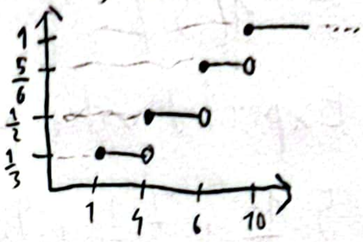
Hw2

Question 1)

a) The pdf is $P(X=x) = p(x) = \begin{cases} \frac{1}{3}, & x=1 \\ \frac{1}{6}, & x=4 \\ \frac{1}{3}, & x=6 \\ \frac{1}{6}, & x=10 \end{cases}$, then

$$E(X) = \frac{1}{3}\cdot 1 + \frac{1}{6}\cdot 4 + \frac{1}{3}\cdot 6 + \frac{1}{6}\cdot 10$$
$$= \frac{14}{3}$$ //

cdf graph



b) The $P(1.5 < X < 7)$ can be expressed

$$P(1.5 < X < 7) = P(X < 7) - P(X < 1.5)$$
$$= \frac{5}{6} - \frac{1}{3} = \frac{5}{6} - \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$$

c) The moment generating function is

$$M_X(t) = E(e^{tx}) = \sum_x e^{tx} p(x)$$
$$= \frac{e^t}{3} + \frac{e^{4t}}{6} + \frac{e^{6t}}{3} + \frac{e^{10t}}{6}$$
$$= \frac{2e^t + e^{4t} + 2e^{6t} + e^{10t}}{6}$$ //

Question 2)

a) Let X be the discrete rv as the # of tries until worn out battery is found

$P(X=1) = \frac{1}{6}$ $P(X=2) = \frac{5}{6}\cdot\frac{1}{5} = \frac{1}{6}$ $P(X=3) = \frac{5}{6}\cdot\frac{4}{5}\cdot\frac{1}{4} = \frac{1}{6}$

$P(X=4) = \frac{5}{6}\cdot\frac{4}{5}\cdot\frac{3}{4}\cdot\frac{1}{4} = \frac{1}{8}$ $P(X=5) = \frac{5}{6}\cdot\frac{4}{5}\cdot\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{3} = \frac{1}{8}$

$P(X=6) = \frac{5}{6}\cdot\frac{4}{5}\cdot\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{2}{3}\cdot\frac{1}{2} = \frac{1}{8}$ $P(X=7) = \frac{5}{6}\cdot\frac{4}{5}\cdot\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{2}{3}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{16}$

$P(X=8) = \frac{5}{6}\cdot\frac{4}{5}\cdot\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{2}{3}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot 1 = \frac{1}{16}$, so the pdf of X is

X	1	2	3	4	5	6	7	8
p(x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{16}$

b) Expected value with unaware workers:

$$\underbrace{5\cdot\frac{1}{6}+7\cdot\frac{1}{6}+9\cdot\frac{1}{6}}_{\frac{7}{2}}+\underbrace{14\cdot\frac{1}{8}+16\cdot\frac{1}{8}+18\cdot\frac{1}{8}}_{6}+\underbrace{23\cdot\frac{1}{16}+25\cdot\frac{1}{16}}_{3}$$

$= 12.5\$$

Expected value without unaware workers:

$$5\cdot\frac{1}{6}+7\cdot\frac{1}{6}+9\cdot\frac{1}{6}+14\cdot\frac{1}{6}+16\cdot\frac{1}{6}+18\cdot\frac{1}{6}=\frac{1}{6}\cdot(69)$$

$= 11.5\$$

There is a 1\$ increase in expected cost.

Question 3)

a) $V=1$ with probability p and $V=0$ with probability $1-p$

$$\Longrightarrow P(V=v)=p(v)=\begin{cases} p, & v=1 \\ 1-p, & v=0 \end{cases} \Rightarrow \text{Bernoulli rv}$$

b) The variance of the Bernoulli rv is

$Var(V) = p\cdot(1-p)$ // and $V=\frac{X-b}{a-b} \Rightarrow V(a-b)+b=X$

$Var(X) = Var[(a-b)V+b] = (a-b)^2 Var(V) = (a-b)^2\cdot p(1-p)$ //

CamScanner ile tarandı

Question 4)

a) To go without collision, one needs to send while the others do not.

Lets define discrete rv X as the # number of messages being transmitted then $X \sim Bin(N,p)$

For only one to sent is

$$P(X=1) = p(1) = \binom{N}{1} p^1 q^{N-1}$$

$$= N.p.(1-p)^{N-1}$$

b) From the part a, the probability of a message going through without collision is found as $N.p(1-p)^{N-1}$

For maximize this we need to take the derivative with respect to p and set it equal to 0

$$\Rightarrow \frac{d(Np(1-p)^{N-1})}{dp} = N \frac{d(p(1-p)^{N-1})}{dp}$$

$$= N[\underbrace{(1-p)^{N-1} - p(N-1)(1-p)^{N-2}}_{0}] = 0$$

$$(1-p)^{N-1} = p(N-1)(1-p)^{N-2} \Rightarrow 1-p = p(N-1)$$

$$\Rightarrow 1 = Np \Rightarrow p = \frac{1}{N}$$

From $E(X) = N.p \Rightarrow E(X) = N.\frac{1}{N} = 1$

So that we have shown that mean value of initiated messages should be exactly one message

Question 9)

a) Let define X as the # of customers that comes until the 3^{rd} white is ordered

So, $X \sim \text{NegBin}(3, \frac{1}{2})$

For the 5^{th} customer to order the 3^{rd}

$P(X=5) = \binom{4}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 \frac{1}{2} = 6 \cdot \left(\frac{1}{2}\right)^5 = \frac{6}{32} = \frac{3}{16}$ //

b) For all whites to be ordered before any browns, all whites must be sold first.

$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$ //

c) For this question the 3^{rd} white must be ordered by 3^{rd} or 4^{th} or 5^{th} customer. From part a) we found $P(X=5) = \frac{3}{16}$ and from part b) $P(X=3) = \frac{1}{8}$

Last of all $P(X=4)$ is $\binom{3}{2} \cdot \left(\frac{1}{2}\right)^2 \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{16}$

$P(X=3) + P(X=4) + P(X=5)$

$\Rightarrow \frac{3}{16} + \frac{3}{16} + \frac{1}{8} = \frac{1}{2}$ //