October 17, 2024

MATH 230 Homework 2

Due on Thursday, October 24, 2024

(Submit your answers to Moodle in a single pdf file)

- Write your **full name**, **section** and **department** on the top-right corner of the first page.
- There are 5 questions to be answered and submitted to Moodle. Each is 20 points.
- Please keep the order of the questions. If you don't have a solution for the question you should write question number with the note "no answer".
- You should show your work to get full credit. Correct answers without sufficient explanation may not get full credit.
- No late submitted homework will be accepted!
- In the end of the homework questions, there are self-study questions. Answers of self-study questions will not be submitted.

Homework Questions:

1. An investment firm offers its customers special bonds that mature after varying number of years. Let X be the number of years to maturity for a randomly selected bond, cumulative distribution function (cdf) of X is given below:

$$F_X(t) = \begin{cases} 0 & t < 1 \\ \frac{1}{3} & 1 \le t < 4 \\ \frac{1}{2} & 4 \le t < 6 \\ \frac{5}{6} & 6 \le t < 10 \\ 1 & t \ge 10 \end{cases}$$

a) Find E(X),
b) use cdf to find $P(1.5 < X < 7)$ and
c) find moment generating function (mgf) of X.

2. In a lot of 6 batteries, one is worn out, this is unknown to the technician and technician tests the batteries one at a time until the worn-out battery is found. Tested batteries are put aside, but after <u>every</u> 3^{rd} test the tester takes a break and another worker, unaware of the test, returns one of the tested batteries to the set of batteries not yet tested.

a) Find probability distribution function (pdf) of X, the number of tests required to identify the worn out battery.

b) Assume <u>the first test</u> of each of three tests costs \$5 and each of the next two tests in each set of three tests costs \$2. (That is costs are \$5, \$2, \$2 for each set of three tests)

Find the increase in expected cost of locating the worn-out battery due to the unaware worker(s).

3. Suppose that $P(X = a) = p$ and $P(X = b) = 1 - p$,

a) Show that $V = \dfrac{X - b}{a - b}$ is a Bernoulli random variable.

b) Find *Var(V)* and use it to find *Var(X)*.

4. A computer network consists of several stations connected by various media (usually cables) There are certain instances when no message is being transmitted. At such “suitable instances”, each station will send a message with probability **p**, independently of other stations. However if two or more stations send messages, a collusion will corrupt messages and they will be discarded. These messages will be retransmitted until they reach their destination. Suppose network consists of **N** stations.

a) What is the probability that at a “suitable instance” a message is initiated by one of the stations and will go through without a collision?

b) Show that, to maximize the probability of a message going through with no collisions, mean value of initiated messages should be exactly one message.

5. An appliance comes in two colors, white and brown which are in equal demand. A certain dealer in these appliances has 3 of each color in stock, although this is not known to the customers. Customers arrive and independently order these appliances. Find the probability that

a) the 3^{rd} white is ordered by the 5^{th} customer,

b) all of the whites are ordered before any of the browns,

c) all of the whites are ordered before all of the browns.

Self-study questions (Do not submit solutions)

Q6 page 114, Q19 and Q23 page 125, Q45 page 137, Q54 and Q55 page 143, Q81 page 155, Q99 page 163 and Q121 page 173.