ETH zürich

Analyzing time-stamped network data: When ordering matters

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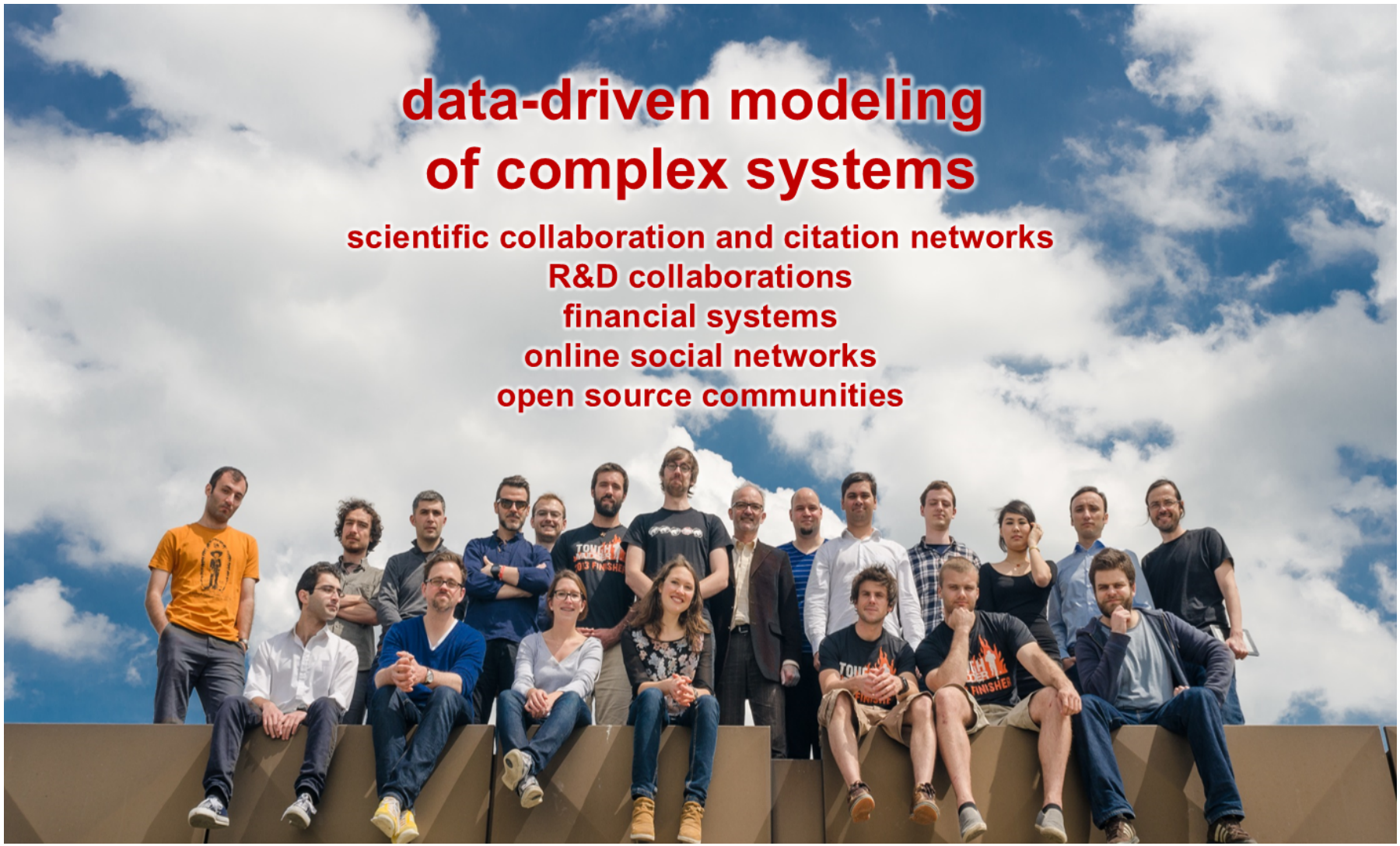
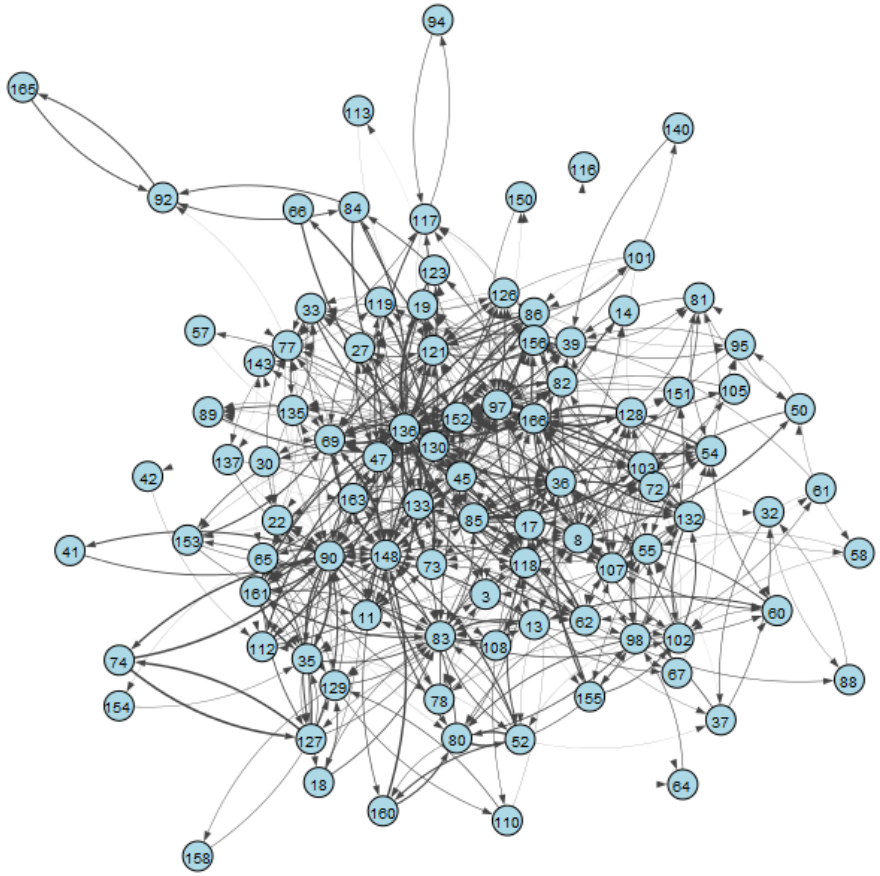


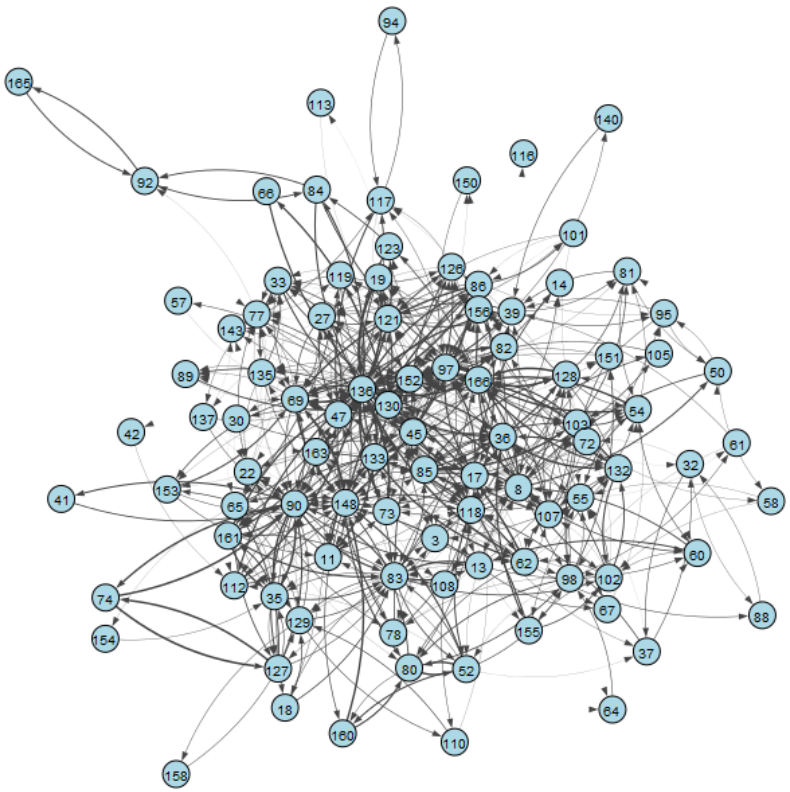
Network perspective on time-stamped data

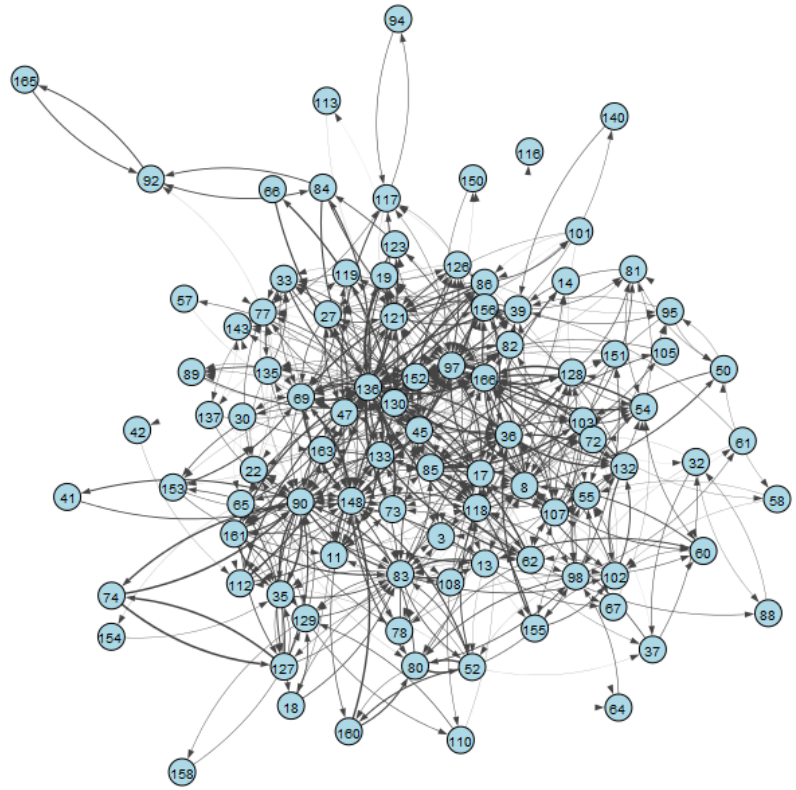
- **network perspective** on complex systems
  - provides wealth of methods
- problem: **over-simplification**
  - multi-layer structures
  - nodes with different roles
  - ...
- what about **dynamic complex systems**?
- **what do we lose** if we aggregate time-stamped links?



example: E-Mail communication network in manufacturing company

Temporal networks: two perspectives





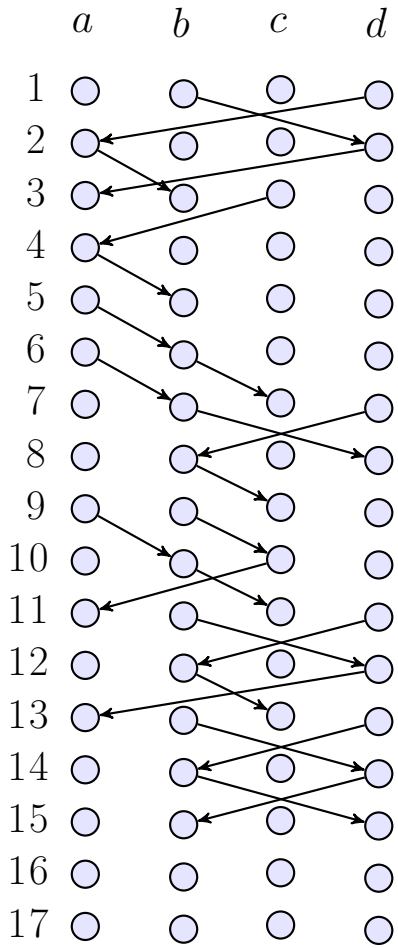
macro perspective: focus on **distribution of activities** in time

micro perspective: focus on **ordering of links** in time

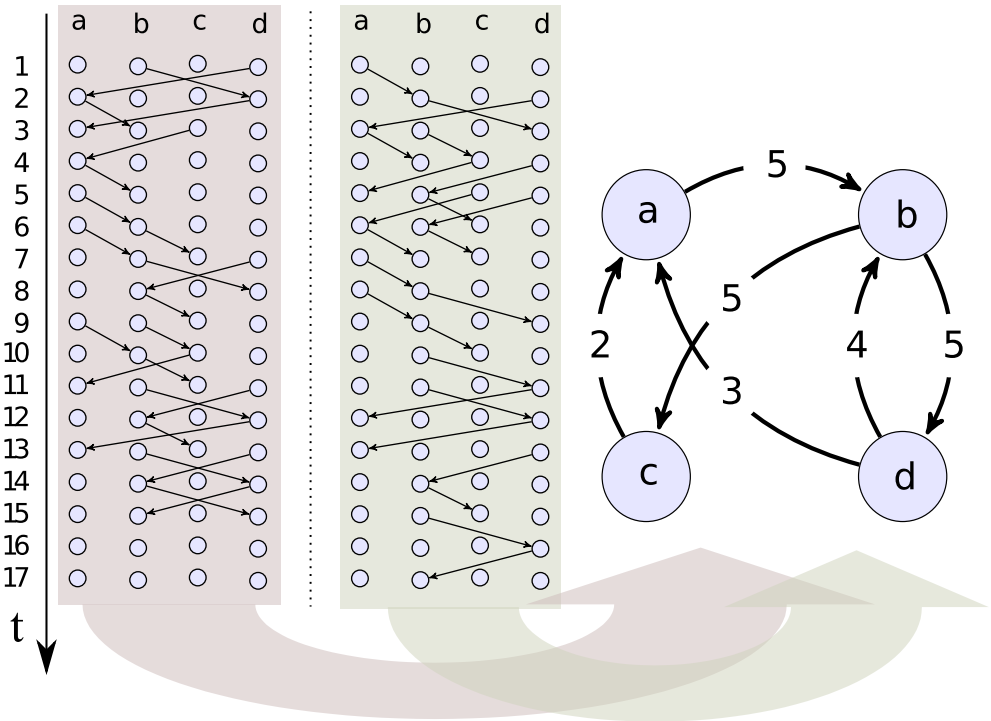
How does **ordering of links** affect network analysis?

The micro perspective: ordering of links

- we focus on the **microscopic ordering of links** and its effect on **causality** in time-varying complex systems
- we consider temporal networks with **time-stamped links** $(i, j; t)$ for discrete times t
- temporal network can be represented as **time-unfolded network**
- example: time-unfolded representation of temporal network with 4 nodes and 24 time-stamped links



Time-aggregated network



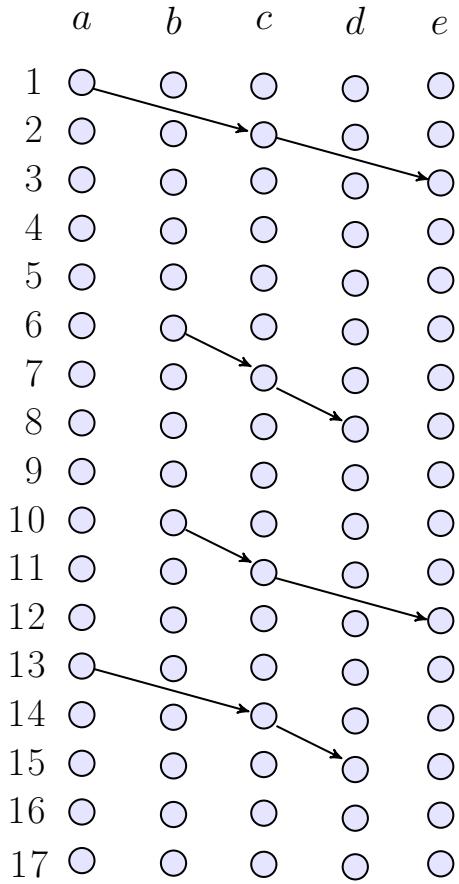
two temporal networks (left) and their corresponding time-aggregated network

- **time-aggregated network** is static representation of temporal network
- we assume that **link weights** count the number of times links have been active
- clear: different **temporal networks** are consistent with same **time-aggregated network**

  $\Rightarrow$ we **lose information on the ordering of links**

Causality: time-respecting paths

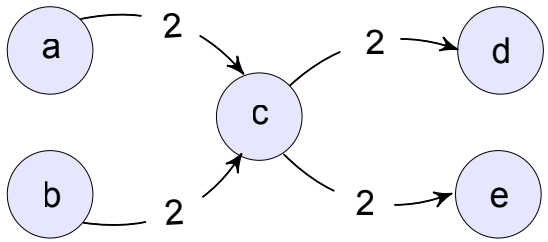
- $v_0 \to v_1 \to \ldots \to v_l$ is a **time-respecting path** if $(v_i, v_{i+1}; t_i)$ are time-stamped links which satisfy **causality**, i.e. $t_i < t_{i+1}$
- we can impose **maximum waiting time** δ, i.e. we require $t_{i+1} - t_i \leq \delta$
- example: $\delta = 1 \Rightarrow$ **four** time-respecting paths
  1. $a \to c \to e$
  2. $b \to c \to d$
  3. $b \to c \to e$
  4. $a \to c \to d$



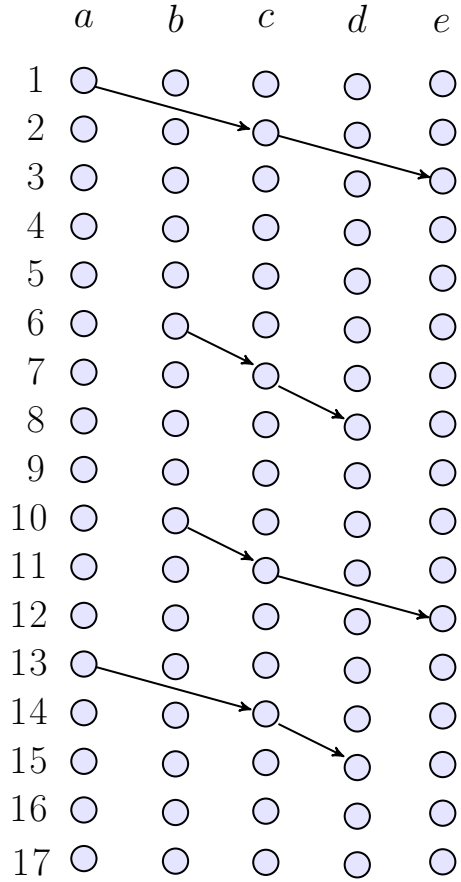
example for temporal network with four time-respecting paths

Expected paths in Markovian temporal networks

- what do we **expect** based on the **time-aggregated network**?



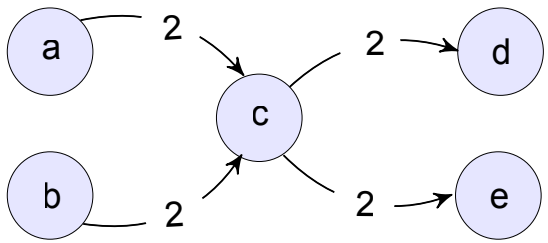
- assumption: consecutive links independent, i.e. temporal network **memoryless** or **Markovian**
- **transitivity**: $u \to v \wedge v \to w \Rightarrow u \to v \to w$
- we expect **four** time-respecting paths
  1. $a \to c \to e$
  2. $b \to c \to d$
  3. $b \to c \to e$
  4. $a \to c \to d$



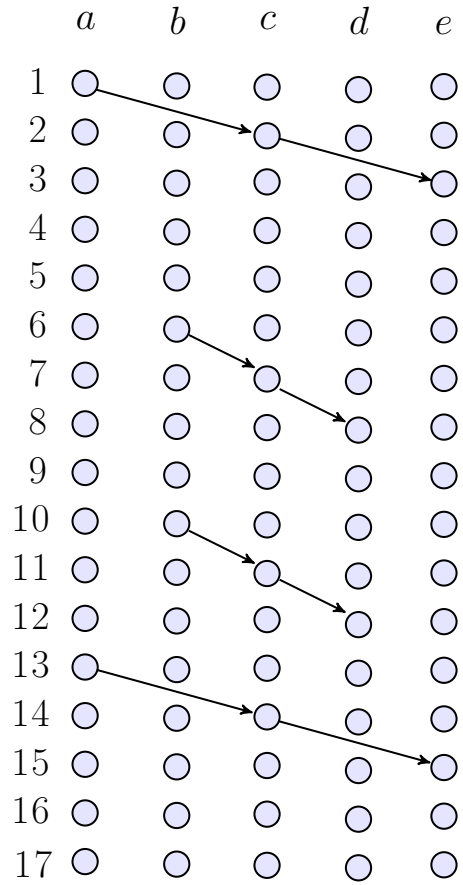
temporal network consistent with time-aggregated network

Order of links can break transitivity!

- we **change the order** of links: swap time-stamps of $(c, e; 11)$ and $(c, d; 14)$
- time-aggregated network remains the same



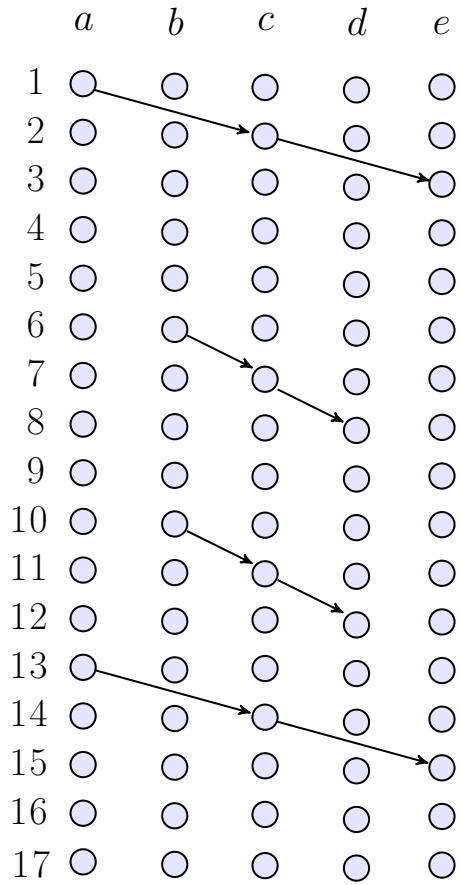
- example: **two** time-respecting paths
  1. $a \to c \to e$
  2. $b \to c \to d$
- **ordering of links can break transitivity** expected from time-aggregated network



ordering of interactions can invalidate transitivity of aggregate network

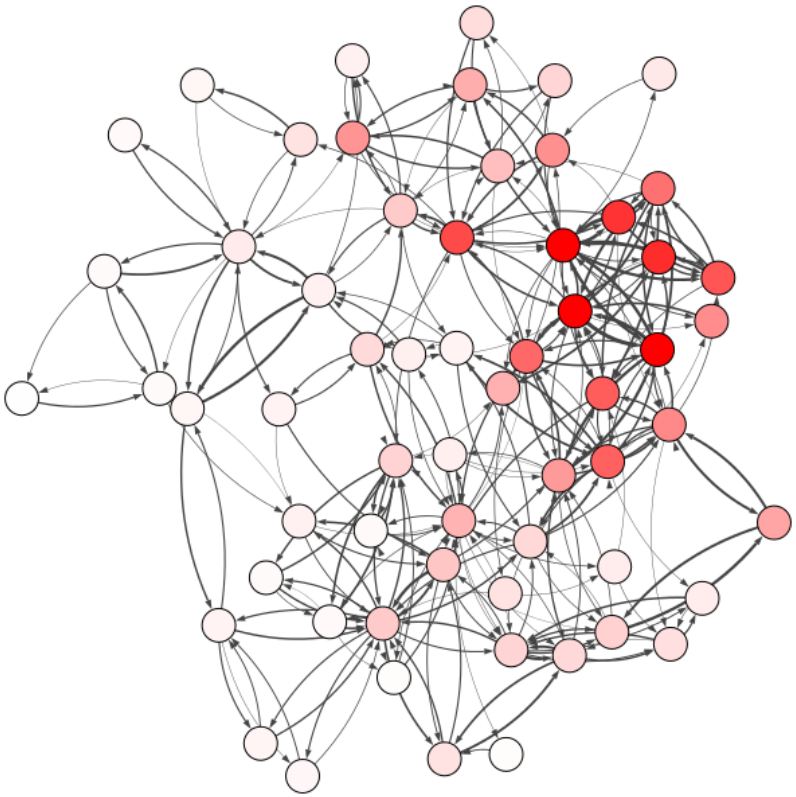
Non-Markovian Temporal Networks

- idea: calculate statistics of time-respecting paths $u \to v \to w$
  1. **Markovian** temporal network
     $P(u \to v \to w) = P(u \to v) \cdot P(v \to w)$
  2. **Non-Markovian** temporal network
     $P(u \to v \to w) \neq P(u \to v) \cdot P(v \to w)$
- different ways to **detect need for higher-order models**
  1. betweenness preference: PRL 110, 198701 (May 2013)
  2. entropy growth rate: Nature Comm. 5:5024 (Sept. 2014)
- observation: **Markovian assumption rarely holds in reality**

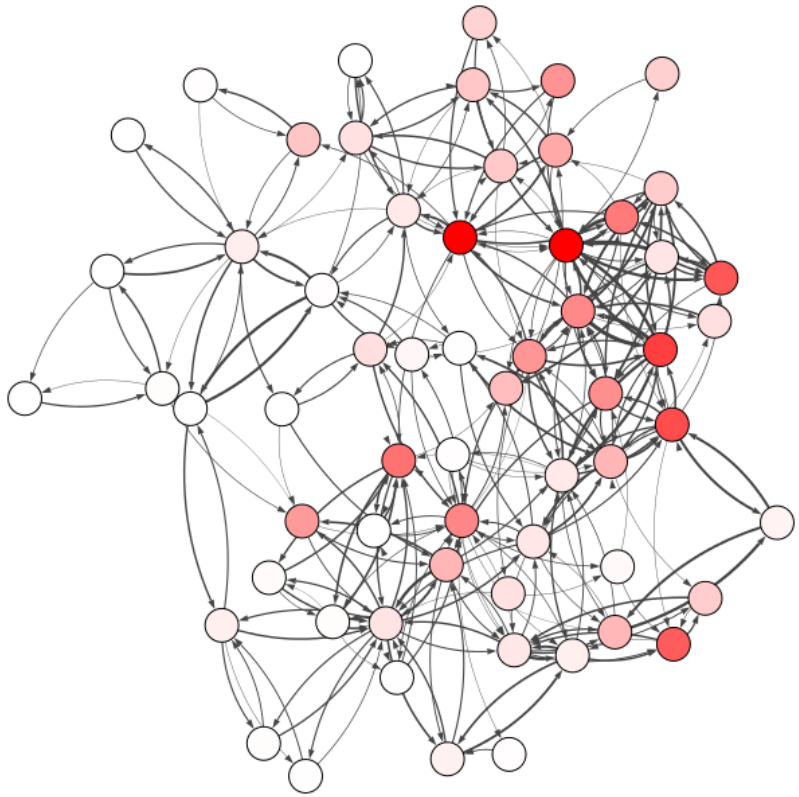


toy example for a non-Markovian temporal network

Non-Markovianity affects diffusion: ant colony



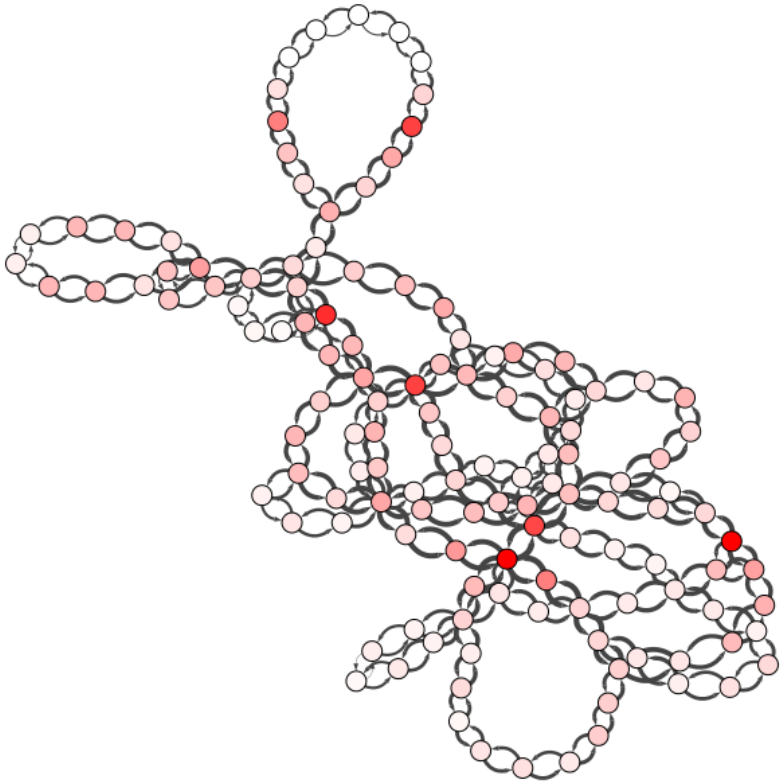
actual diffusion dynamics in
non-Markovian temporal network



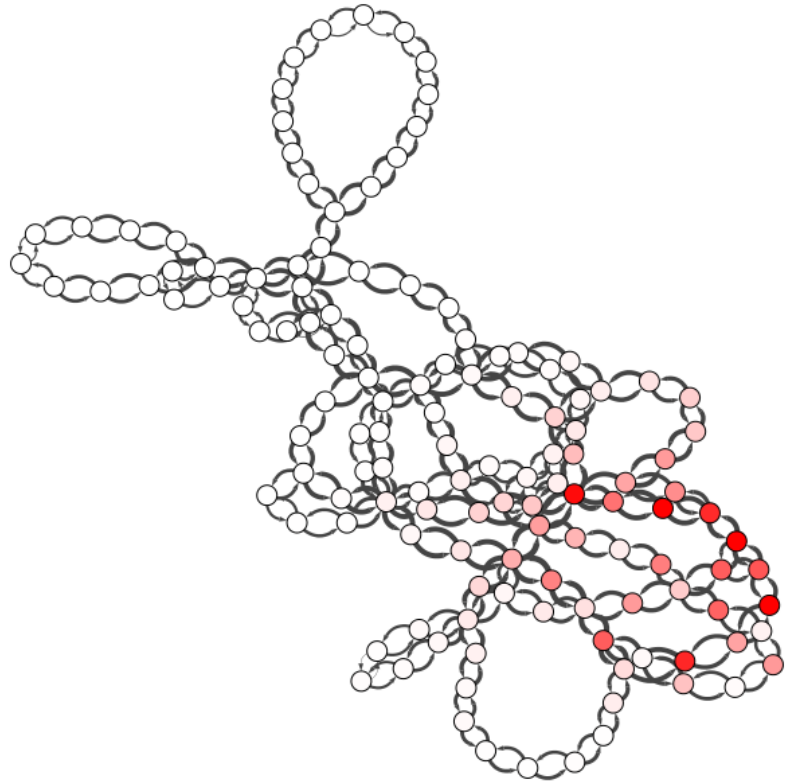
expected diffusion dynamics in
Markovian temporal network

non-Markovianity **slows down** dynamical process

Non-Markovianity affects diffusion: London Tube



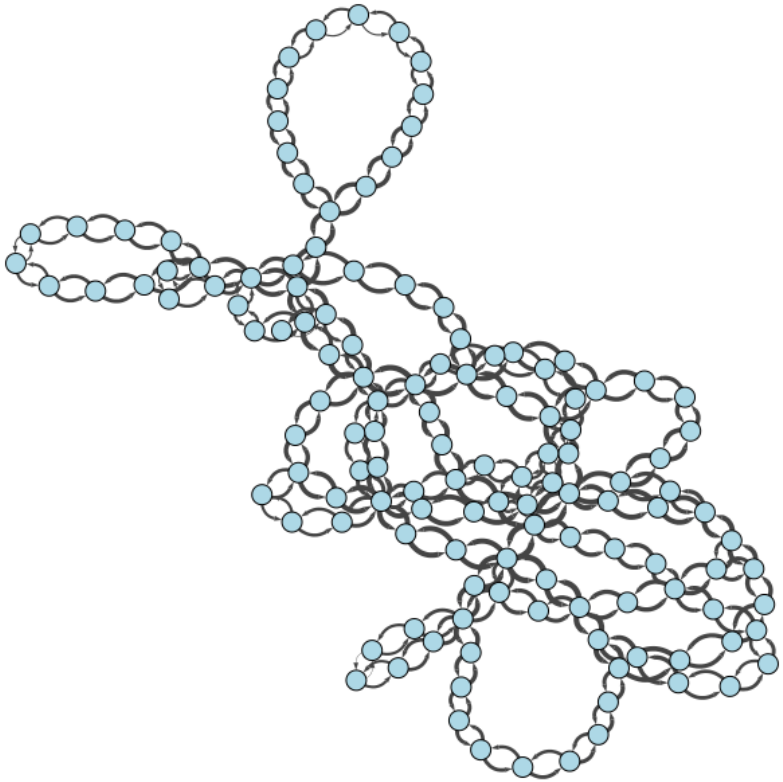
actual diffusion dynamics in
non-Markovian temporal network



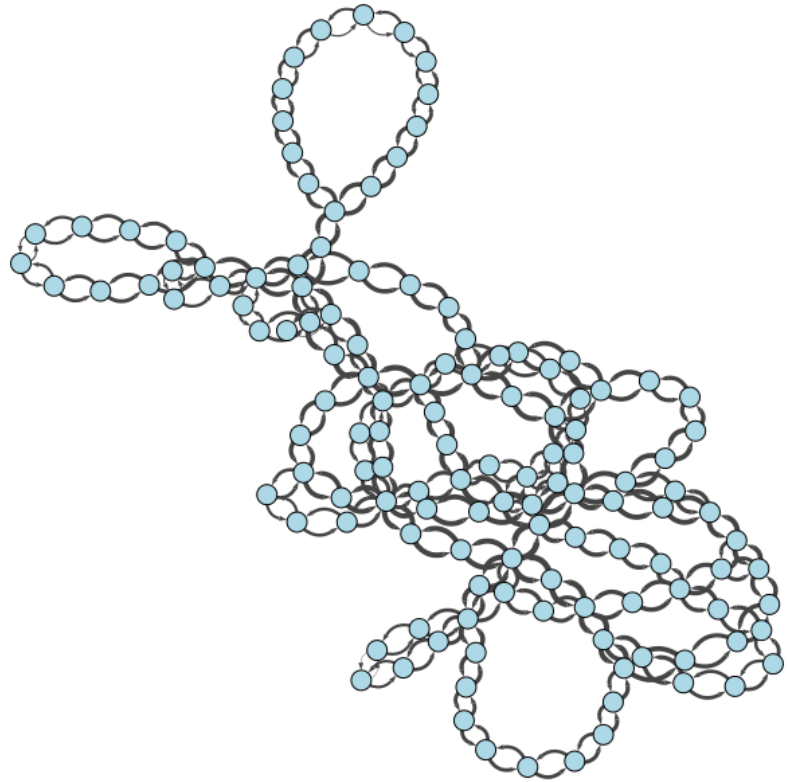
expected diffusion dynamics in
Markovian temporal network

non-Markovianity **speeds up** dynamical process

non-Markovian vs. Markovian temporal network

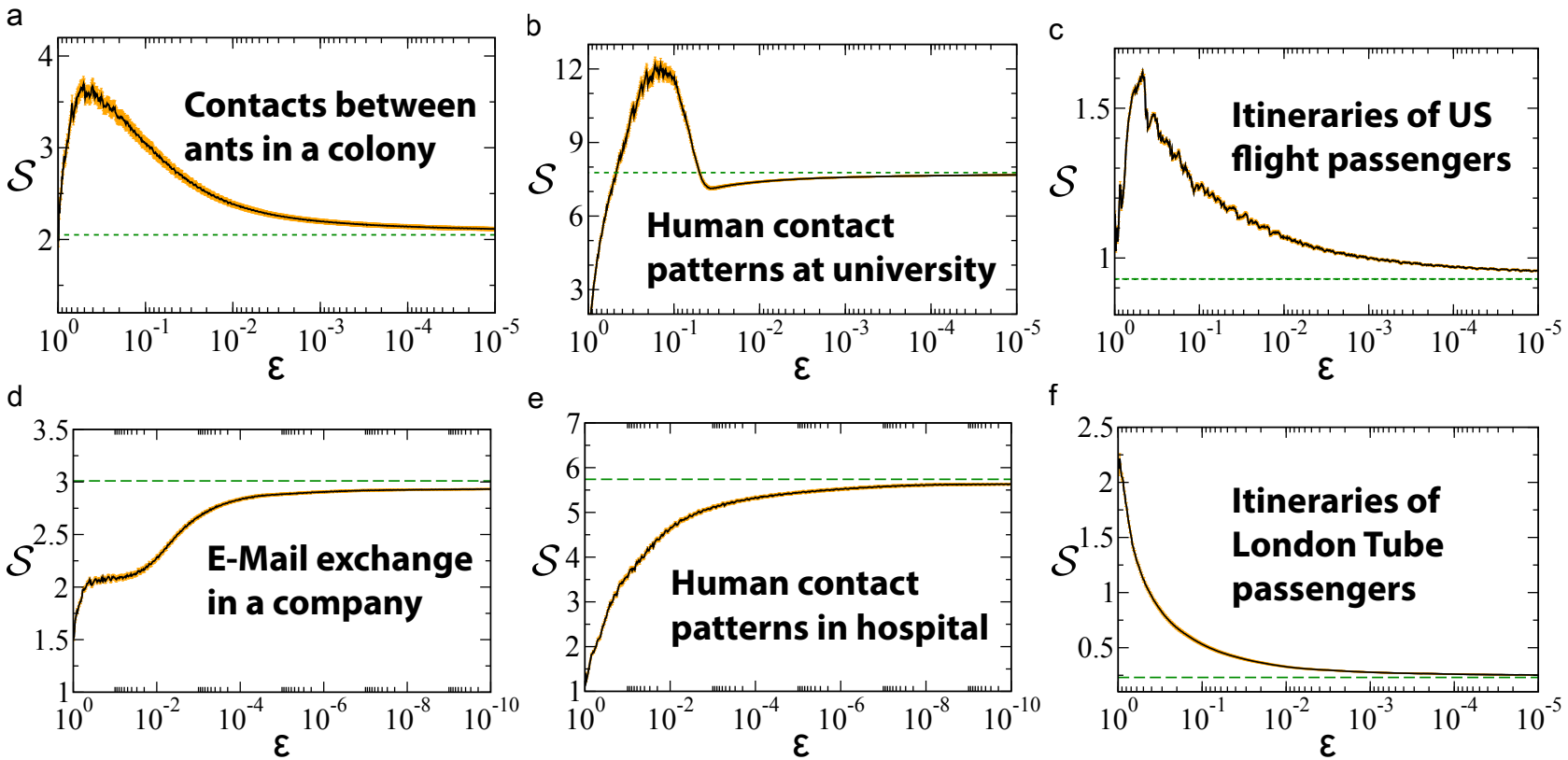


empirical sequence of passenger trips: **non-Markovian temporal network**



shuffled sequence of passenger trips: **Markovian temporal network**

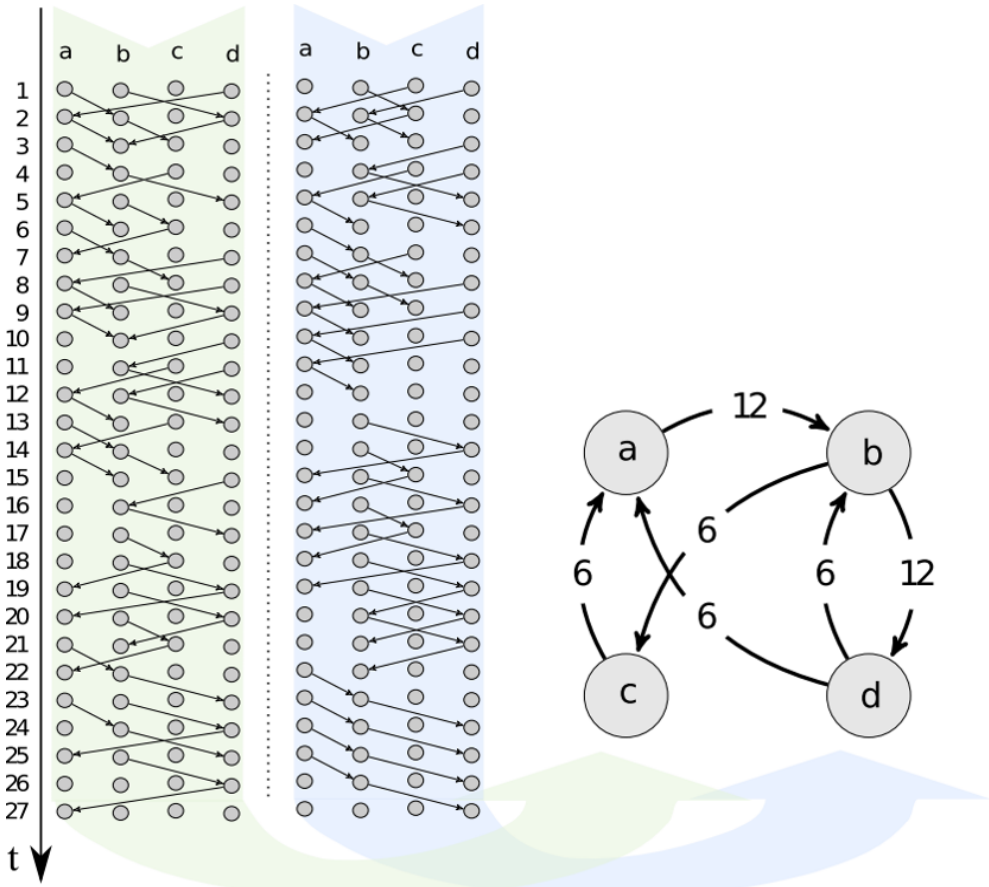
Non-Markovianity: effect on diffusion speed



London Tube: $S \approx 0.25$, i.e. non-Markovianity **speeds up** diffusion

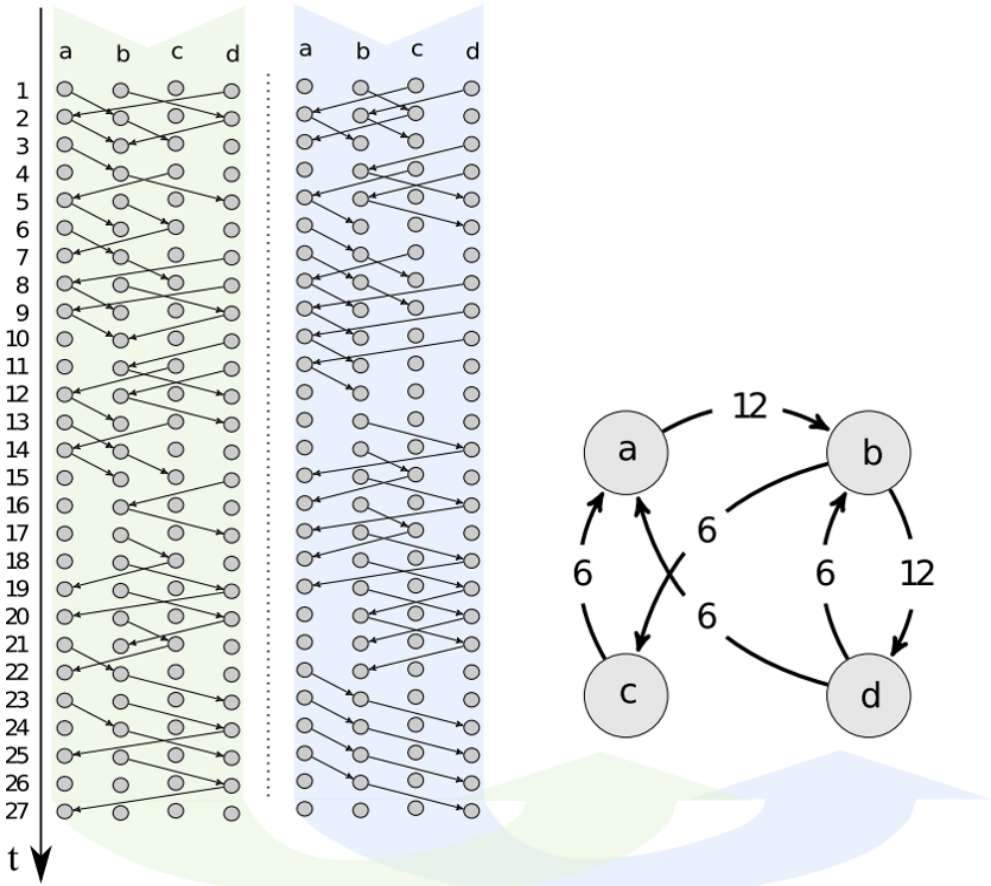
Ant colony: $S \approx 2$, i.e. non-Markovianity **slows down** diffusion

Time-aggregated networks: problem



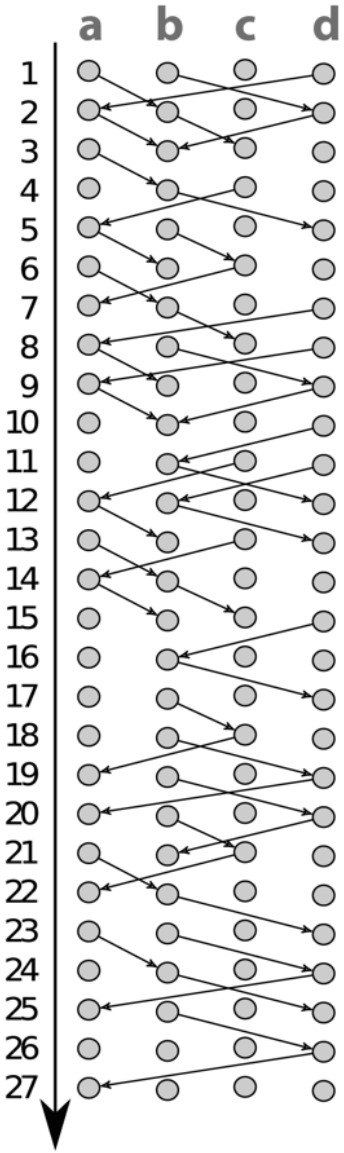
- weights in time-aggregated network capture **link statistics** in temporal network
- information on **ordering of links** is lost
- **limits our understanding** of dynamical processes, node centralities, community structures, etc.

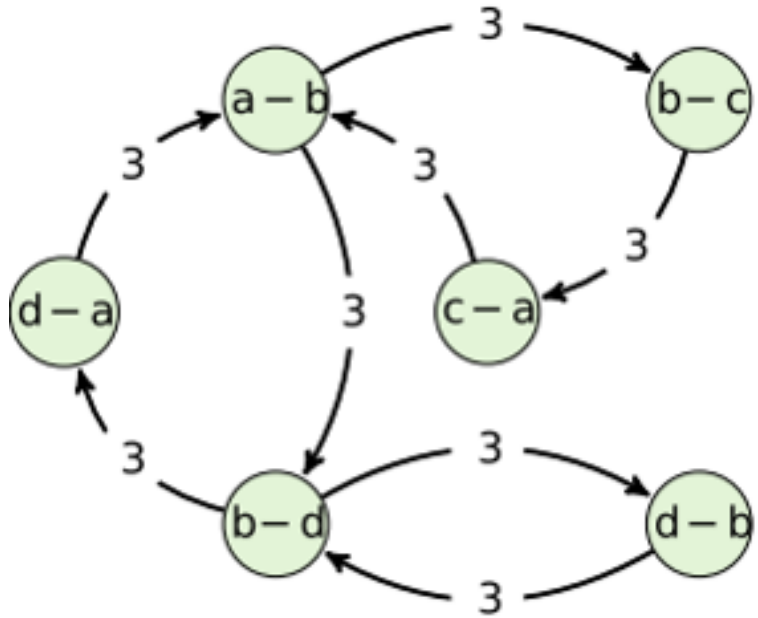
Higher-order aggregate networks?



- link = **time-respecting path of length one**
- time-aggregated networks are **first-order static representation** of temporal networks
- can we construct **higher-order static representations** which capture statistics of longer time-respecting paths?

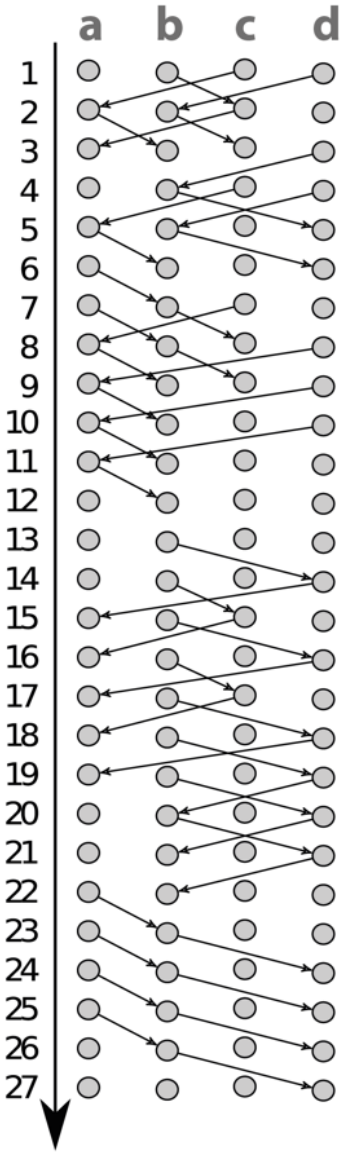
Second-order aggregate network

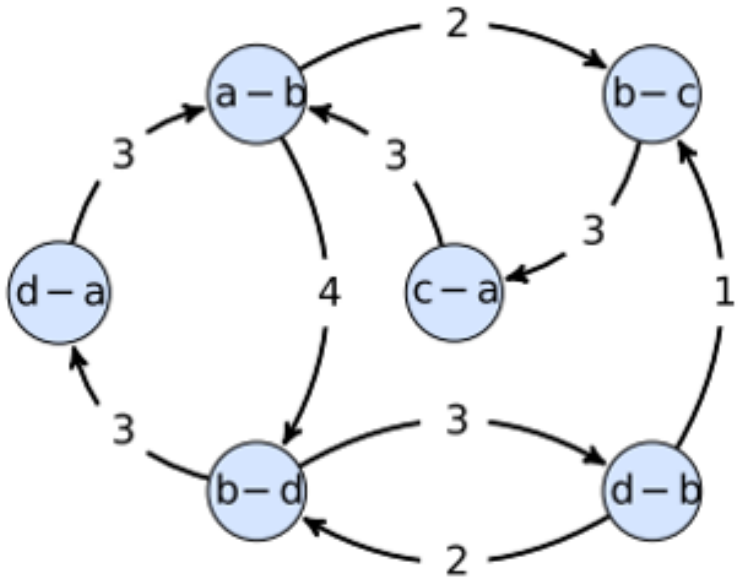




- nodes = links in temporal network
- link weight = statistics of time-respecting paths of length two
- link (e_1, e_2) implies: e_1 before e_2

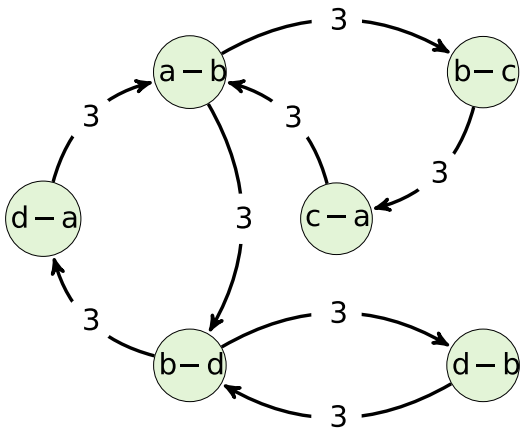
Reordering links . . .



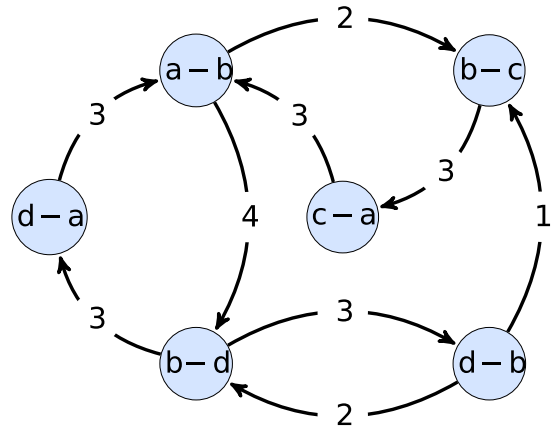


- second-order aggregate networks are **sensitive to ordering** of interactions
- we obtain the **causal topology** of temporal networks
- how does this help us?

Algebraic connectivity of temporal networks



real causal topology



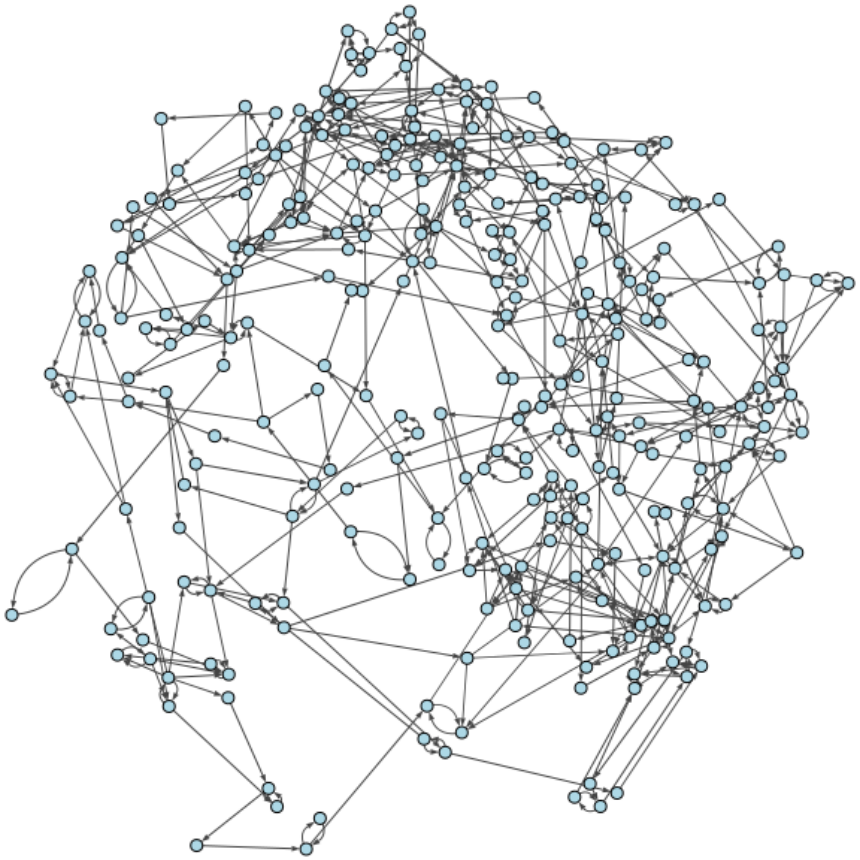
expected causal topology

$$\mathcal{L} = \begin{array}{c} \\ a;b \\ b;c \\ d;a \\ c;a \\ b;d \\ d;b \end{array} \begin{array}{c} \begin{array}{cccccc} a;b & b;c & d;a & c;a & b;d & d;b \end{array} \\ \begin{bmatrix} 1 & -\frac{1}{2} & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \end{array}$$

$$\mathcal{L} = \begin{array}{c} \\ a;b \\ b;c \\ d;a \\ c;a \\ b;d \\ d;b \end{array} \begin{array}{c} \begin{array}{cccccc} a;b & b;c & d;a & c;a & b;d & d;b \end{array} \\ \begin{bmatrix} 1 & -\frac{1}{3} & 0 & 0 & -\frac{2}{3} & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & 1 & -\frac{1}{2} \\ 0 & -\frac{1}{3} & 0 & 0 & -\frac{2}{3} & 1 \end{bmatrix} \end{array}$$

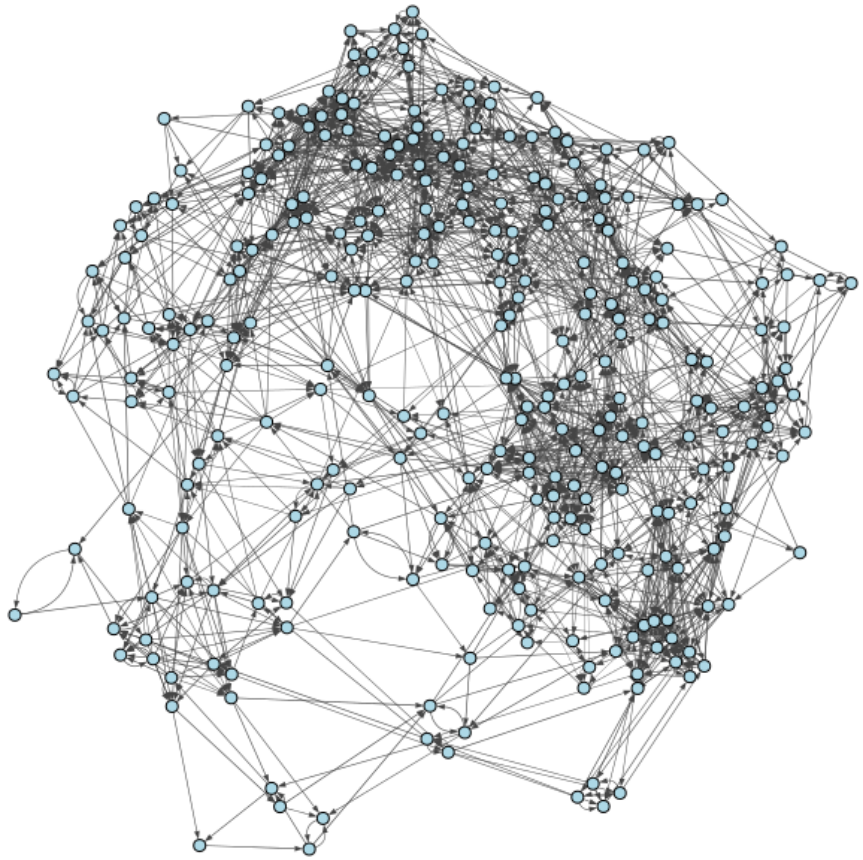
$\Rightarrow$ **spectral analysis of causal topology** induced by order of links

Ant colony: ordering slows down diffusion



real causal topology

$\lambda_2(\mathbf{L}) \approx 0.043$

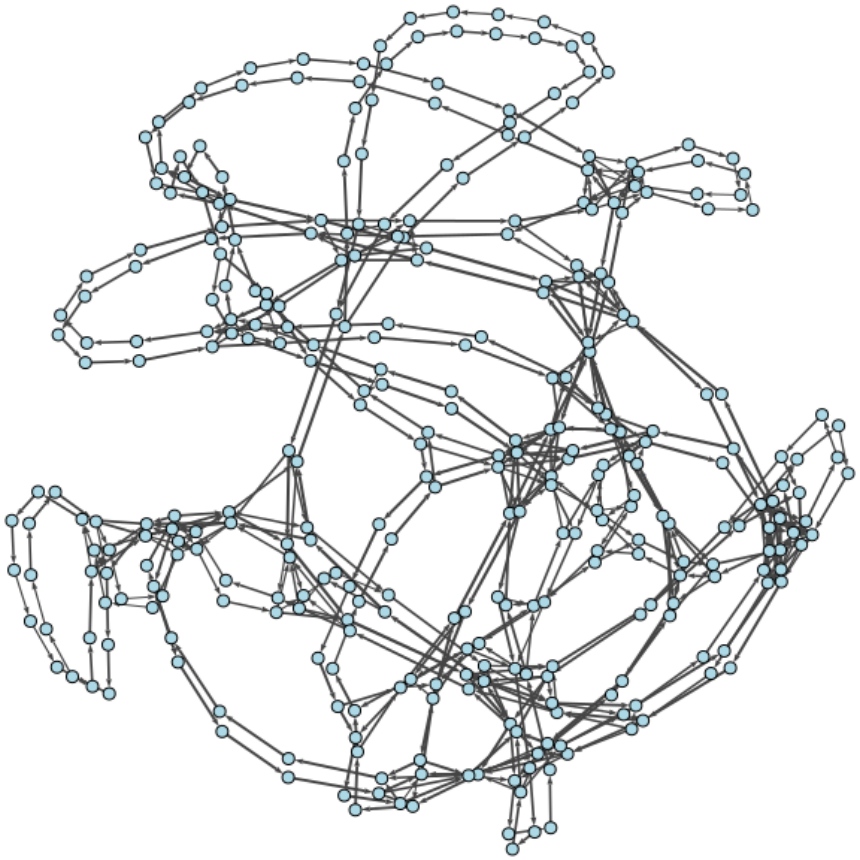


expected causal topology

$\tilde{\lambda}_2(\mathbf{L}) \approx 0.086$

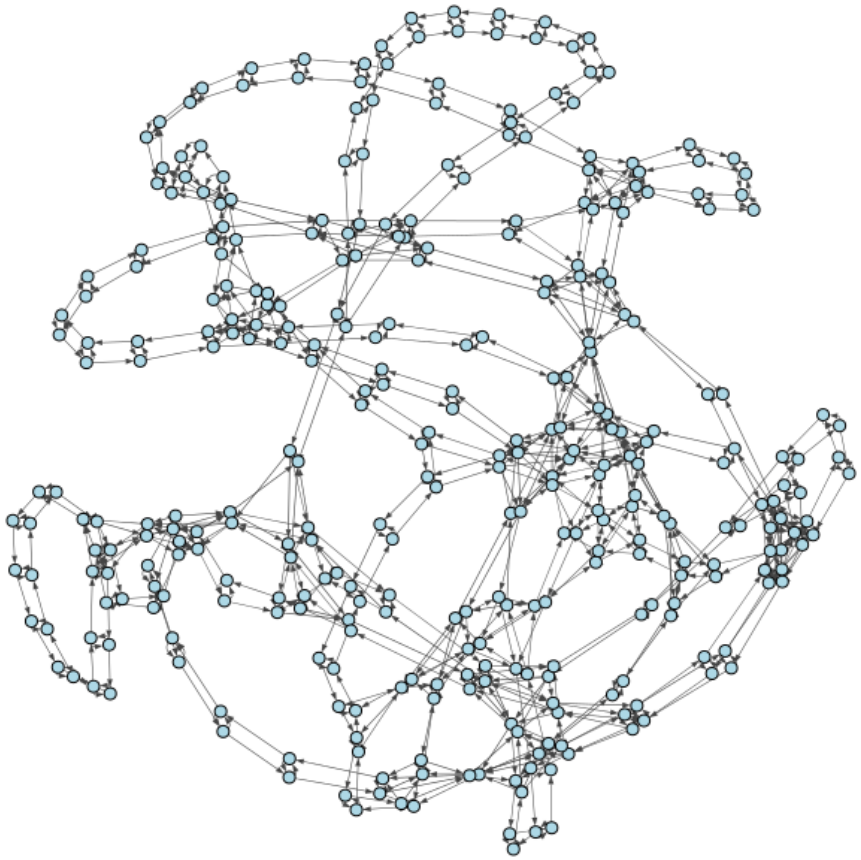
$$S = \frac{\ln|\tilde{\lambda}_2(\mathbf{T})|}{\ln|\lambda_2(\mathbf{T})|} = 2.05$$

London Tube: ordering speeds up diffusion



real causal topology

$\lambda_2(\mathbf{L}) \approx 0.125$



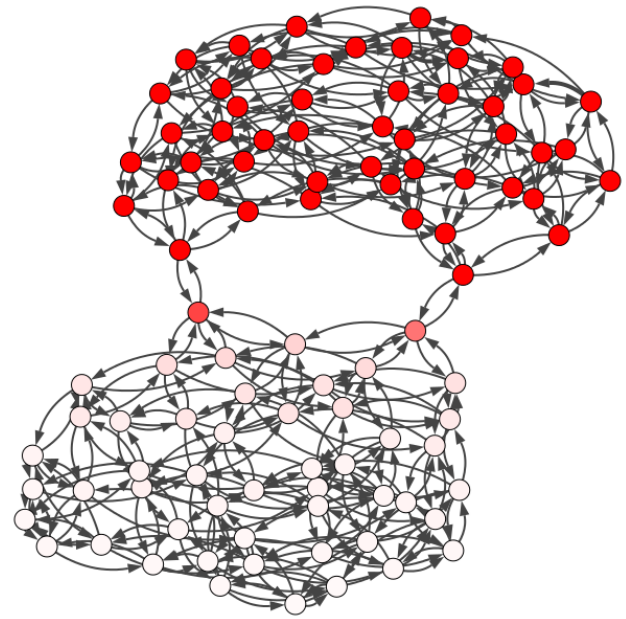
expected causal topology

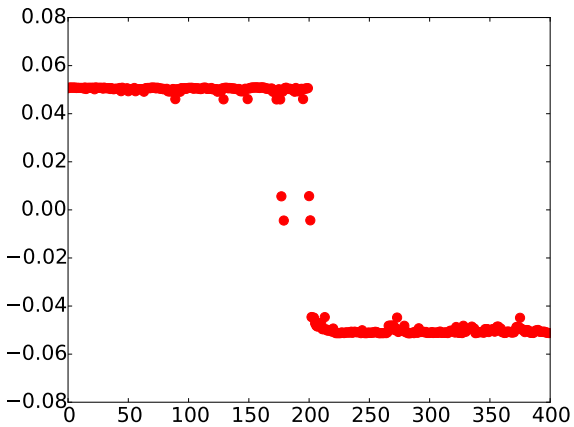
$\tilde{\lambda}_2(\mathbf{L}) \approx 0.011$

$$S = \frac{\ln|\tilde{\lambda}_2(\mathbf{T})|}{\ln|\lambda_2(\mathbf{T})|} = 0.24$$

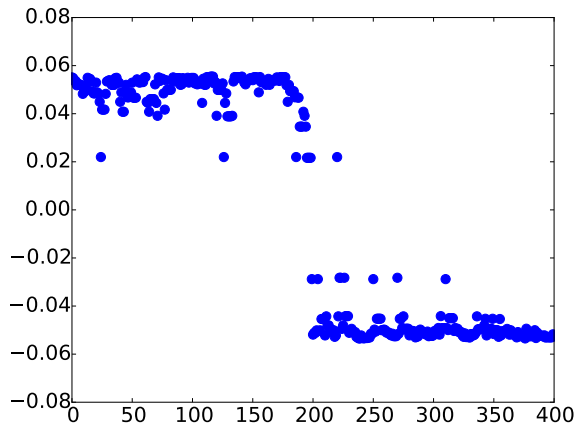
Shameless plug: spectral partitioning

- higher-order networks allow **spectral partitioning**
- example: Fiedler vector of second-order aggregate network
- order correlations can **enforce** or **mitigate** community structures
- details: temporal networks session (THU)

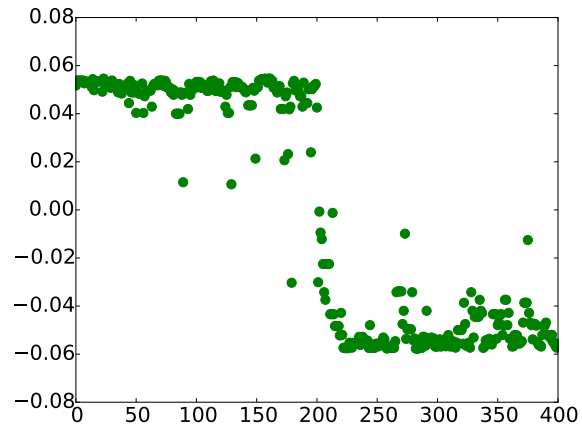




non-Markovian



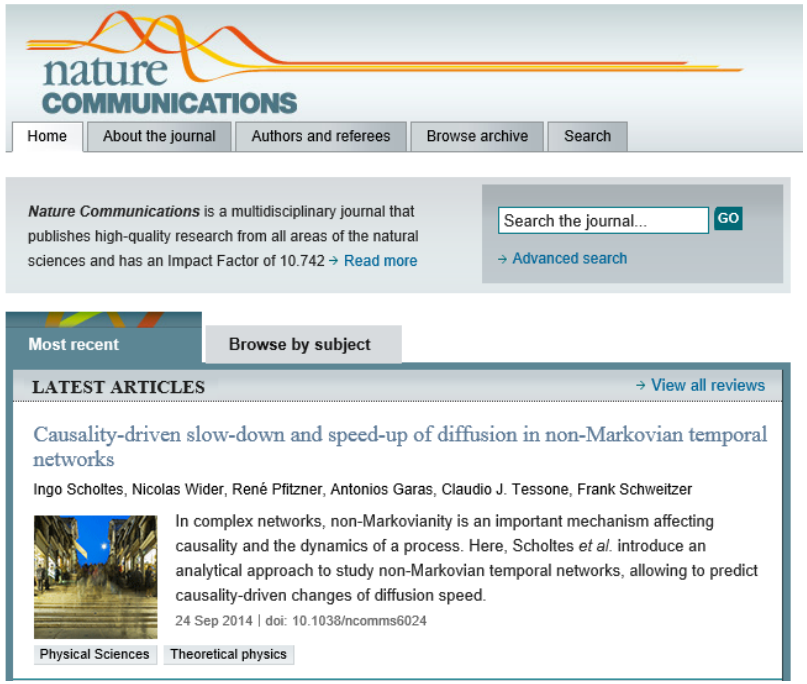
Markovian



non-Markovian

Conclusion

- **ordering matters in temporal networks!**
- **complex interplay with network topology** can lead both to slow down and speed up of dynamical processes
- interplay can be studied analytically using **higher-order aggregate networks**
- allows to explore **additional dimension of complexity** in temporal networks
- **pyTempNets**: full-fledged python module to reproduce results



Scholtes, I., Wider, N., Pfitzner, R., Garas A., Tessone, C.J., Schweitzer, F. (2014) **Causality-driven slow-down and speed-up of diffusion in non-Markovian temporal networks**. Nature Communications 5, September 24 2014

Thank You!

R Pfitzner, I Scholtes, A Garas, CJ Tessone, F Schweitzer: **Betweenness Preference: Quantifying Correlations in the Topological Dynamics of Temporal Networks**, Phys Rev Lett, Vol. 110, 198701, May 10 2013

I Scholtes, N Wider, R Pfitzner, A Garas, CJ Tessone, F Schweitzer: **Slow-down and Speed-up of Diffusion in Non-Markovian Temporal Networks**, Nature Communications, Vol. 5, September 24 2014

I Scholtes et al.: **Force-Directed Layout of Non-Markovian Temporal Networks**, working paper, available online at http://www.sg.ethz.ch, May 06 2014

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