September 26, 2024

MATH 230 Homework 1

Due on October 3, 2024

(Submit your answers to Moodle in a single pdf file)

- Write your **full name**, **section** and **department** on the top-right corner of the first page.
- There are 5 questions to be answered and submitted to Moodle. Each is 20 points.
- Please keep the order of the questions. If you don't have a solution for the question you should write question number with the note "no answer".
- You should show your work to get full credit. Correct answers without sufficient explanation may not get full credit.
- No late submitted homework will be accepted!
- In the end of the homework questions, there are self-study questions. Answers of self-study questions will not be submitted.

Homework Questions:

1. There are two boxes, each has **N** balls numbered **1** through **N**. A random sample of **n** balls is drawn, without replacement, from each box. Find the probability that the samples contain exactly **k ($k \leq n \leq N$)** balls having the same numbers in common.

2. In a lottery, players pick 6 different integers between 1 and 49, inclusive, order of selection is irrelevant. The lottery commission then randomly selects 6 of these as the winning numbers. A player wins the grand prize if all 6 numbers that they are selected, match the winning numbers. They win the second prize if exactly 5, and the third prize if exactly 4 of the 6 numbers chosen match with the winning ones. Find the probability that a randomly selected player wins
a) the grand prize, **b)** the second prize , and **c)** the third prize.

3. a) Let A_1, A_2, A_3 be (mutually) independent events. Show that A_1' , A_2' and A_3' are also (mutually) independent events, where A_1', A_2', A_3' are complements of A_1, A_2 , A_3, respectively.
b) Let A_1, A_2, A_3 be (mutually) independent events and $P(A_i) = 0.i$, i = 1, 2, 3. Find the probability that at least one A_i occurs.
c) An orbiting satellite has 3 panels of solar cells, all of which must be activate to provide an adequate power output. The panels function independently of one another. The chance that single panel will fail during the mission is 0.02. What is the probability that there will be adequate power output during the entire mission time?

4. Measurements at a Super Computing Center (SCC) on a certain day, indicated that the source of incoming jobs is 15% from university U_1, 35% from university U_2, and 50% from university U_3. Suppose that a job initiated from university U_1 is a multitasking job with probability 0.01, the same probability for universities U_2 and U_3 are 0.05 and 0.02, respectively.
a) Find the probability that a job chosen at random at SCC is a multitasking job.
b) Find the probability that randomly chosen job comes from U_2, given that it is a multitasking job**.**

5. The game of "craps" is played as follows. A pair of fair dice is rolled. If the sum is 7 or 11 you win, if the sum is 2, 3, or 12 you lose. With any other sum, you continue to roll until you roll the initial number again (then you win) or you roll a 7 (in which case you lose). Let X be the rv and X=0, if you lose on the first roll, X=1, if you win on the first roll and X=2 if you continue to roll the dice. Find the probability distribution function (pdf) of X.

Self-study questions (Do not submit solutions)
Q4 page 54, Q12 page 55, Q28 page 65, Q45 page 74, Q67 page 86, Q87 page 93 and Q18 page 125.