NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

# CE/CZ4052 Cloud Computing

## Pagerank Algorithm


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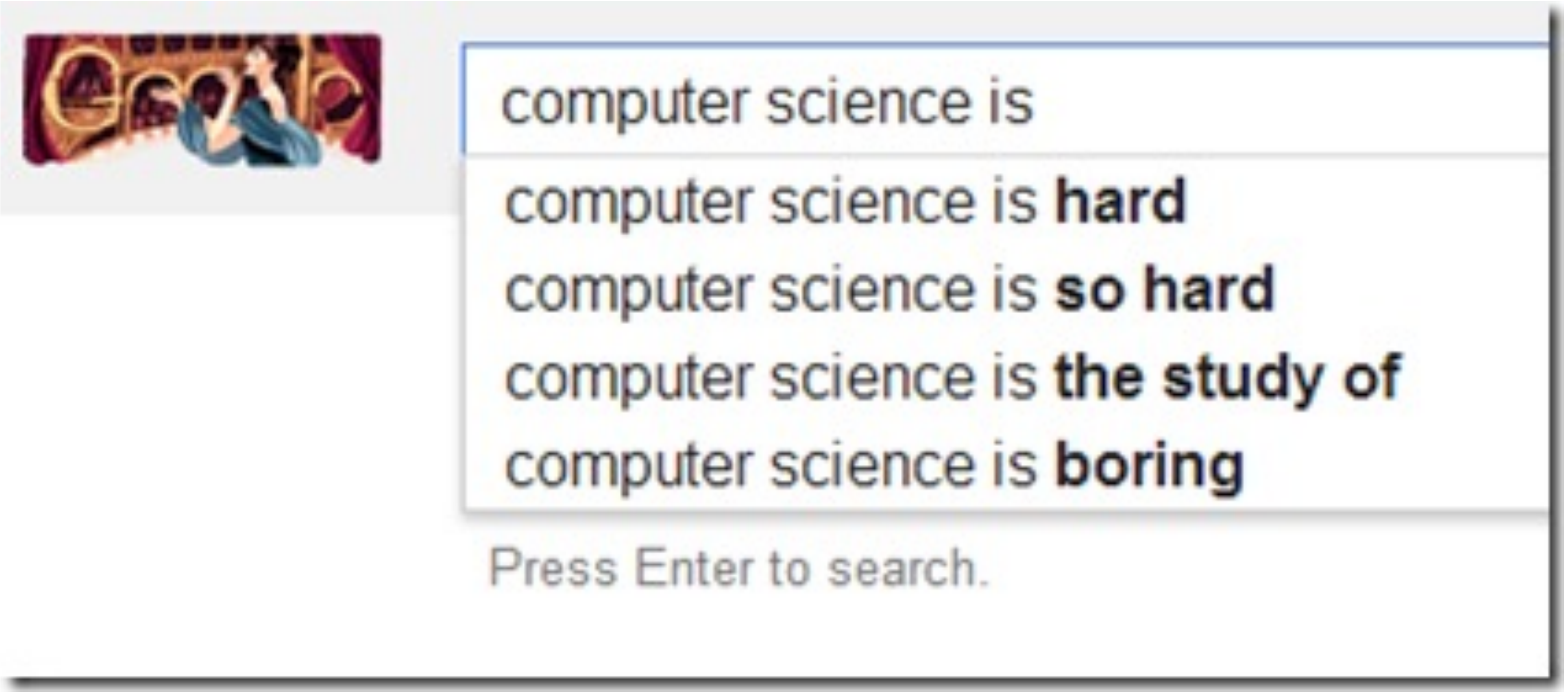
# How does Google rank the Web?

Acknowledgement:
https://www.cazencott.info/dotclear/public/lectures/lsml19/2019-03-28_lsml19_systems.pdf

# Search Engine Technologies

- **Computer Science is …**



# Search Engine Technologies

## Definition of **google** in English

### google

Pronunciation: /ˈguːgl/

Translate **google** | into French | into Italian | into Spanish

*verb*

[*with object*]

search for information about (someone or something) on the Internet using the search engine Google:

*on Sunday she googled an ex-boyfriend*

[*no object*]:

*I* ***googled for*** *a cheap hotel/flight deal*

**Derivatives**

**googleable**

(also **googlable**) *adjective*

# The History of PageRank

- PageRank was developed by Larry Page (hence the name *Page*-Rank) and Sergey Brin at Stanford in 1999.
- Shortly after, Page and Brin founded Google.
- Challenges: Web contains many sources of information – including spam.
  - What is the best answer to a web query “google” in 1998?
  - A good web search algorithm enables trust
- Use links as votes to rank pages
- Are all links equally important?
  - Links from important pages count more.
  - This question is recursive.

# The PageRank in Search Engines (1997)

Location: http://walrus.stanford.edu/diglib/pub/people.html

What's New? | What's Cool? | Destinations | Net Search | People | Software

## Stanford University Participants

*The Database group*

Prof. Hector Garcia-Molina , Misturu Akizawa (Visiting Scholar from Hitachi) , Edward Chang , Chen-Chuan K. Chang , Arturo Crespo , Luis Gravano , Matt Jacobsen , Steven Ketchpel , Yusuke Mishina (Visiting Scholar from Hitachi) , Narayanan Shivakumar

*The Project on People Computers and Design*

Prof. Terry Winograd , Michelle Q Wang Baldonado , Steve Cousins , Mauria Finley , Frankie James , Larry Page , Christian P. Rohrer , Martin Röscheisen , Alan Steremberg , Trace Wax

*The Nobots group*

Prof. Daphne Koller , Prof. Yoav Shoham , Marko Balabanovic , Avi Pfeffer , Mehran Sahami , Katsumi Tanaka (Visiting Scholar)

*The Testbed group*

Scott Hassan , Andy Kacsman , Andreas Paepcke , Tom Schirmer

*Stanford Libraries and Academic Information Resources*

Rebecca Lasher , Vicky Reich

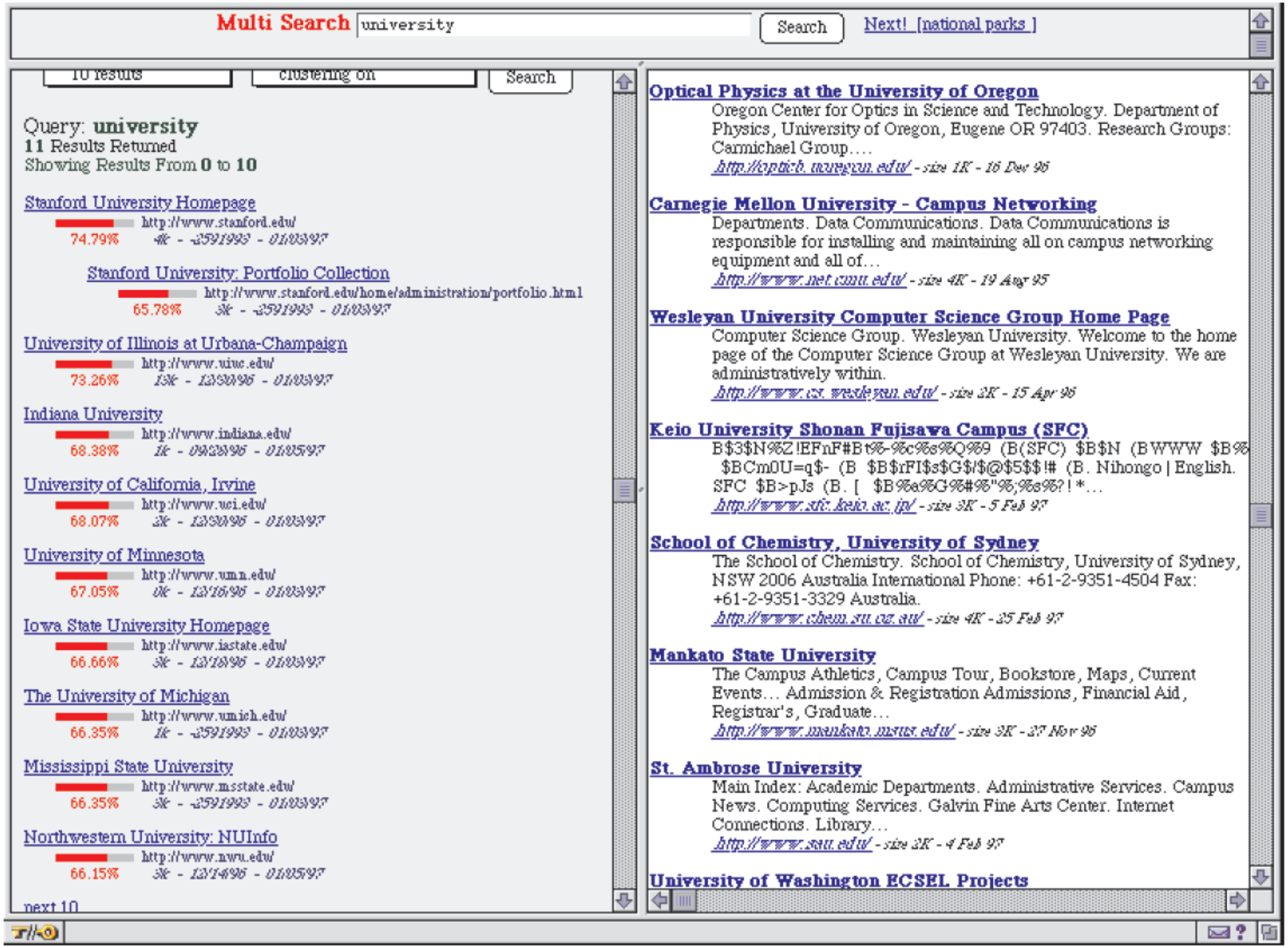
*Engineering-Economic Systems*

Tim Stanley

*Alumni from the Stanford Digital Libraries Project*

Perry Arnold , Kenichi Kamiya , James Kittock , Christian Mogensen , Tak Yan

# Searching with PageRank (1997)



# Searching with PageRank (1997)

| Web Page | PageRank (average is 1.0) |
|---|---|
| Download Netscape Software | 11589.00 |
| http://www.w3.org/ | 10717.70 |
| Welcome to Netscape | 8673.51 |
| Point: It's What You're Searching For | 7930.92 |
| Web-Counter Home Page | 7254.97 |
| The Blue Ribbon Campaign for Online Free Speech | 7010.39 |
| CERN Welcome | 6562.49 |
| Yahoo! | 6561.80 |
| Welcome to Netscape | 6203.47 |
| Wusage 4.1: A Usage Statistics System For Web Servers | 5963.27 |
| The World Wide Web Consortium (W3C) | 5672.21 |
| Lycos, Inc. Home Page | 4683.31 |
| Starting Point | 4501.98 |
| Welcome to Magellan! | 3866.82 |
| Oracle Corporation | 3587.63 |

Top 15 Page Ranks: July 1996

# The PageRank in Search Engines (2017)


https://www.alexa.com/siteinfo https://moonsy.com/alexa_rank/

**Guess, who has top ranking, i.e., number 1?**

# Link Analysis

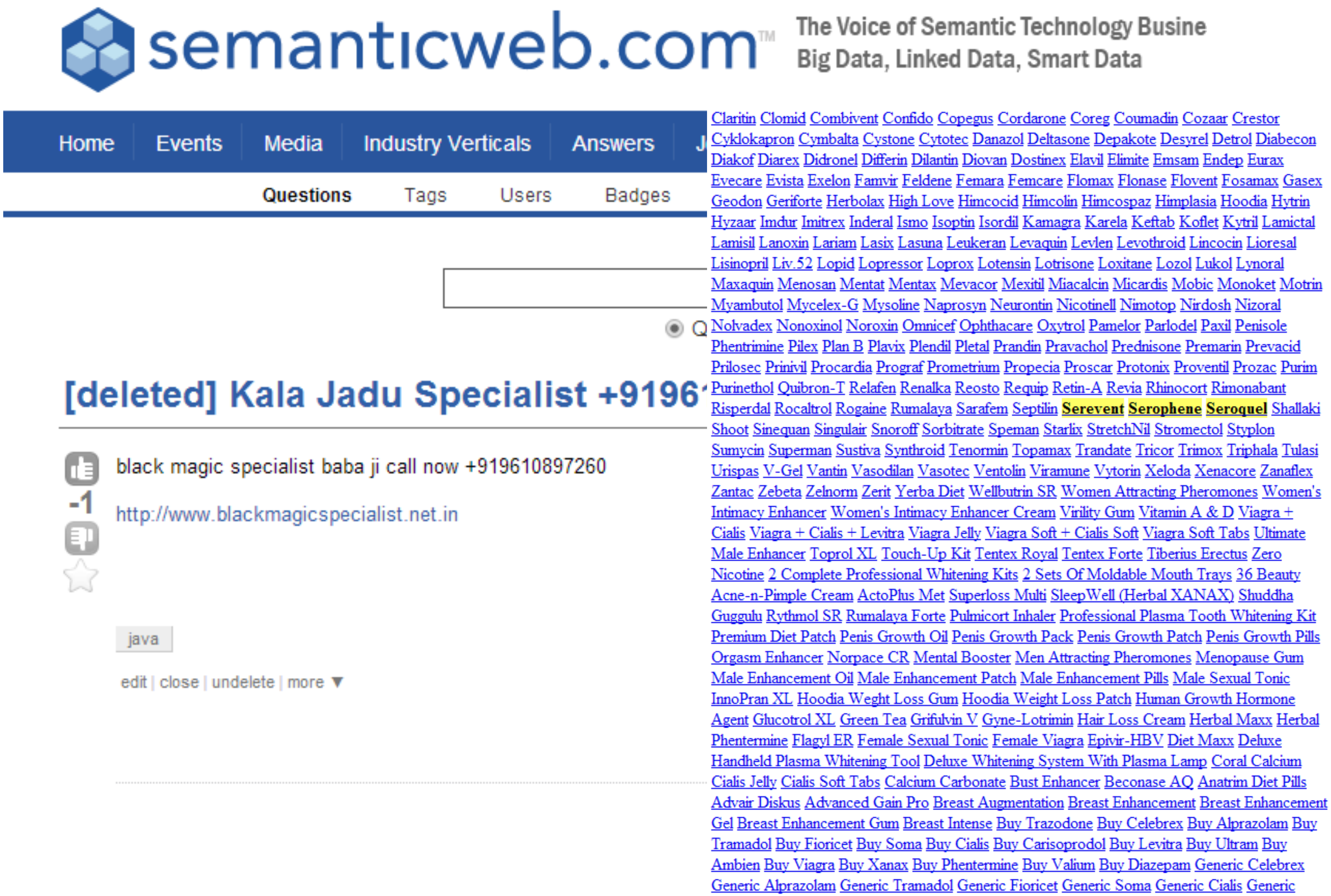
- Consider links as votes of confidence in a page
- A hyperlink is the open Web's version of …

(… even if the page is linked in a negative way.)

# Link Analysis

So if we just count the number of inlinks a web-page receives we know its importance, right?

# Link Spamming



# PageRank

# PageRank

- Not just a count of inlinks
  - A link from a more important page is more important
  - A link from a page with fewer links is more important

  ∴ A page with lots of inlinks from important pages (which have few outlinks) is more important

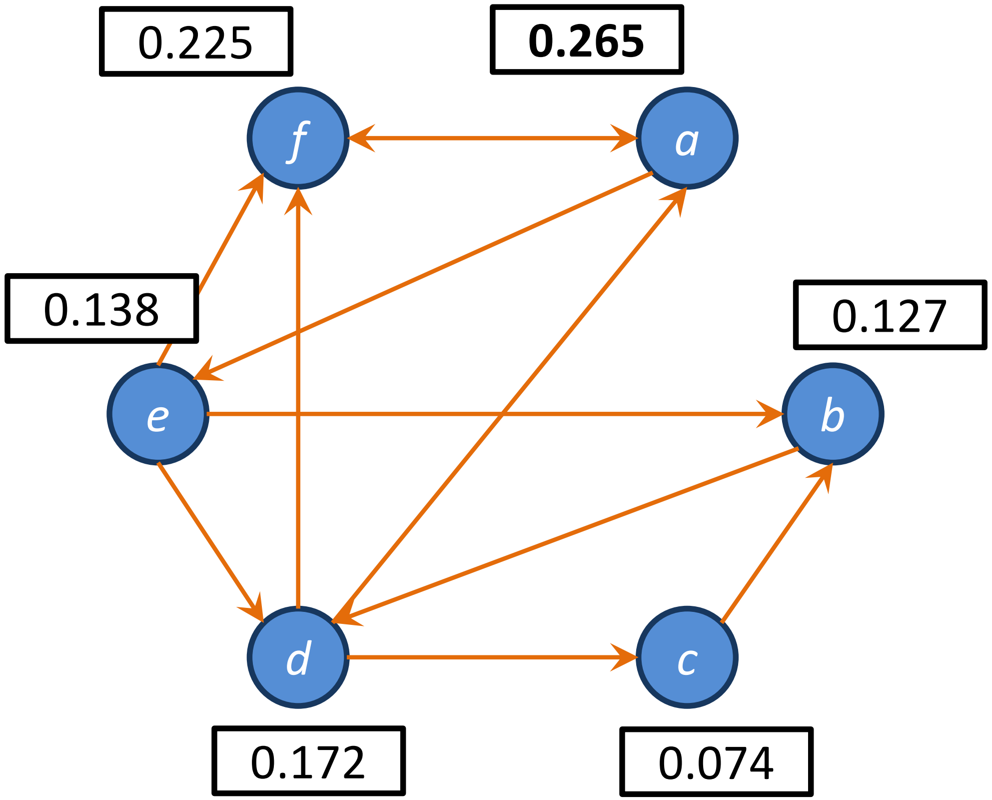
# PageRank Model

- The Web: a directed graph

$G = (\boxed{V}, \boxed{E})$

Vertices (*pages*)

Edges (*links*)




Which is the most "important" vertex?

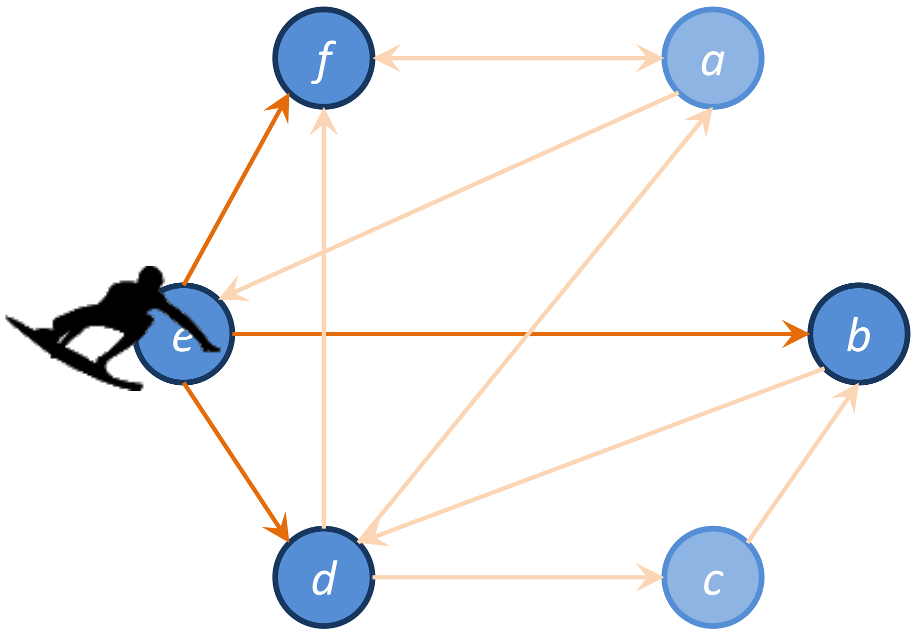
$V = \{a, b, c, d, e, f\}$

$E = \{(a, e), (a, f), (b, d), (c, b), (d, a), (d, c), (d, f), (e, b), (e, d), (e, f), (f, a)\}$

# PageRank: Random Surfer Model

= someone surfing the web, clicking links randomly

- What is the probability of being at page *x* after *n* hops?



# PageRank: Random Surfer Model

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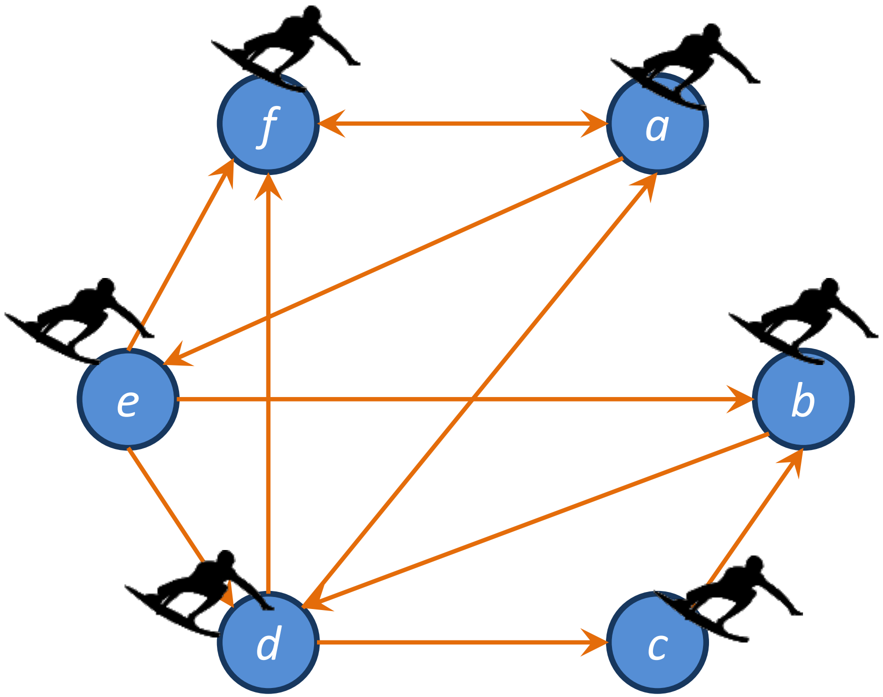


- What is the probability of being at page *x* after *n* hops?
- *Initial state:* surfer equally likely to start at any node

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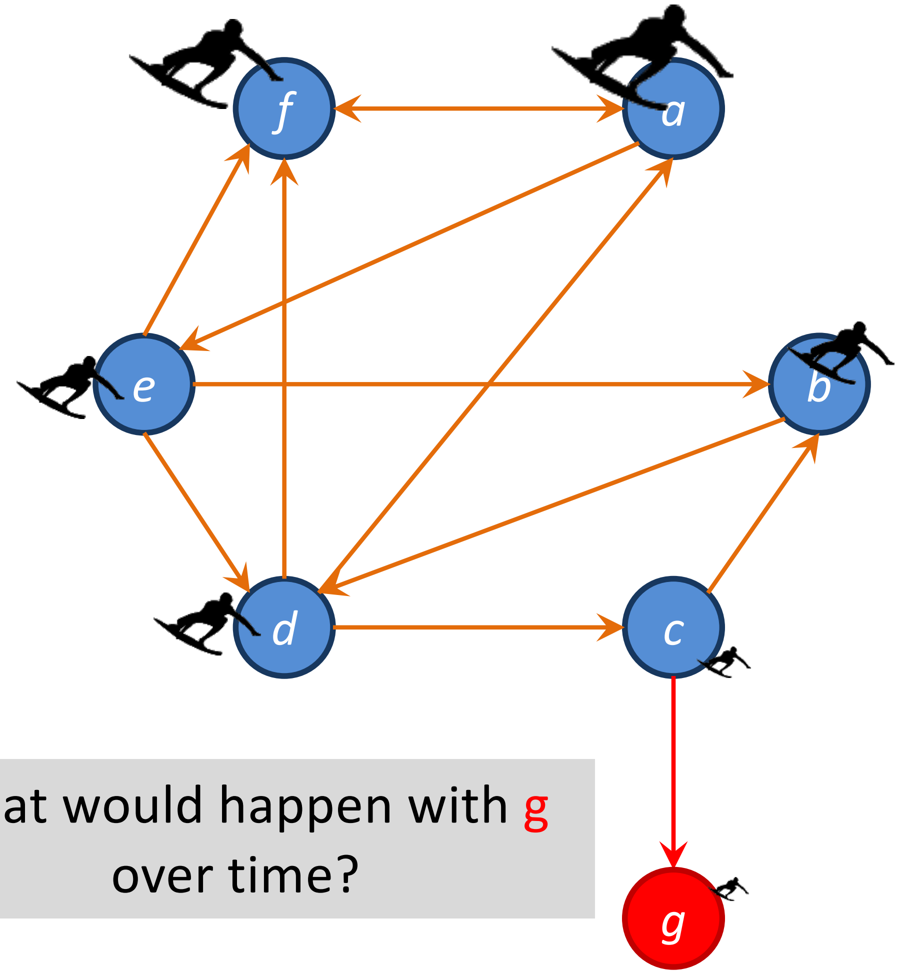


What would happen with g over time?

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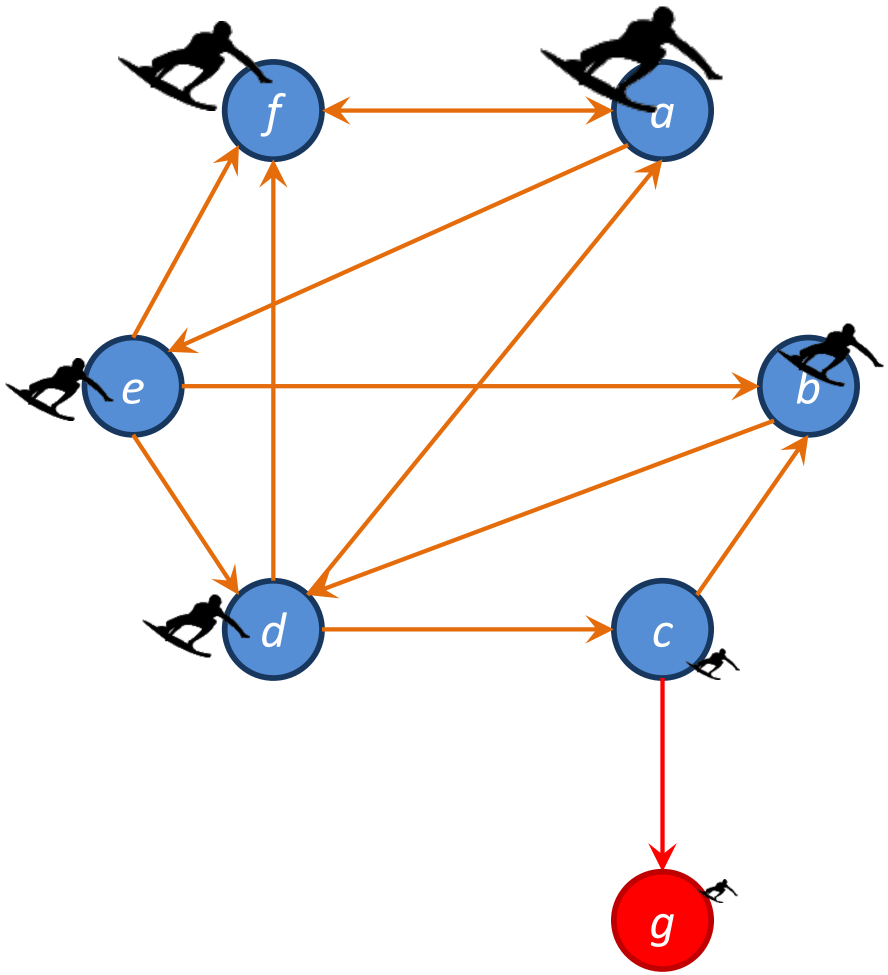


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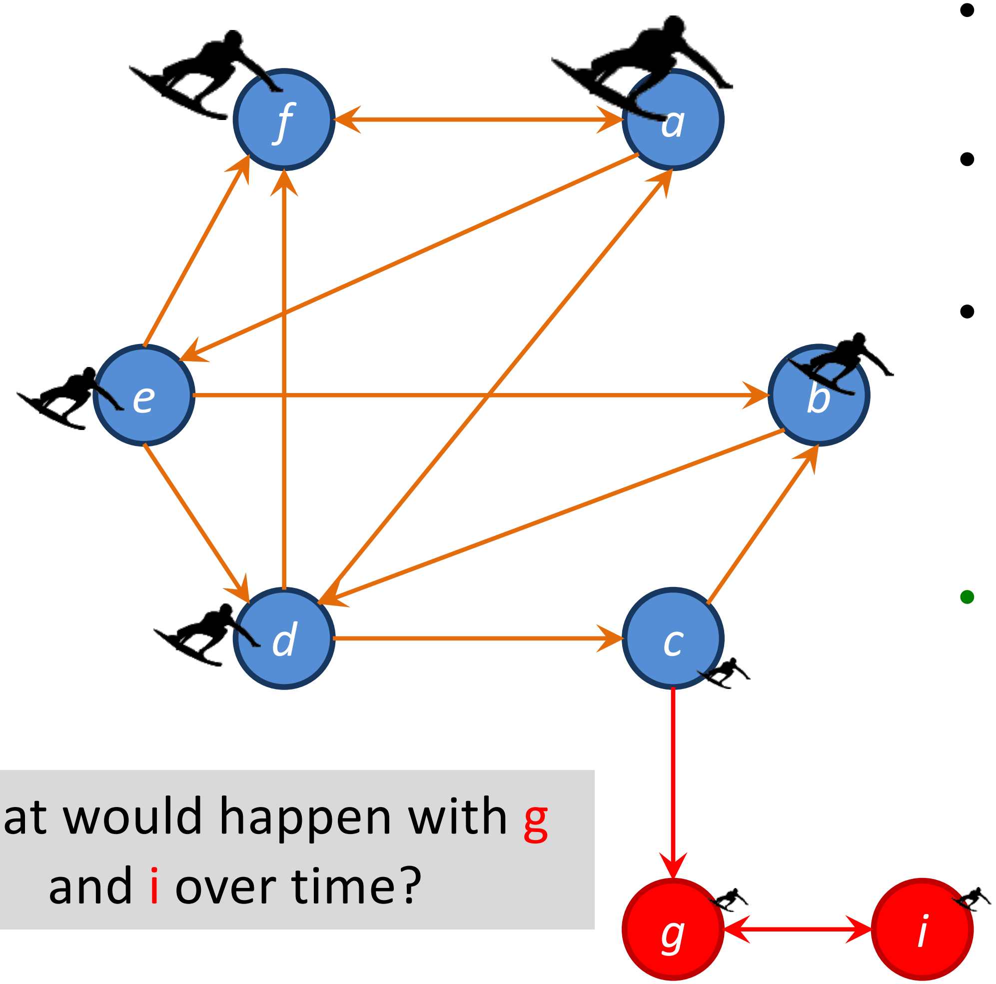


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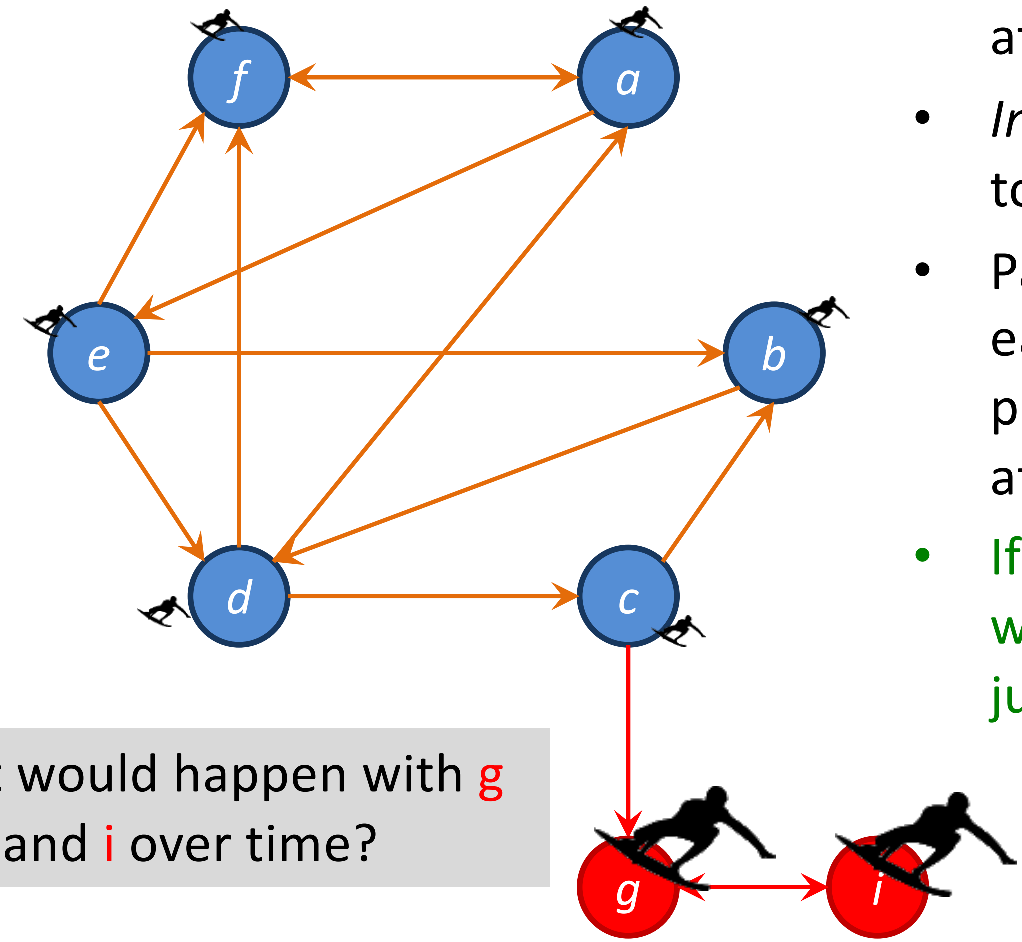


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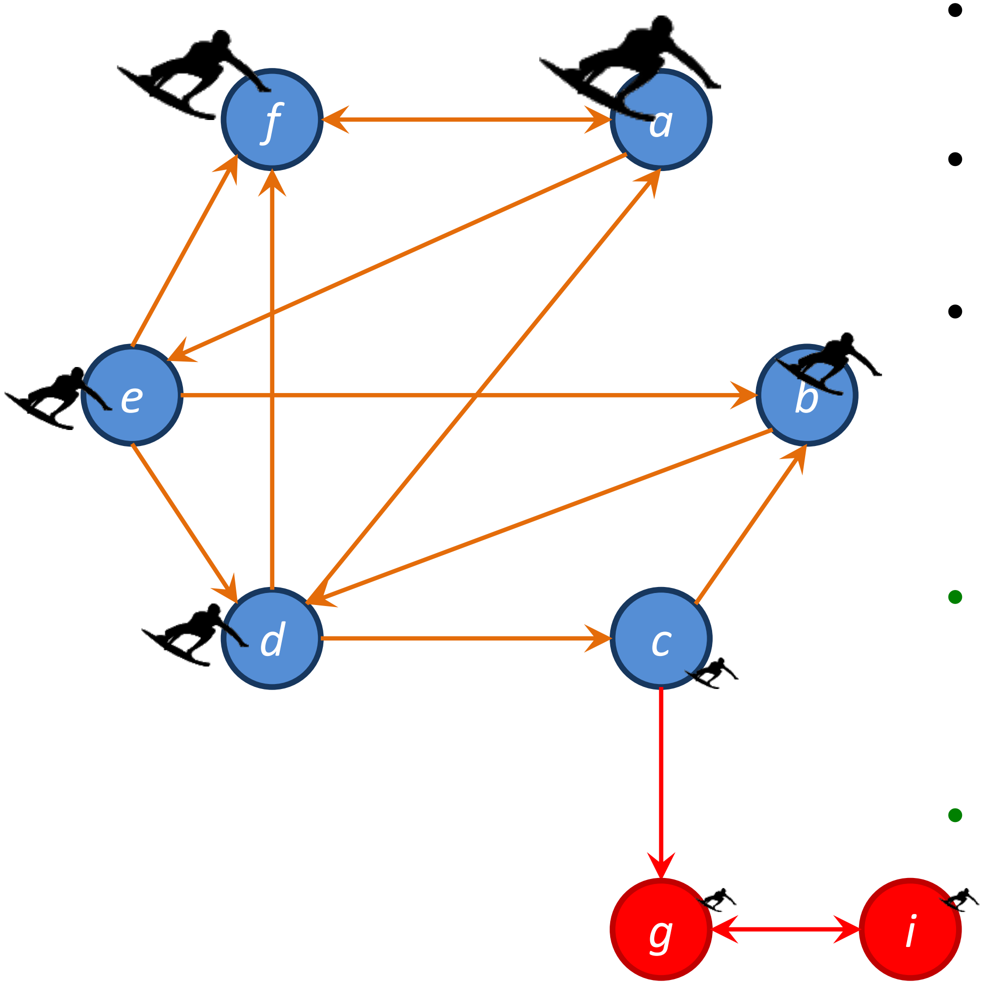
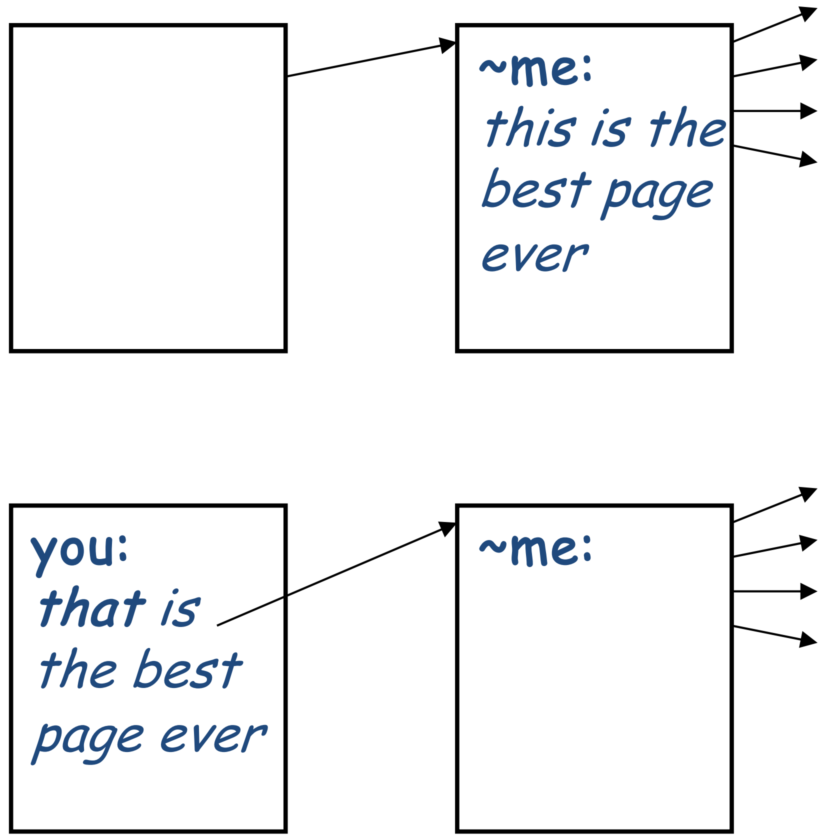



- What is the probability of being at page *x* after *n* hops?
- *Initial state:* surfer equally likely to start at any node
- PageRank applied iteratively for each hop: score indicates probability of being at that page after than many hops
- If the surfer reaches a page without links, the surfer randomly jumps to another page
- The surfer will jump to a random page at any time with a probability 1 – *d … this avoids traps and ensures convergence!*

# Google search: anchor text

- Pagerank
- Anchor text



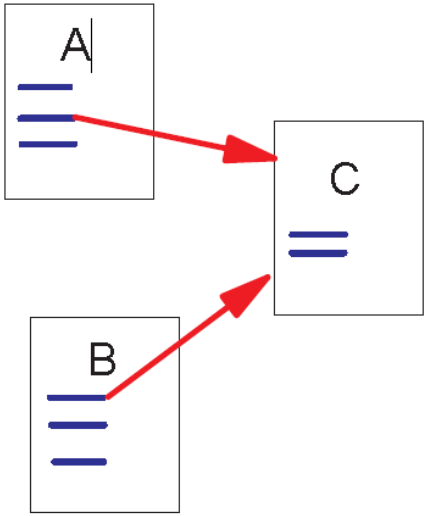

## Google uses:

- In anchor text?
- In URL?
- Title
- Meta tags
- <h> level
- Rel font size
- Capitalization
- Word pos in doc
- Secret ingredients

… and weighs them according to a secret recipe

# Link Structure of the Web

- 150 million web pages → 1.7 billion links




Backlinks and Forward links:

- A and B are C's backlinks
- C is A and B's forward link

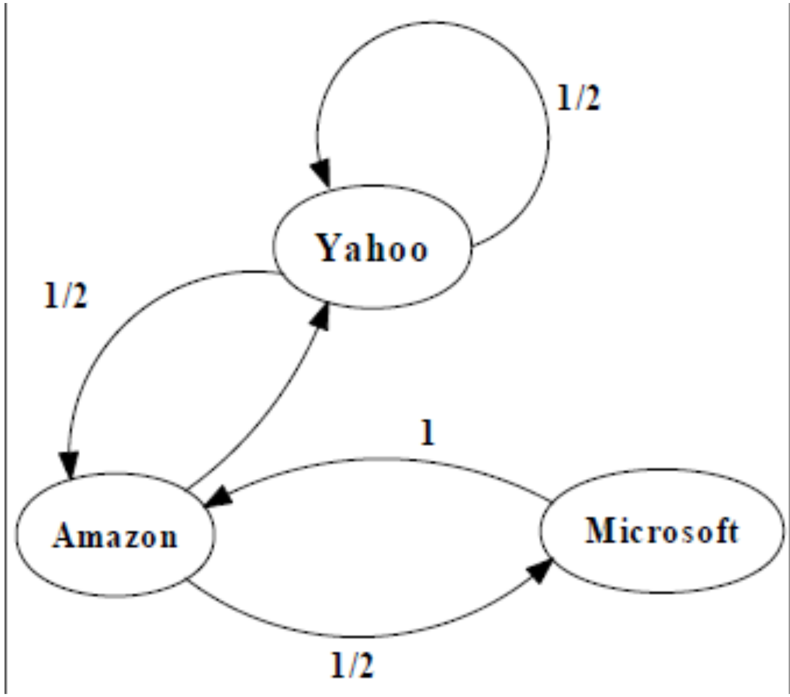
Intuitively, a webpage is important if it has a lot of backlinks.

# A Simple Version of PageRank

$$R(u) = c \sum_{v \in B_u} \frac{R(v)}{N_v}$$

- u: a web page
- $B_u$: the set of u’s backlinks
- $N_v$: the number of forward links of page v
- c: the normalization factor to make R(1) + … + R(T) = 1 where there are T pages in total

# An example of Simplified PageRank



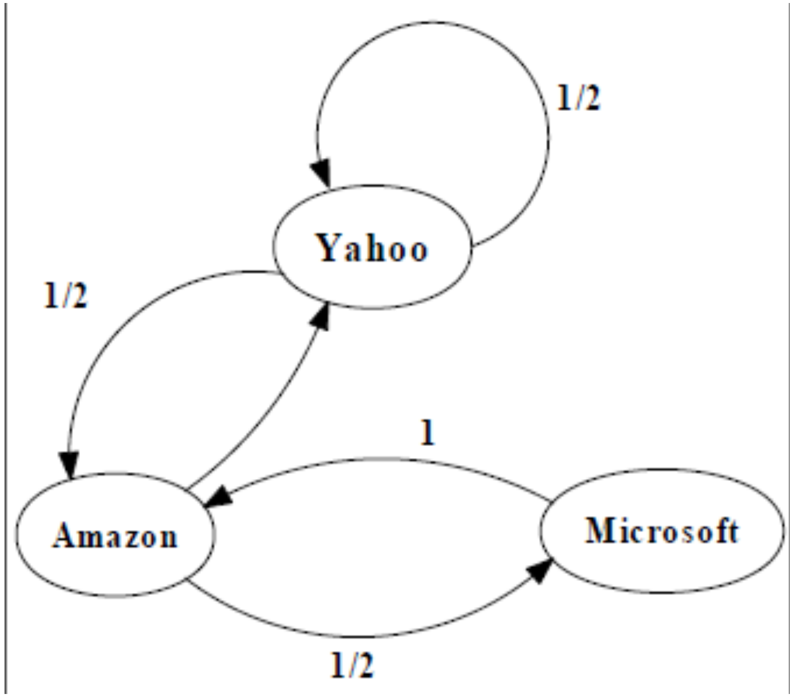

$$M = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1 \\ 0 & 1/2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 1/3 \\ 1/2 \\ 1/6 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1 \\ 0 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

PageRank Calculation: first iteration

# An example of Simplified PageRank

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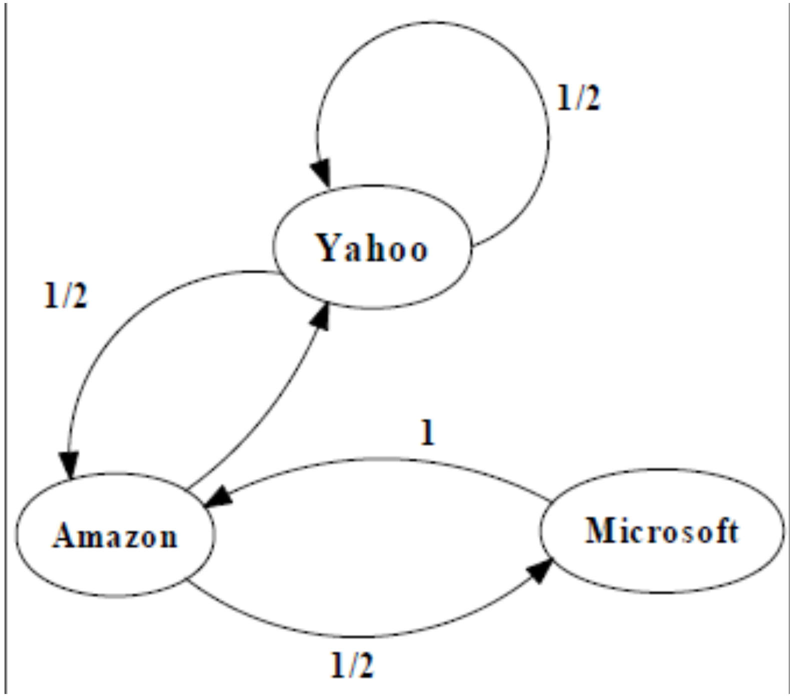
$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 5/12 \\ 1/3 \\ 1/4 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1 \\ 0 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/2 \\ 1/6 \end{bmatrix}$$

PageRank Calculation: second iteration

# An example of Simplified PageRank




$$M = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1 \\ 0 & 1/2 & 0 \end{bmatrix}$$

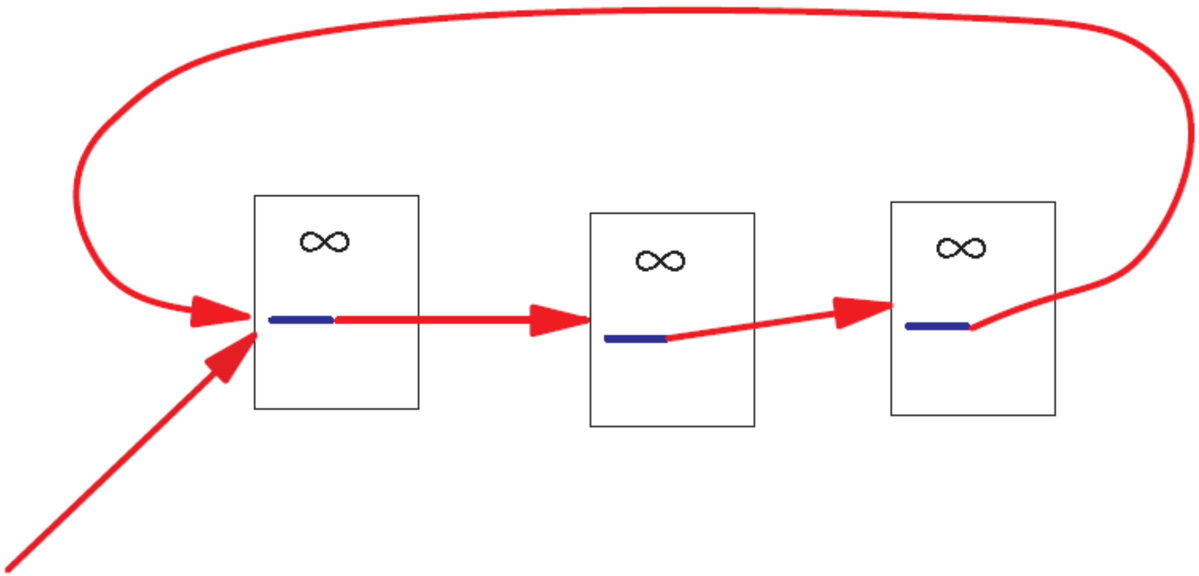
$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 3/8 \\ 11/24 \\ 1/6 \end{bmatrix} \begin{bmatrix} 5/12 \\ 17/48 \\ 11/48 \end{bmatrix} \ldots \begin{bmatrix} 2/5 \\ 2/5 \\ 1/5 \end{bmatrix}$$

Convergence after some iterations

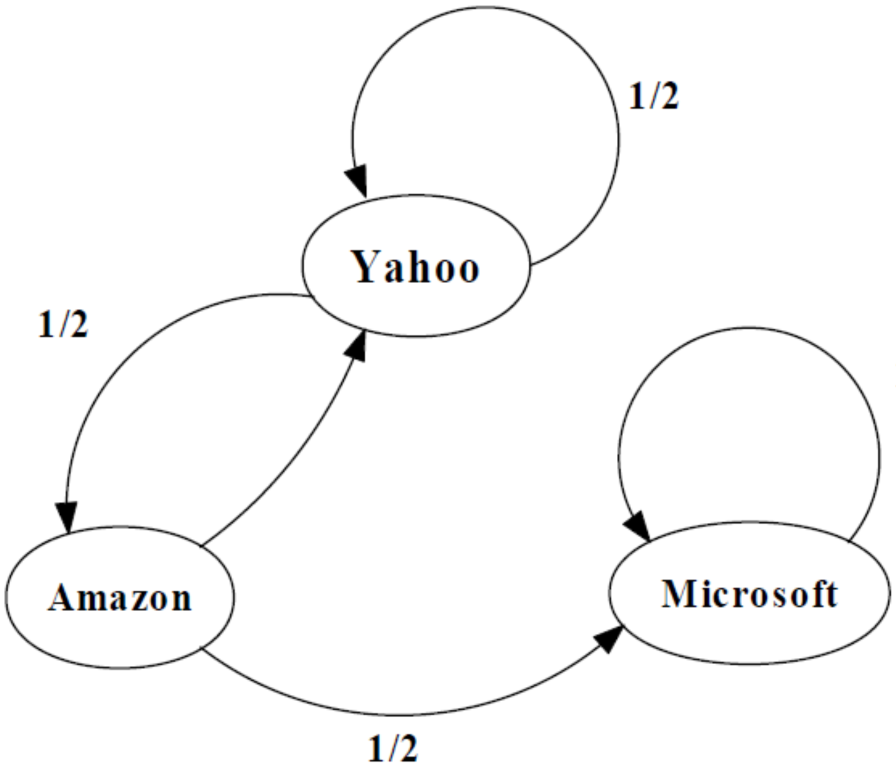
# A Problem with Simplified PageRank

A loop:




During each iteration, the loop accumulates rank but never distributes rank to other pages!

# An example of the Problem




$$M = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 1/3 \\ 1/6 \\ 1/2 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

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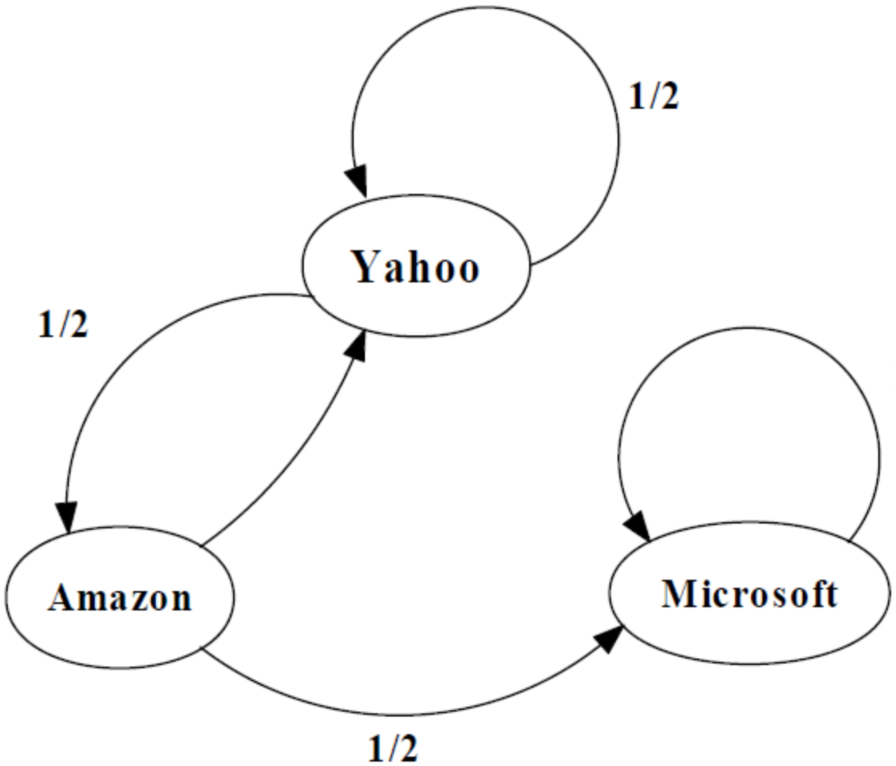
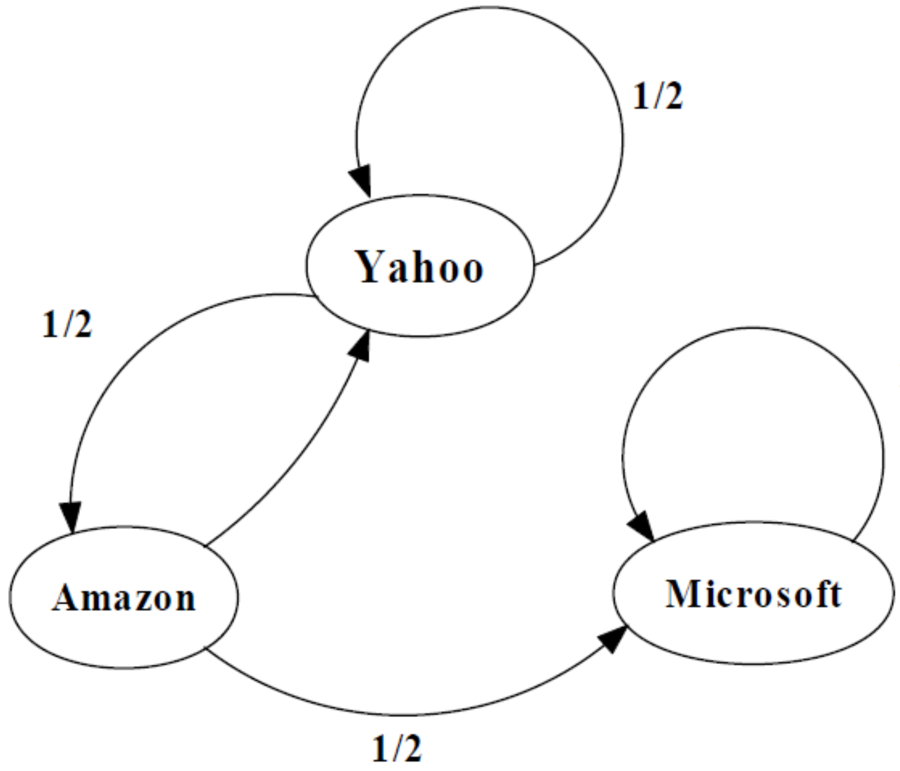



$$M = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 1/4 \\ 1/6 \\ 7/12 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/6 \\ 1/2 \end{bmatrix}$$

# An example of the Problem

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$$\begin{bmatrix} 5/24 \\ 1/8 \\ 2/3 \end{bmatrix} \begin{bmatrix} 1/6 \\ 5/48 \\ 35/48 \end{bmatrix} \quad .... \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

# Random Walks in Graphs

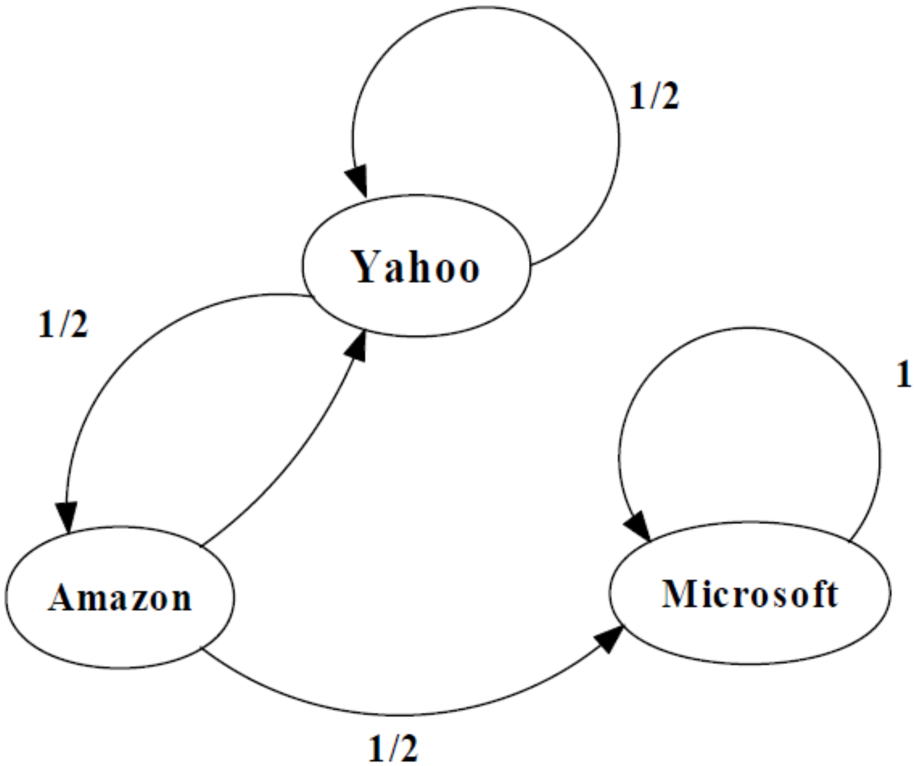
- The Random Surfer Model
  - The simplified model: the standing probability distribution of a random walk on the graph of the web. simply keeps clicking successive links at random
- The Modified Model
  - The modified model: the “random surfer” simply keeps clicking successive links at random, but periodically “gets bored” and jumps to a random page based on the distribution of E

# Modified Version of PageRank

$$R'(u) = c_1 \sum_{v \in B_u} \frac{R'(v)}{N_v} + c_2 E(u)$$

E(u): a distribution of ranks of web pages that “users” jump to when  they “gets bored” after successive links at random.

# An example of Modified PageRank




$$M = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$C_1 = 0.8 \qquad C_2 = 0.2$

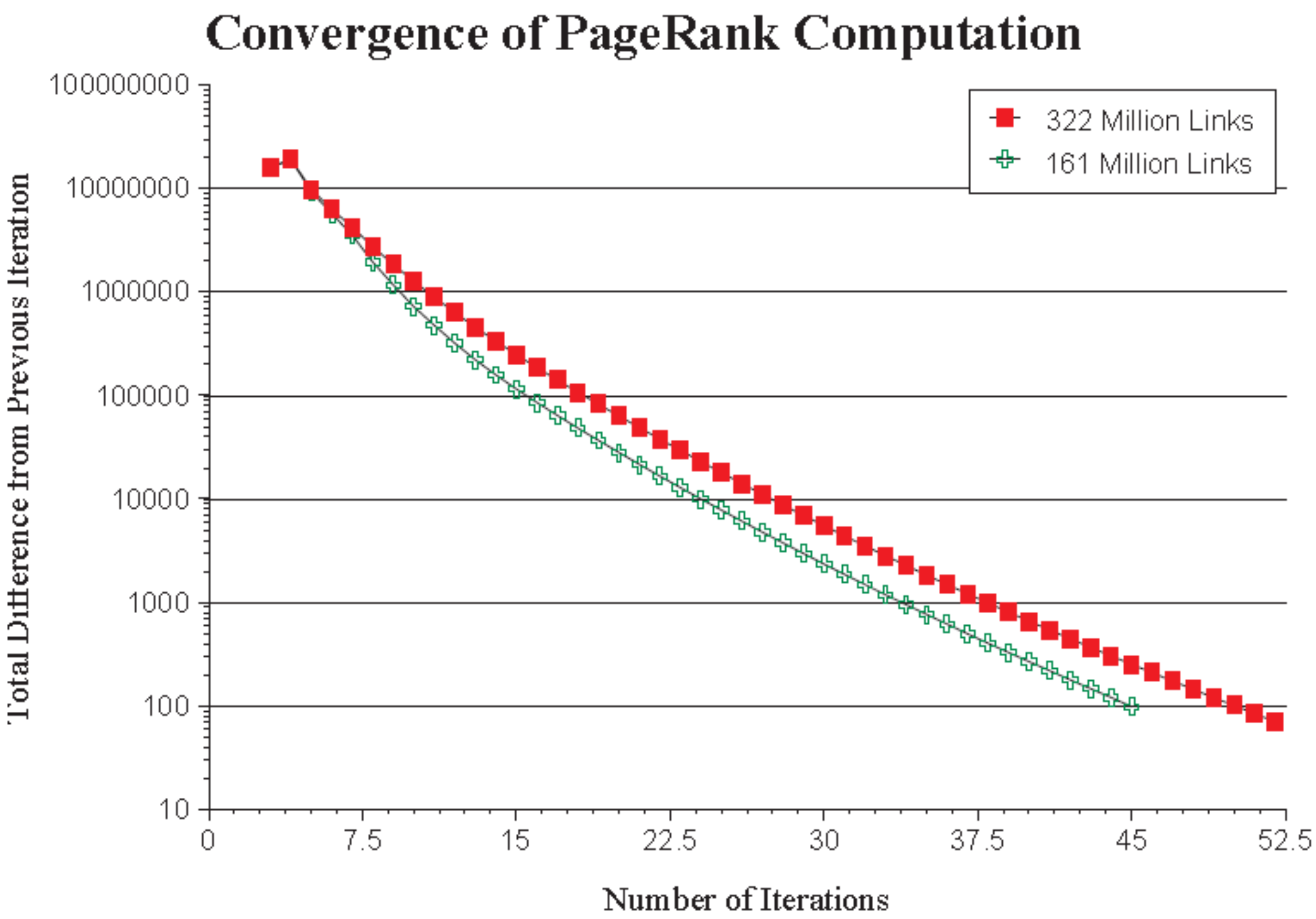
$$\begin{bmatrix} 0.333 \\ 0.333 \\ 0.333 \end{bmatrix} \begin{bmatrix} 0.333 \\ 0.200 \\ 0.467 \end{bmatrix} \begin{bmatrix} 0.280 \\ 0.200 \\ 0.520 \end{bmatrix} \begin{bmatrix} 0.259 \\ 0.179 \\ 0.563 \end{bmatrix} \dots \begin{bmatrix} 7/33 \\ 5/33 \\ 21/33 \end{bmatrix}$$

# Dangling Links

- Links that point to any page with no outgoing links
- Most are pages that have not been downloaded yet
- Affect the model since it is not clear where their weight should be distributed
- Do not affect the ranking of any other page directly
- Can be simply removed before pagerank calculation and added back afterwards

# Convergence Property

- PR (322 Million Links): 52 iterations
- PR (161 Million Links): 45 iterations
- Scaling factor is roughly linear in *logn*



# Convergence Property

- The Web is an expander-like graph
  - Theory of random walk: a random walk on a graph is said to be rapidly-mixing if it quickly converges to a limiting distribution on the set of nodes in the graph. A random walk is rapidly-mixing on a graph if and only if the graph is an expander graph.
  - Expander graph: every subset of nodes S has a neighborhood (set of vertices accessible via outedges emanating from nodes in S) that is larger than some factor $\alpha$ times of $|S|$. A graph has a good expansion factor if and only if the largest eigenvalue is sufficiently larger than the second-largest eigenvalue.

# PageRank vs. Web Traffic

- Important component of PageRank calculation is *E*
  - A vector over the web pages (used as source of rank)
  - Powerful parameter to adjust the page ranks
- The vector E corresponds to the distribution of web pages that a random surfer periodically jumps to from the search engine
- Some highly accessed web pages have low page rank possibly because
  - People do not want to link to these pages from their own web pages (the example in 1998 PageRank paper is pornographic sites…)
  - Some important backlinks are omitted
  - Use web usage data as a start vector for PageRank.

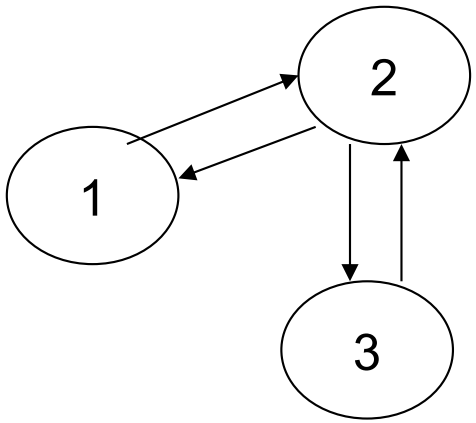
# Web Spamming by Gaming Pagerank

- Since 2000, Google Search has become the default gateway to the web
- Very high premium to appear on the first few pages of search results
  - E-commerce sites
  - Advertising-driven sites

- Spamming: Manipulating the text of web pages in order to appear relevant to queries
- Approximately 10-15% of web pages are spam
- Spammers' goal: Maximize the page rank of a target page *t*
- Spammers' technique: Manipulating the text of web pages so as to appear relevant to queries and get as many links from accessible pages as possible to target page *t*

# Exercise on PageRank

- Consider a Web graph with three nodes 1, 2, and 3. The links are as follows: 1->2, 3->2, 2->1, 2->3. Write down the transition probability matrices P for the surfer's walk with teleporting, with the value of teleport probability α=0.5.




A=

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 0 |

Each 1 divided by the number of ones in this row

(1- α)*

| | | |
|---|---|---|
| 0 | 1 | 0 |
| ½ | 0 | ½ |
| 0 | 1 | 0 |

+

α*

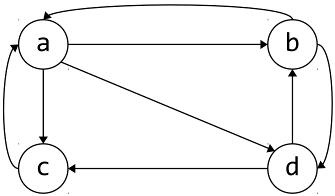
| | | |
|---|---|---|
| 1/3 | 1/3 | 1/3 |
| 1/3 | 1/3 | 1/3 |
| 1/3 | 1/3 | 1/3 |

=

| | | |
|---|---|---|
| 1/6 | 2/3 | 1/6 |
| 5/12 | 1/6 | 5/12 |
| 1/6 | 2/3 | 1/6 |

# PageRank example

$$r_i = \sum_{j:j \to i \in \mathcal{E}} \frac{r_j}{d_j}$$




**Equations:**

- $r_a = \frac{r_b}{2} + r_c$.
- $r_b = \frac{r_a}{3} + \frac{r_d}{2}$.
- $r_c = \frac{r_a}{3} + \frac{r_d}{2}$.
- $r_d = \frac{r_a}{3} + \frac{r_b}{2}$.

- 4 equations, 4 unknowns, no constants.

  No **unique solution:** all solutions are equivalent modulo a scale factor.

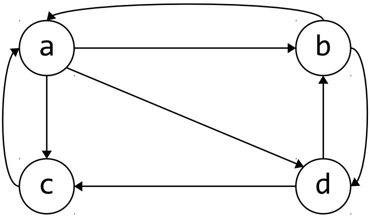
- Additional **constraint** for uniqueness:

$$\sum_i r_i = 1.$$

- **Solution** by **Gaussian elimination:**
  - $r_a = \frac{1}{3}$.
  - $r_b = r_c = r_d = \frac{2}{9}$.

# Random walkers

- For **large graphs,** solving linear systems of equations is intractable.
- **Random surfers:** Where do you end if you follow links at random?

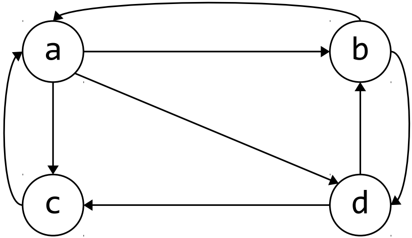



Start at node a: after one step, end up in b, c, or d with probability $\frac{1}{3}$.

- **Transition matrix:** $M_{ij} = \frac{1}{d_j}$ if $j \to i \in \mathcal{E}$ and 0 otherwise.

  The transition matrix is **column-stochastic:** columns sum to 1.

# Random walkers: Transition matrix example




– **Transition matrix:**

$$\begin{bmatrix} 0 & 1/2 & 1 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix}$$

# PageRank with random walkers

- Start random surfers **at all pages** with **equal probability** $\frac{1}{n}$

$$\vec{v}_0 = [1/n, 1/n, \ldots, 1/n] \,.$$

- **After one step,** the distribution will be

$$\vec{v}_1 = M\vec{v}_0 \,.$$

- **After** $k$ **steps:**

$$\vec{v}_k = M^k\vec{v}_0 \,.$$

- **Markov process:** The distribution approaches a limiting distribution $\vec{v}$ such that $\vec{v} = M\vec{v}$ if
  - The graph is **strongly connected:** can get from a node to any other node.
  - No **dead ends:** nodes that have no out-links.

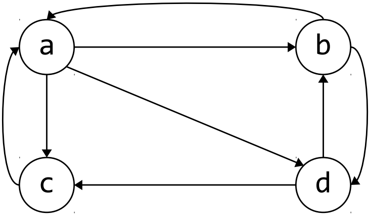
# PageRank with random walkers

$\vec{v} = M\vec{v}$.

- Surfers are **stationary.**
- The more important a page, and the more likely it is to have a surfer.
- $\vec{v}$ is … **the principal eigenvector** of M. (M stochastic has largest eigenval 1.)
- **Power iteration:** compute $\vec{v}$ by iterative **matrix-vector multiplications.**
  - Stop when $||\vec{v}_t - \vec{v}_{t-1}|| \leq \epsilon$.
  - How eigenvectors are computed in large dimensions (eg. Lanczos method.)
  - Amenable to **MapReduce** parallelization.
- Equivalent to previous PageRank formulation:

$$v_i = \sum_{j: j \to i \in \mathcal{E}} \frac{v_j}{d_j}$$

# Example




**Transition matrix:**

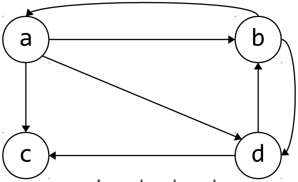
$$\begin{bmatrix} 0 & 1/2 & 1 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix}$$

- **Initialization:** $\vec{v}_0 = [1/4,\ 1/4,\ 1/4,\ 1/4]$.
- **After one step:** $\vec{v}_1 = [9/24,\ 5/24,\ 5/24,\ 5/24]$.
- **After two steps:** $\vec{v}_2 = [15/48,\ 11/48,\ 11/48,\ 11/48]$.

  ...
- **Converges to:** $\vec{v} = [1/3,\ 2/9,\ 2/9,\ 2/9]$.

# Dead ends

- **Dead ends:** nodes that have no out-links.



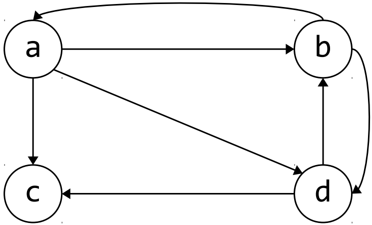

c is a dead end

**Transition matrix:**

$$\begin{bmatrix} 0 & 1/2 & 0 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix}$$

- The **transition matrix** does not have full rank.
- It cannot be **inverted**, i.e. our linear system of equations has **no solution.**
- The **power method** converges to $\vec{v} = \vec{0}$.
- **Solutions:**
  - Recursively **remove** dead ends and their incoming links.
  - When at a dead end, **teleport** (with equal probability) to another node.

# Example




**Transition matrix:**

$$\begin{bmatrix} 0 & 1/2 & \mathbf{0} & 0 \\ 1/3 & 0 & \mathbf{0} & 1/2 \\ 1/3 & 0 & \mathbf{0} & 1/2 \\ 1/3 & 1/2 & \mathbf{0} & 0 \end{bmatrix}$$

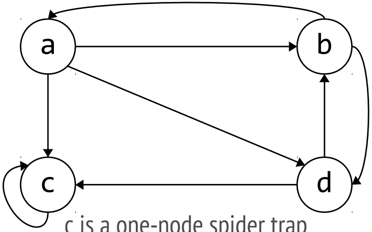
- New **transition matrix:**

$$\begin{bmatrix} 0 & 1/2 & \mathbf{1/4} & 0 \\ 1/3 & 0 & \mathbf{1/4} & 1/2 \\ 1/3 & 0 & \mathbf{1/4} & 1/2 \\ 1/3 & 1/2 & \mathbf{1/4} & 0 \end{bmatrix}$$

- Eventually, $\vec{v} = [1/5,\ 4/15,\ 4/15,\ 4/15]$ .

# Spider traps

- **Spider trap:** set of nodes with no dead ends but no links out.
- **Problem:**
  - All random surfers end up in the spider trap.




c is a one-node spider trap

- **Transition matrix:**

$$\begin{bmatrix} 0 & 1/2 & \mathbf{0} & 0 \\ 1/3 & 0 & \mathbf{0} & 1/2 \\ 1/3 & 0 & \mathbf{1} & 1/2 \\ 1/3 & 1/2 & \mathbf{0} & 0 \end{bmatrix}$$

- $\vec{v}$ **converges to** $\vec{v} = [0,\ 0,\ 1,\ 0]$ .

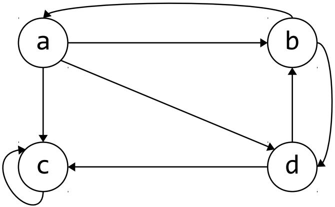
# Taxation

- How to get out of **spider traps?**
  - A random surfer can **leave the graph** at any moment.
  - **New surfers** can be started at any page at any moment.
- **Taxation:** Allow each random surfer a probability $1-\beta$ of **teleporting** to a random page

$$\vec{v} = \beta M\vec{v} + \frac{(1-\beta)}{n}\vec{1}.$$

Typically, $\beta \in [0.8 - 0.9]$.

# Example




**Transition matrix:**

$$\begin{bmatrix} 0 & 1/2 & \mathbf{0} & 0 \\ 1/3 & 0 & \mathbf{0} & 1/2 \\ 1/3 & 0 & \mathbf{1} & 1/2 \\ 1/3 & 1/2 & \mathbf{0} & 0 \end{bmatrix}$$

$$\vec{v} = \beta M \vec{v} + \frac{(1-\beta)}{n}\vec{1}$$

- $\beta = 0.8 = 4/5$

$$\vec{v} = \begin{bmatrix} 0 & 2/5 & 0 & 0 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 0 & 4/5 & 2/5 \\ 4/15 & 2/5 & 0 & 0 \end{bmatrix} \vec{v} + \begin{bmatrix} 1/20 \\ 1/20 \\ 1/20 \\ 1/20 \end{bmatrix}, \quad \vec{v}_0 = \left[\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right].$$

- **Solution:** $\vec{v} = \left[\frac{15}{148}, \frac{19}{148}, \frac{95}{148}, \frac{19}{148}\right].$

# Summary

- **Large-scale data** poses new technical problems for:
  - **storage** $\Rightarrow$ distributed file systems.
  - **computations** $\Rightarrow$ MapReduce programming model.
    - Split the data in chunks.
    - Map workers all execute the same operation on a chunk and return a key-val pair.
    - Reduce workers process all key-val pairs with the same key at once.
- **Algorithmic costs** of MapReduce:
  - **Communication costs** vs. **computation costs.**
  - **Reducer size** and **replication rate.**
- Extensions of MapReduce: **Spark** and **TensorFlow.**
- MapReduce for **machine learning.**
- **Link analysis** with **PageRank.**

# PageRank Summary

- Robust and scalable algorithm with proven convergence guarantees
- Distributed algorithm in Google’s data center-drive breakthroughs in compute (Google MapReduce) and storage (Google File System)
- Amenable to distributed computation via parallel computation (MapReduce in next Lecture)
- MapReduce Code walkthrough:
- http://web.archive.org/web/20221216071408/https://michaelnielsen.org/blog/using-mapreduce-to-compute-pagerank