  (m=2) state-space model
% Find controller gain K_p to keep current limited to 200 A and no
% overshoot
% Analyze observability of m1
% Analyze observability of m2

% Motor parameters
R = 0.1;                    % Ohms
L = 0.5;                    % Henrys
Km = 0.5;                   % motor constant
J = 10;                     % kg.m^2/s^2
Kp = 2;
Ts = 0.05;
% Healthy system m=1
A = [0 Km/J 1/J; -Kp/L -(R+Km)/L 0; 0 0 0];
B = [0; Kp/L; 0];
C = [1 0 0;0 1 0];
D = 0;
sys_healthy = ss(A,B,C,D,Ts);

% Faulty system m=2

A_f = A;
B_f = B;
C_f = [1 0 0;0 0 0];
D_f = 0;
sys_faulty = ss(A_f,B_f,C_f,D_f,Ts);

% Observability
disp("Observability of healthy system")
```

```
Observability of healthy system
```

```
Ob_healthy = obsv(A,C);
unobsv = length(A) - rank(Ob_healthy)
```

```
unobsv = 0
```

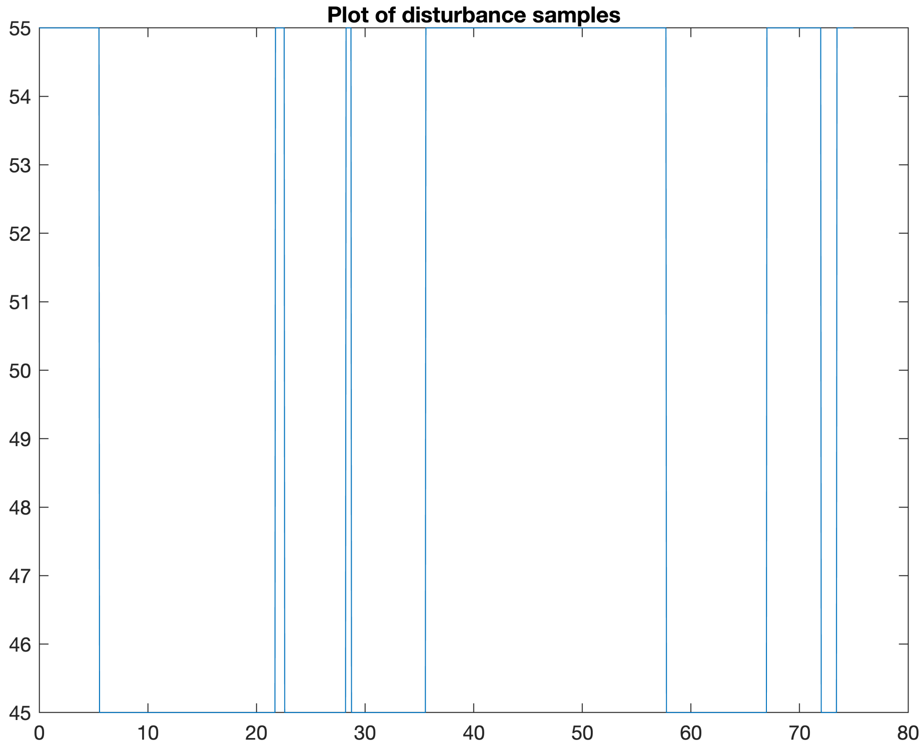
```
disp("Observability of faulty system")
```

```
Observability of faulty system
```

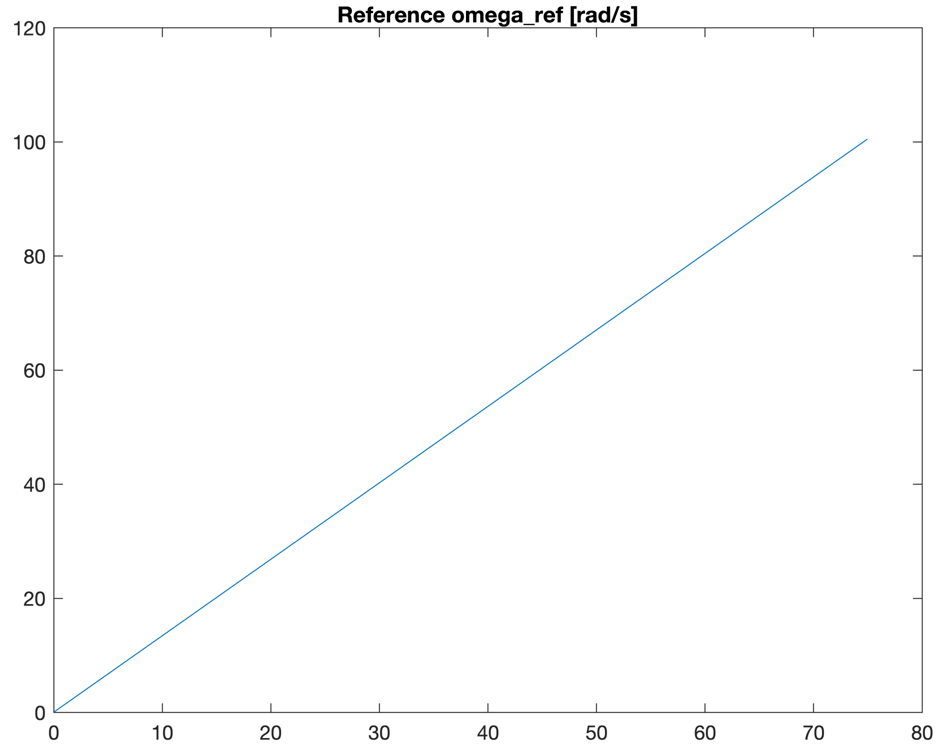
```
Ob_faulty = obsv(A_f,C_f);
unobsv = length(A_f) - rank(Ob_faulty)
```

```
unobsv = 0
```

```



% disturbance


N = 1500;
T = Ts*(N-1);
t = 0:Ts:T;


load = 50;        % Nm
deviation = 5;
Td = load*ones(1,N); % load disturbance
p_switch = 0.005;
UU = ones(N,1);
for i=2:(N-1)
     if rand(1)<p_switch, UU(i)=-UU(i-1); else UU(i)=UU(i-1); end
end
Td = Td + deviation*UU';
figure(1);
plot(t,Td);
title("Plot of disturbance samples");
```




```
% refference signal
w_ref = zeros(1,N);
for i=1:length(t)
    w_ref(i) = w_ref(i) + 0.067*i;
end
figure(2);
plot(t,w_ref);
title("Reference omega\_ref [rad/s]");
```




```
% u = [w_ref;Td];
% response = lsim(sys_healthy,u,t);
% figure(3);
% plot(t,response(:,1));
% title("omega");
% figure(4);
% plot(t,response(:,2));
% title("current");
```

Problem 2:

```
% Select and justify process noise properties
% Find input-output data with multiple sensor faults

% noise properties
Q = [0.001 0 0; 0 0.001 0; 0 0 0.001];
Q_f = Q;
R_cov = [100 0; 0 100];
Tc = 0.01; % Tc should be smaller than Ts
etta = (Ts-Tc)/Ts;

% healthy
Ah = expm(A*Ts);
Ch = C*expm(A*etta*Ts);
```

```
funB = @(x) expm(A*x);
funD = @(x) expm(A*x);
funQ = @(x) expm(A*x)*Q*expm(A'*x);
funS = @(x) expm(A*x)*Q*expm(A'*x)*C';
funR = @(x) C*expm(A*x)*Q*expm(A'*x)*C';

Bh = integral(funB,0,Ts,'ArrayValued', true);
Dh = integral(funD,0,etta*Ts,'ArrayValued', true);
Qh = integral(funQ,0,Ts,'ArrayValued', true);
Sh = integral(funS,0,etta*Ts,'ArrayValued', true);
Rh = integral(funR,0,etta*Ts,'ArrayValued', true);
Bh = Bh*B;
Dh = C*Dh*B;
Rh = R_cov + Rh;

% faulty

Af = expm(A_f*Ts);
Cf = C_f*expm(A_f*etta*Ts);

funBf = @(x) expm(A_f*x);
funDf = @(x) expm(A_f*x);
funQf = @(x) expm(A_f*x)*Q_f*expm(A_f'*x);
funSf = @(x) expm(A_f*x)*Q_f*expm(A_f'*x)*C_f';
funRf = @(x) C_f*expm(A_f*x)*Q_f*expm(A_f'*x)*C_f';

Bf = integral(funBf,0,Ts,'ArrayValued', true);
Df = integral(funDf,0,etta*Ts,'ArrayValued', true);
Qf = integral(funQf,0,Ts,'ArrayValued', true);
Sf = integral(funSf,0,etta*Ts,'ArrayValued', true);
Rf = integral(funRf,0,etta*Ts,'ArrayValued', true);
Bf = Bf*B_f;
Df = C_f*Df*B_f;
Rf = R_cov + Rf;


x = zeros(3,N);
x(3,:) = Td;
y = zeros(2,N);

fault = zeros(1,N);
u = zeros(1,N);

for i=1:(N-1)
    u(:,i) = w_ref(i);
    if ((i>100) && (i<300))||((i>500) && (i<700))||((i>1000) && (i<1200))
        % faulty
        x(:,i+1) = Af*x(:,i) + Bf*u(:,i) + sqrtm(Qf)*randn(3,1);
        y(:,i)   = Cf*x(:,i) + Df*u(:,i) + sqrtm(Rf)*randn(2,1);
        fault(i) = 1;
    else
        % normal
        x(:,i+1) = Ah*x(:,i) + Bh*u(:,i) + sqrtm(Qh)*randn(3,1);
        y(:,i)   = Ch*x(:,i) + Dh*u(:,i) + sqrtm(Rh)*randn(2,1);
```

```
        fault(i) = 0;
    end
end

figure(3);
plot(y(1,:));
hold on
plot(y(2,:));
legend("omega","current")
title("[Faulty] tracking with noise properties")
hold off
```


```

```

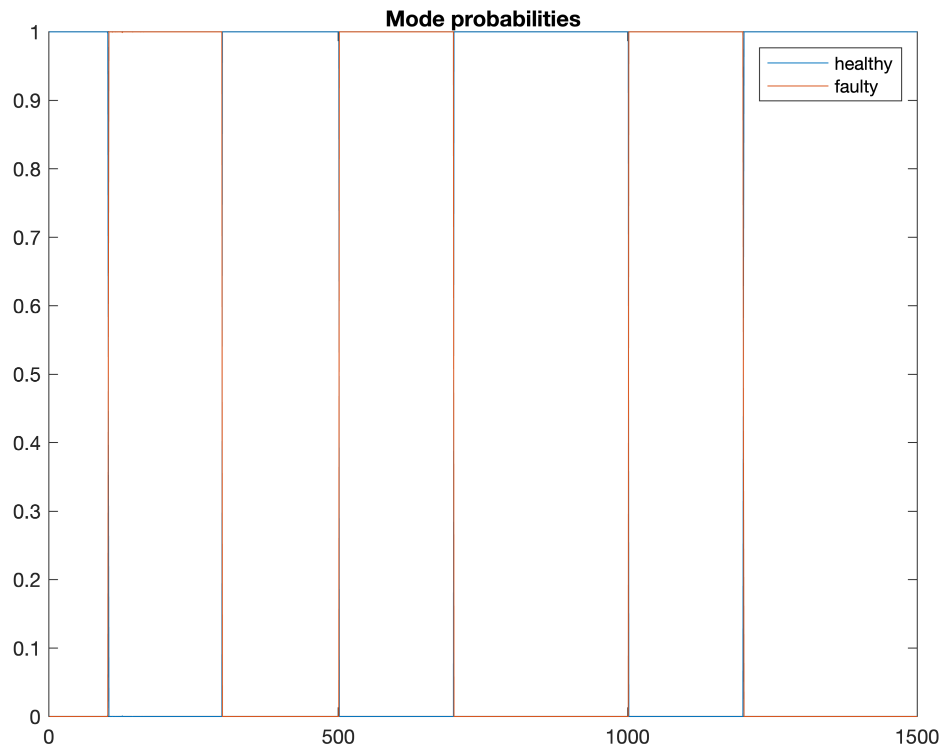
Problem 3:

```
% Design IMM algorithm
% Evaluate unmeasurable load
% Detect faults of current sensor
% p12 = 0.001 transition to fault
% p21 = 0.005 recovery


x0 = [0; 0; 50];
prob_init = [1; 0];
xhat0 = cell(3, 1);

xhat = cell(1, 2);
```

```
xhat{1} = zeros(3, N);
xhat{2} = zeros(3, N);
xhat{1}(:, 1) = x0;
xhat{2}(:, 1) = x0;

xhatc = zeros(3, N);
H = cell(2, 1);
H{1} = Ch;
H{2} = Cf;

P = cell(2, N);
P{1,1} = Qh;
P{2,1} = Qf;

Qc = cell(2, 1);
Qc{1} = Qh;
Qc{2} = Qf;

Rc = cell(2, 1);
Rc{1} = Rh;
Rc{2} = Rf;

prob_vec = zeros(2, N);
result_vec = zeros(1, N);
prob_vec(:, 1) = prob_init;
Lk = cell(2, N);
Sk = cell(2, N);

r = cell(2, N);
W = cell(2, N);

% Interaction
Pt = [0.999, 0.001;
      0.005, 0.995];

mu = [1;0];
c_bar = zeros(2, 1);
mu_mix = zeros(2, 2);

for i = 2:N
    c_bar(1) = Pt(1, 1)*mu(1) + Pt(2, 1)*mu(2);
    c_bar(2) = Pt(1, 2)*mu(1) + Pt(2, 2)*mu(2);

    mu_mix = [(1/c_bar(1))*Pt(1,1)*mu(1), (1/c_bar(2))*Pt(1,2)*mu(1);
              (1/c_bar(1))*Pt(2,1)*mu(2), (1/c_bar(2))*Pt(2,2)*mu(2)];
    % filter 1 init state
    xhat0{1} = xhat{1}(:, i-1)*mu_mix(1,1) + xhat{2}(:, i-1)*mu_mix(2,1);
    % filter 2 init state
    xhat0{2} = xhat{1}(:, i-1)*mu_mix(1,2) + xhat{2}(:, i-1)*mu_mix(2,2);


    % filter 1 init cov
    P0{1} = (P{1, i-1} + (xhat{1}(:, i-1) - xhat0{1})*(xhat{1}(:, i-1) - xhat0{1})')*mu
            (P{2, i-1} + (xhat{2}(:, i-1) - xhat0{1})*(xhat{2}(:, i-1) - xhat0{1})')*mu
```

```
    % filter 2 init cov
    P0{2} = (P{1, i-1} + (xhat{1}(:, i-1) - xhat0{2})*(xhat{1}(:, i-1) - xhat0{2})')*mu
            (P{2, i-1} + (xhat{2}(:, i-1) - xhat0{2})*(xhat{2}(:, i-1) - xhat0{2})')*mu

    % Filtering
    % Prediction step
    % filter 1
    xhat{1}(:, i) = Ah*xhat0{1} + Bh*u(:,i-1);
    % filter 2
    xhat{2}(:, i) = Af*xhat0{2} + Bf*u(:,i-1);

    %% filter 1
    P{1, i} = Ah*P0{1}*Ah' + Qc{1};
    %% filter 2
    P{2, i} = Af*P0{2}*Af' + Qc{2};

    % Correction step
    % innovation
    r{1, i} = y(:,i) - H{1}*xhat{1}(:, i);
    r{2, i} = y(:,i) - H{2}*xhat{2}(:, i);

    % residual covariance
    Sk{1, i} = H{1}*P{1, i}*H{1}' + Rc{1};
    Sk{2, i} = H{2}*P{2, i}*H{2}' + Rc{2};

    % Kalman gain
    W{1, i} = P{1, i}*H{1}'*inv((Sk{1, i}));
    W{2, i} = P{2, i}*H{2}'*inv((Sk{2, i}));

    % update
    xhat{1}(:, i) = xhat{1}(:, i) + W{1, i}*r{1, i};
    xhat{2}(:, i) = xhat{2}(:, i) + W{2, i}*r{2, i};

    P{1, i} = P{1, i} - W{1, i}*H{1}*P{1, i};
    P{2, i} = P{2, i} - W{2, i}*H{2}*P{2, i};

    % Likelihood
    Lk{1, i} = mvnpdf(r{1, i}, 0, Sk{1, i});
    Lk{2, i} = mvnpdf(r{2, i}, 0, Sk{2, i});

    mu(1) = Lk{1, i}*(Pt(1,1)*mu(1)+Pt(2,1)*mu(2));
    mu(2) = Lk{2, i}*(Pt(1,2)*mu(1)+Pt(2,2)*mu(2));

    c = sum(mu, 'all');
    mu = mu*(1/c);
    prob1 = mu(1);
    prob2 = mu(2);

    % Combination of two filters
    prob_vec(:, i) = mu;
    result_vec(i) = prob_vec(2,i);
    xhatc(:, i) = xhat{1}(:, i)*prob1 + xhat{2}(:, i)*prob2;
    Pc{i} = (P{1, i} + (xhat{1}(:, i) - xhatc(:, i))*(xhat{1}(:, i) - xhatc(:, i))')*p
```

```
            (P{2, i} + (xhat{2}(:, i) - xhatc(:, i))*(xhat{2}(:, i) - xhatc(:, i))')*p
end


figure(4);
plot(prob_vec(1,:));
hold on
plot(prob_vec(2,:));
title("Mode probabilities");
legend("healthy","faulty");
hold off
```



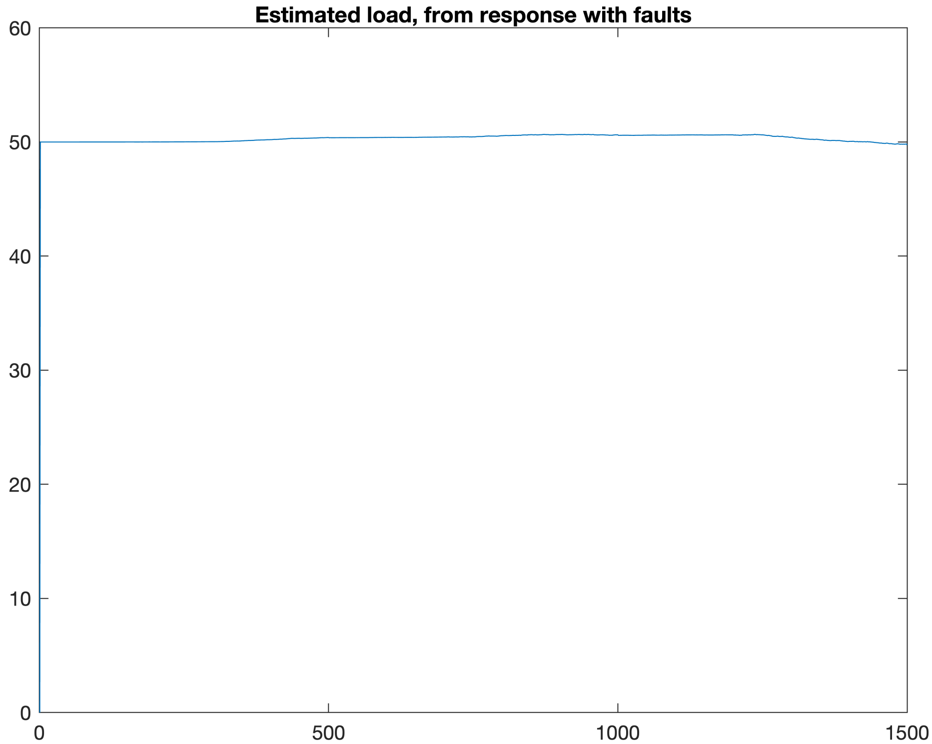

```
% Analysis of fault detection
disp("Mean Square Error:")
```

```
Mean Square Error:
```

```
disp(calculateMSE(fault,[0 result_vec]))
```

```
    0.0830
```

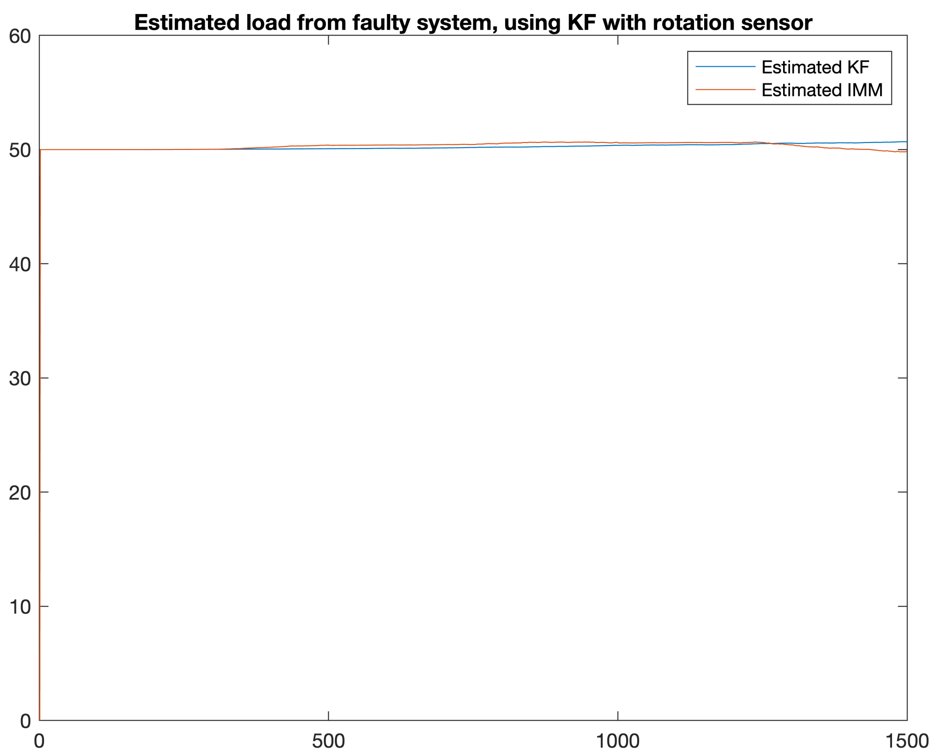
```
figure(9);
plot(xhatc(3,:));
title("Estimated load, from response with faults");
```



Problem 4:

```
% Analyse the performance of fault detection
% Compare IMM to KF using only rotation speed sensor
% Is it possible to justify use of current sensor?
x0 = [0; 0; load];

xhat = zeros(3, N);
xhat(:, 1) = x0;
xTLoad = zeros(1, N);
xTLoad(1) = load;
H = Cf;
P = Qf;
Qc = Qf;
Rc = Rf;
Sk = Sf;

for i = 2:N
    % filter 1 init state
    xhat0 = xhat(:,i-1);
    P0 = P;
    % Filtering
    % Prediction step
    xhat(:, i) = Af*xhat0 + Bf*u(:,i-1);
    P = Af*P0*Af' + Qc;
```

```
    % Correction step
    % innovation
    r = y(:,i) - H*xhat(:, i);
    % residual covariance
    Sk = H*P*H' + Rc;
    % Kalman gain
    W = P*H'/(Sk);
    % update
    xhat(:, i) = xhat(:, i) + W*r;
    P = P - W*H*P;

    xTLoad(i) = xhat(3,i);
end

figure(5);
plot(xTLoad);
hold on
plot(xhatc(3,:))
title("Estimated load from faulty system, using KF with rotation sensor");
legend("Estimated KF","Estimated IMM");
hold off
```




```
disp("Mean Square Error:")
```

```
Mean Square Error:
```

```
disp(calculateMSE(xTLoad,xhatc(3,:)))
```

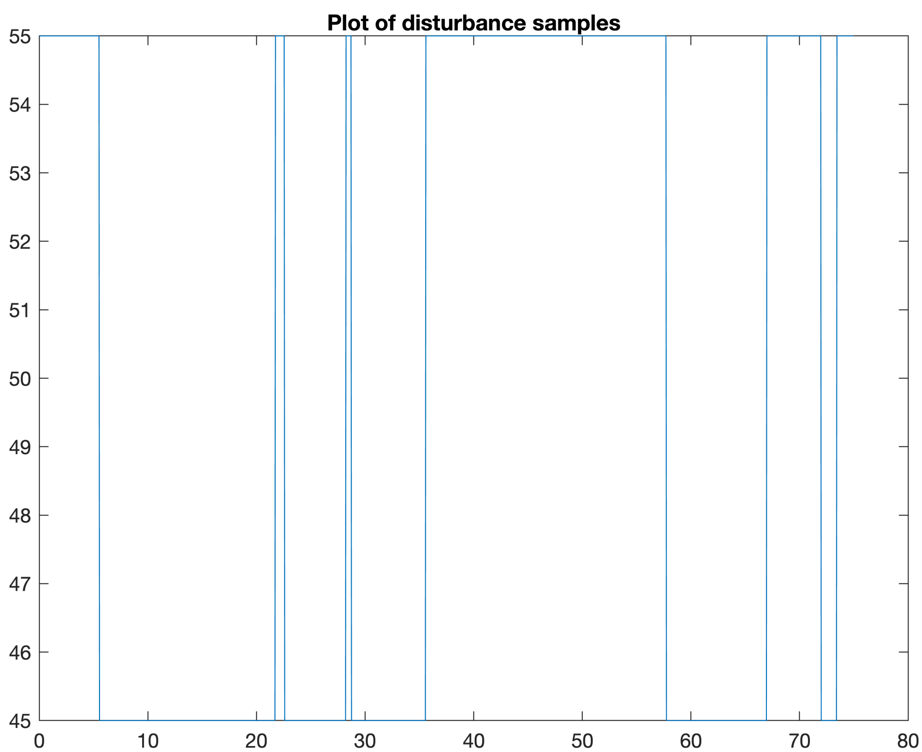
```
1.3246
```

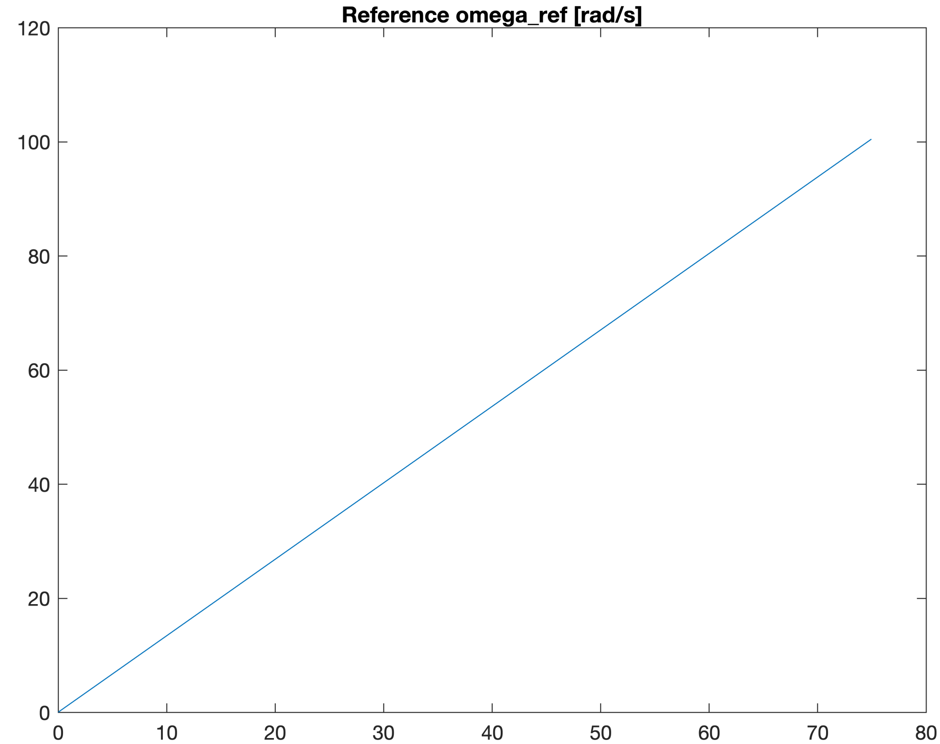
Problem 5:

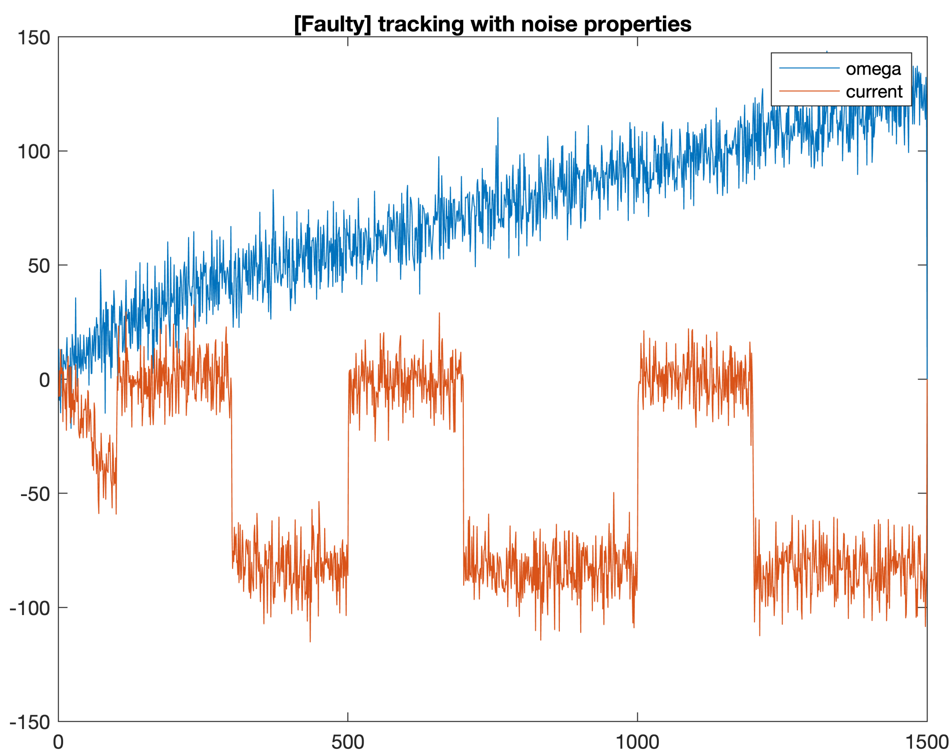
```
% Lessons learned
% IMM is best suited for fault detection
% KF using only rotation speed sensor can be enough for estimation of load
```

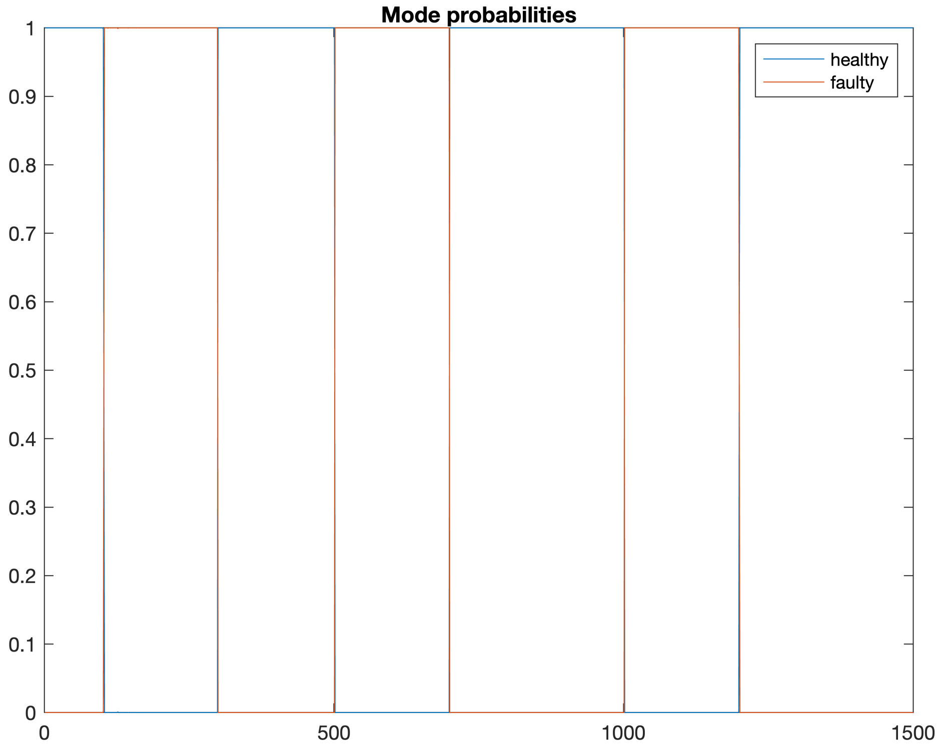
Additional tools

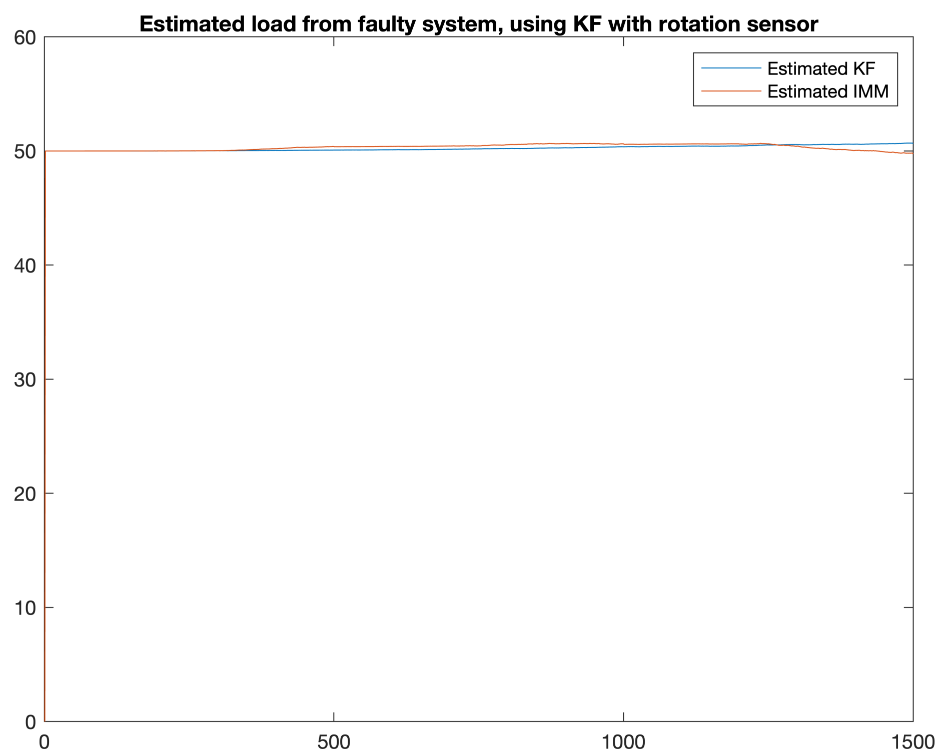
```
%Storing figures
% if storeFigures
%     for i=1:5
%         filename = strcat('figure_',num2str(i));
%         foldername = './figures/';
%         saveas(figure(i),fullfile(foldername,filename),'jpg');
%     end
% end
```




Reference omega_ref [rad/s]
120
100
80
60
40
20
0
0
10
20
30
40
50
60
70
80


[Faulty] tracking with noise properties
omega
current
150
100
50
0
-50
-100
-150
0
500
1000
1500


Mode probabilities
healthy
faulty
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
500
1000
1500


Estimated load from faulty system, using KF with rotation sensor
Estimated KF
Estimated IMM
60
50
40
30
20
10
0
0
500
1000
1500