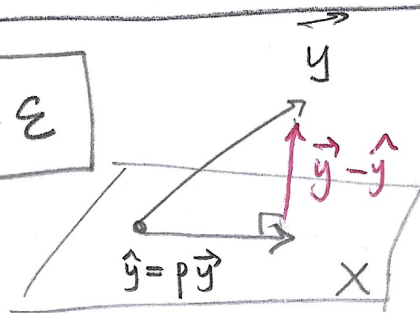


# Minimum Norm Least Square Solution

$$\vec{y} = Xw + \varepsilon$$



■  $p$  projection of  $\vec{y}$  on  $X$

■  $\hat{y} = p\vec{y}$  minimizes  $\|\vec{y} - \hat{y}\|$

■  $\hat{y}$  is in the vector space of  $X$ . So  $\hat{y} = Xw$

■  $\vec{y} - \hat{y} \perp \hat{y}$

■  $\hat{y}^T (\vec{y} - \hat{y}) = 0 \Rightarrow (Xw)^T (\vec{y} - Xw) = 0$

$$\Rightarrow w^T X^T \vec{y} - w^T X^T X w = 0$$

$$\Rightarrow w^T ( \underbrace{X^T \vec{y} - X^T X w}_0 ) = 0$$

$$\Rightarrow X^T X w = X^T \vec{y}$$

$$\Rightarrow w = \underbrace{(X^T X)^{-1}}_{d \times d} \underbrace{X^T \vec{y}}_{d \times 1}$$

dimension check

Disadvantage: Too Many Matrix-Matrix product / computation