



NANYANG  
TECHNOLOGICAL  
UNIVERSITY  
SINGAPORE

# CE/CZ4052 Cloud Computing

## Pagerank Algorithm

Dr. Tan, Chee Wei

*Email:* [cheewei.tan@ntu.edu.sg](mailto:cheewei.tan@ntu.edu.sg)

*Office:* N4-02c-104





# How does Google rank the Web?

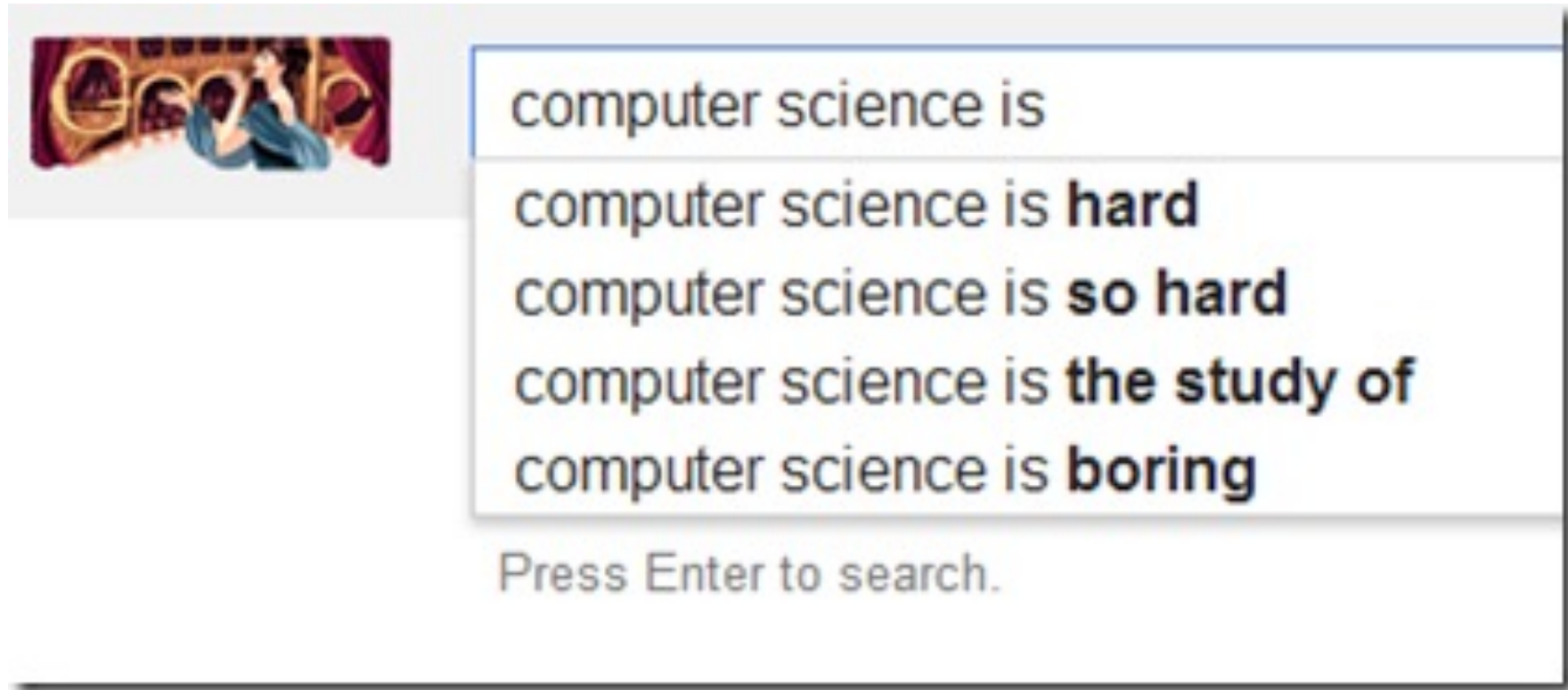


Acknowledgement:

[https://www.cazencott.info/dotclear/public/lectures/lsm19/2019-03-28\\_lsm19\\_systems.pdf](https://www.cazencott.info/dotclear/public/lectures/lsm19/2019-03-28_lsm19_systems.pdf)

# Search Engine Technologies

- ◆ **Computer Science is ...**



# Search Engine Technologies

## Definition of **google** in English

**google**

Pronunciation: /'gu:gl/

Translate **google** | [into French](#) | [into Italian](#) | [into Spanish](#)

*verb*

[*with object*]

search for information about (someone or something) on the Internet  
using the search engine Google:

*on Sunday she googled an ex-boyfriend*

[*no object*]:

*I **googled for** a cheap hotel/flight deal*

### **Derivatives**

**googleable**

(also **googlable**) *adjective*

# The History of PageRank

- PageRank was developed by Larry Page (hence the name *Page*-Rank) and Sergey Brin at Stanford in 1999.
- Shortly after, Page and Brin founded Google.
- Challenges: Web contains many sources of information – including spam.
  - What is the *best* answer to a web query “google” in 1998?
  - A good web search algorithm enables *trust*
- Use links as *votes* to rank pages
- Are all links equally important?
  - Links from important pages count more.
  - This question is *recursive*.



# The PageRank in Search Engines (1997)



# Searching with PageRank (1997)

**Multi Search** university  [Next! \[national parks\]](#)

10 results clustering on Search

Query: **university**  
11 Results Returned  
Showing Results From 0 to 10

**Stanford University Homepage**  
http://www.stanford.edu/  
74.79% 4k - 3/5/1993 - 01/03/97

**Stanford University Portfolio Collection**  
http://www.stanford.edu/home/administration/portfolio.html  
65.78% 3k - 3/5/1993 - 01/03/97

**University of Illinois at Urbana-Champaign**  
http://www.uiuc.edu/  
73.26% 13k - 12/30/96 - 01/03/97

**Indiana University**  
http://www.indiana.edu/  
68.38% 1k - 09/28/96 - 01/05/97

**University of California, Irvine**  
http://www.uci.edu/  
68.07% 3k - 12/30/96 - 01/03/97

**University of Minnesota**  
http://www.umn.edu/  
67.05% 0k - 12/16/96 - 01/03/97

**Iowa State University Homepage**  
http://www.iastate.edu/  
66.66% 3k - 12/18/96 - 01/03/97

**The University of Michigan**  
http://www.umich.edu/  
66.35% 1k - 3/5/1993 - 01/03/97

**Mississippi State University**  
http://www.msstate.edu/  
66.35% 3k - 3/5/1993 - 01/03/97

**Northwestern University: NUInfo**  
http://www.nwu.edu/  
66.15% 3k - 12/14/96 - 01/05/97

next 10

**Optical Physics at the University of Oregon**  
Oregon Center for Optics in Science and Technology. Department of Physics, University of Oregon, Eugene OR 97403. Research Groups: Carmichael Group....  
<http://optics.uoregon.edu/> - size 1K - 16 Dec 96

**Carnegie Mellon University - Campus Networking**  
Departments. Data Communications. Data Communications is responsible for installing and maintaining all on campus networking equipment and all of...  
<http://www.net.cmu.edu/> - size 4K - 19 Aug 95

**Wesleyan University Computer Science Group Home Page**  
Computer Science Group. Wesleyan University. Welcome to the home page of the Computer Science Group at Wesleyan University. We are administratively within.  
<http://www.cs.wesleyan.edu/> - size 3K - 15 Apr 96

**Keio University Shonan Fujisawa Campus (SFC)**  
B\$3\$N%ZIEFnF#Bt%-%c%\$%Q%9 (B(SFC) \$B\$N (BWWW \$B% \$BCmOU=q\$- (B \$B\$IFi\$G\$/\$@5\$ \$ (B. Nihongo | English. SFC \$B>pJs (B. [ \$B%a%G%#% "%;%s%? | \*...  
<http://www.sfc.keio.ac.jp/> - size 3K - 5 Feb 97

**School of Chemistry, University of Sydney**  
The School of Chemistry. School of Chemistry, University of Sydney, NSW 2006 Australia International Phone: +61-2-9351-4504 Fax: +61-2-9351-3329 Australia.  
<http://www.chem.su.oz.au/> - size 4K - 25 Feb 97

**Mankato State University**  
The Campus Athletics, Campus Tour, Bookstore, Maps, Current Events... Admission & Registration Admissions, Financial Aid, Registrar's, Graduate...  
<http://www.mankato.mnsc.edu/> - size 3K - 27 Nov 96

**St. Ambrose University**  
Main Index: Academic Departments. Administrative Services. Campus News. Computing Services. Galvin Fine Arts Center. Internet Connections. Library...  
<http://www.sau.edu/> - size 3K - 4 Feb 97

**University of Washington ECSEL Projects**

# Searching with PageRank (1997)

| Web Page  | PageRank (average is 1.0) |
|---|---------------------------|
| Download Netscape Software                            | 11589.00                  |
| <a href="http://www.w3.org/">http://www.w3.org/</a>   | 10717.70                  |
| Welcome to Netscape                                   | 8673.51                   |
| Point: It's What You're Searching For                 | 7930.92                   |
| Web-Counter Home Page                                 | 7254.97                   |
| The Blue Ribbon Campaign for Online Free Speech       | 7010.39                   |
| CERN Welcome  | 6562.49                   |
| Yahoo!  | 6561.80                   |
| Welcome to Netscape                                   | 6203.47                   |
| Wusage 4.1: A Usage Statistics System For Web Servers | 5963.27                   |
| The World Wide Web Consortium (W3C)                   | 5672.21                   |
| Lycos, Inc. Home Page                                 | 4683.31                   |
| Starting Point  | 4501.98                   |
| Welcome to Magellan!                                  | 3866.82                   |
| Oracle Corporation                                    | 3587.63                   |

Top 15 Page Ranks: July 1996



# The PageRank in Search Engines (2017)

← → ↻ Secure | <https://www.alexasiteinfo>

**@Alexa** An amazon.com company Features ▾ Resources ▾ Pricing Log in

## Find Website Traffic, Statistics, and Analytics

Enter a website. Example: site.com Find

The dashboard displays several key metrics for a website:

- Popularity:** Global Rank 2,346, US Rank 1,354
- Engagement:** Daily Time on Site 1:30 mins, Bounce Rate 15.7%
- Unique Visitors:** Monthly Unique Visitors 2,354,567, Monthly 12,85
- Search Traffic:** Search Visits 25.7%
- Audience Geography:** Top Countries United States

**Increase Website Traffic** Using Competitive Analytics

Alexa is more than just the traffic rank you know and love from its early days. Checking website traffic and rank is the basis for uncovering actionable ideas to grow your business.

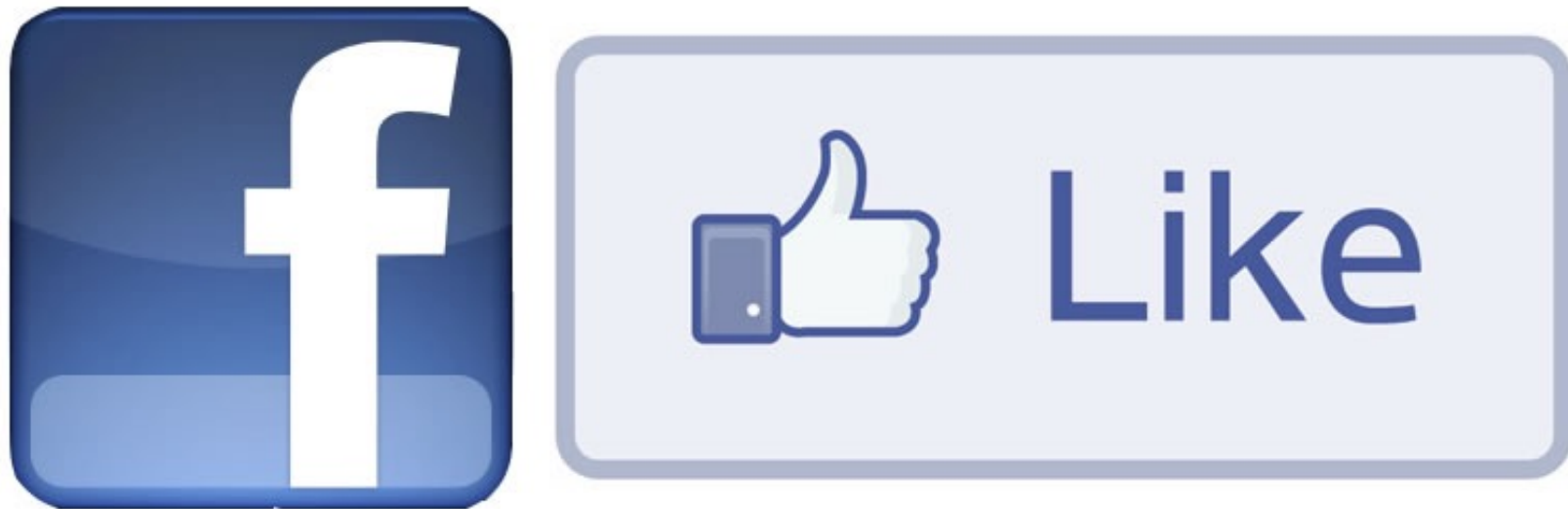
<https://www.alexasiteinfo>

<https://moonsy.com/alexarank/>

**Guess, who has top ranking, i.e., number 1?**

# Link Analysis

- Consider links as votes of confidence in a page
- A hyperlink is the open Web's version of ...



(... even if the page is linked in a negative way.)

# Link Analysis

So if we just count the number of inlinks a web-page receives we know its importance, right?



# Link Spamming



semanticweb.com™

The Voice of Semantic Technology Business  
Big Data, Linked Data, Smart Data

Home

Events

Media

Industry Verticals

Answers

Questions

Tags

Users

Badges

[deleted] Kala Jadu Specialist +9196



-1



black magic specialist baba ji call now +919610897260

<http://www.blackmagicspecialist.net.in>

java

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[Diakof](#) [Diarex](#) [Didronel](#) [Differin](#) [Dilantin](#) [Diovan](#) [Dostinex](#) [Elavil](#) [Elimite](#) [Emsam](#) [Endep](#) [Eurax](#)  
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[Male Enhancement Oil](#) [Male Enhancement Patch](#) [Male Enhancement Pills](#) [Male Sexual Tonic](#)  
[InnoPran XL](#) [Hoodia Weight Loss Gum](#) [Hoodia Weight Loss Patch](#) [Human Growth Hormone](#)  
[Agent Glucotrol XL](#) [Green Tea Grifulvin V](#) [Gyne-Lotrimin](#) [Hair Loss Cream](#) [Herbal Maxx Herbal](#)  
[Phentermine](#) [Flagyl ER](#) [Female Sexual Tonic](#) [Female Viagra](#) [Epivir-HBV](#) [Diet Maxx](#) [Deluxe](#)  
[Handheld Plasma Whitening Tool](#) [Deluxe Whitening System With Plasma Lamp](#) [Coral Calcium](#)  
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PageRank

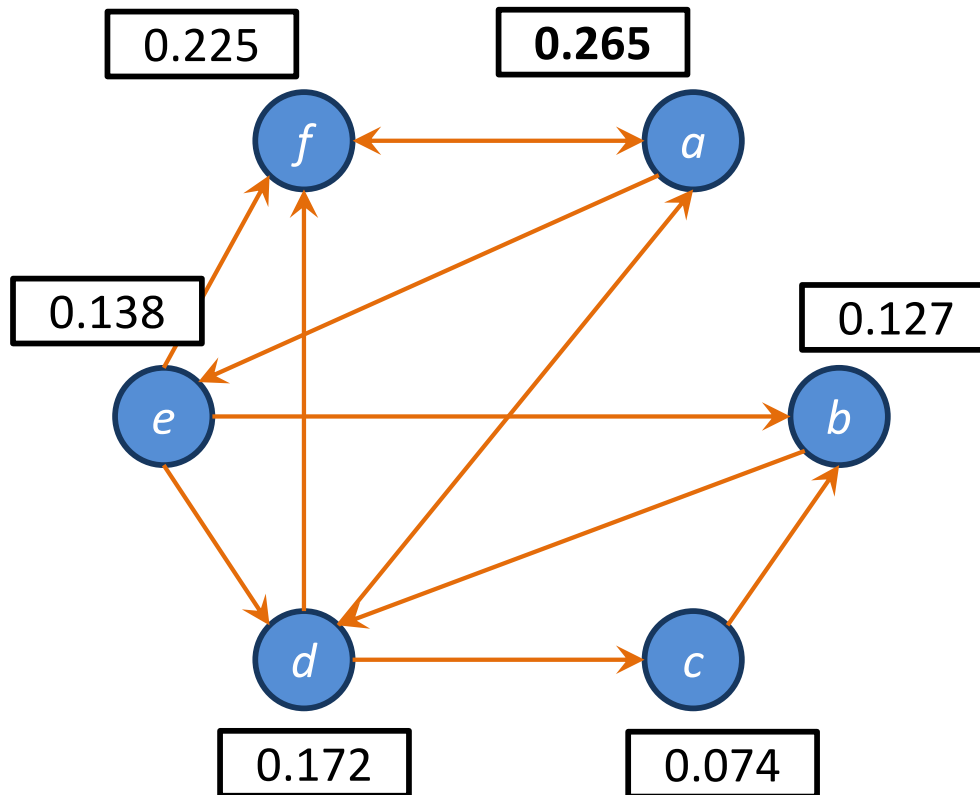


# PageRank

- Not just a count of inlinks
  - A link from a more important page is more important
  - A link from a page with fewer links is more important
  - ∴ A page with lots of inlinks from important pages (which have few outlinks) is more important

# PageRank Model

- The Web: a directed graph



$$G = [V, E]$$

Vertices  
(pages)

Edges  
(links)

Which is the most  
“important” vertex?

$$V = \{a, b, c, d, e, f\}$$

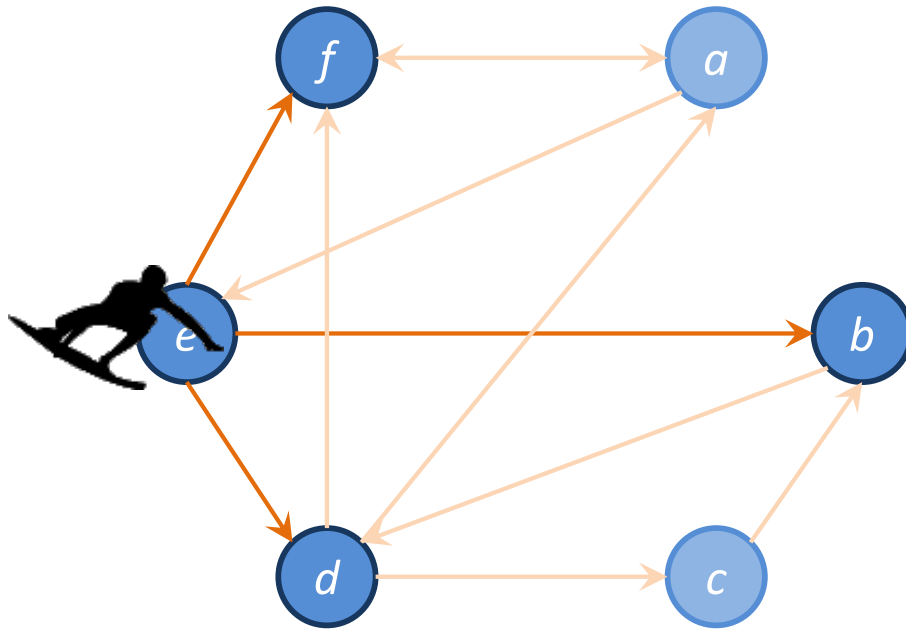
$$E = \{(a, e), (a, f), (b, d), (c, b), (d, a), (d, c), (d, f), (e, b), (e, d), (e, f), (f, a)\}$$

# PageRank: Random Surfer Model



= someone surfing the web,  
clicking links randomly

- What is the probability of being at page  $x$  after  $n$  hops?

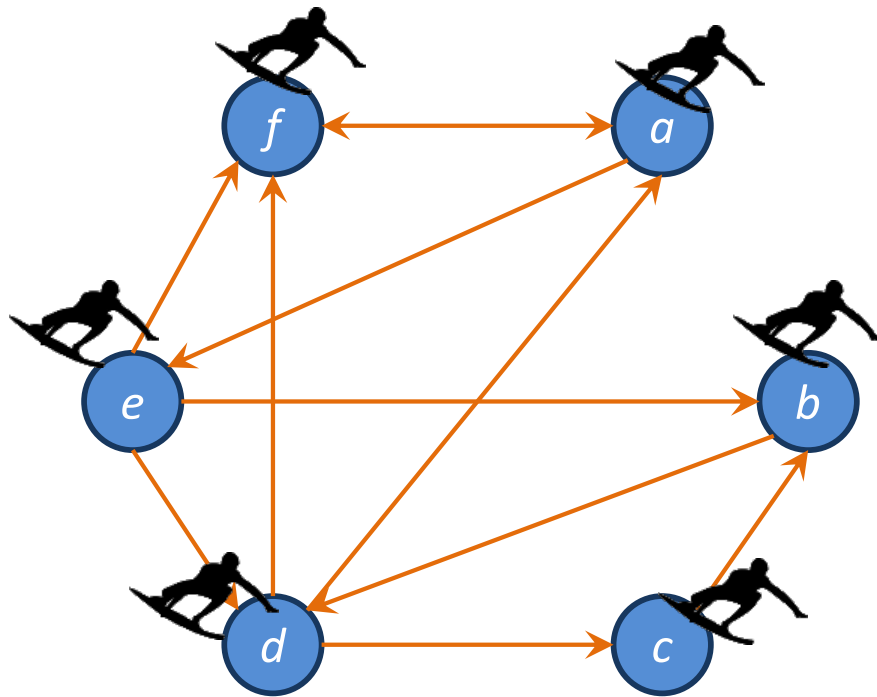




# PageRank: Random Surfer Model



= someone surfing the web,  
clicking links randomly

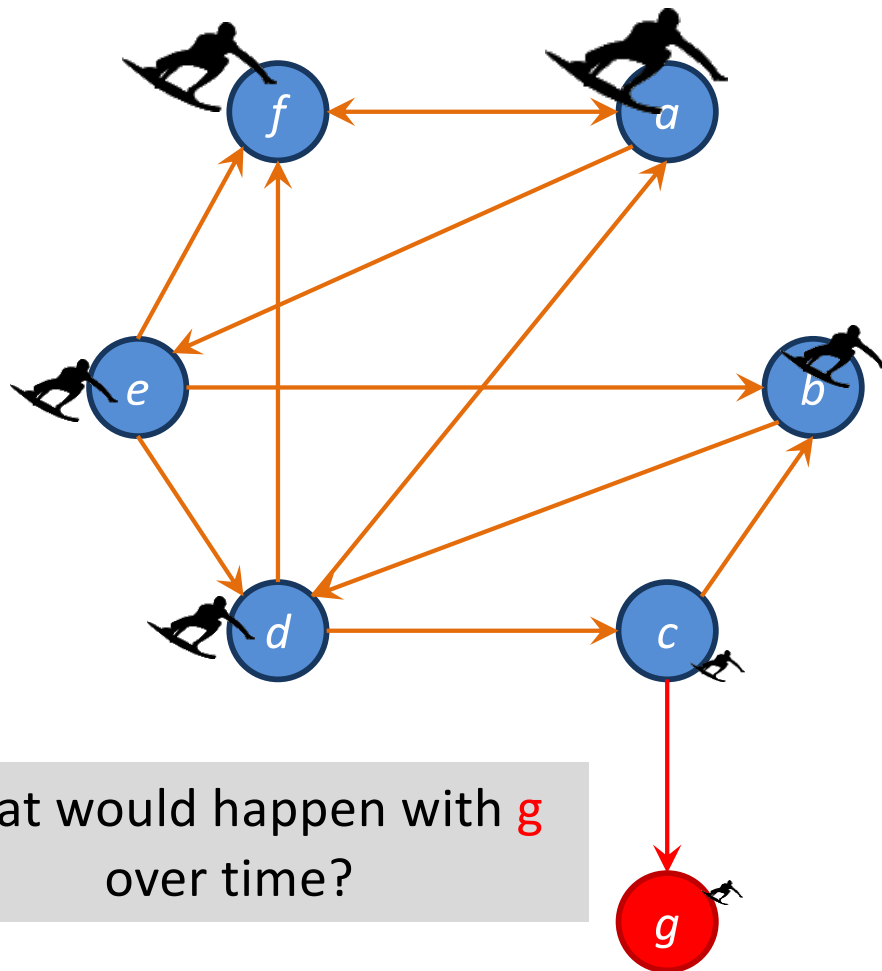


- What is the probability of being at page  $x$  after  $n$  hops?
- *Initial state*: surfer equally likely to start at any node

# PageRank: Random Surfer Model



= someone surfing the web,  
clicking links randomly



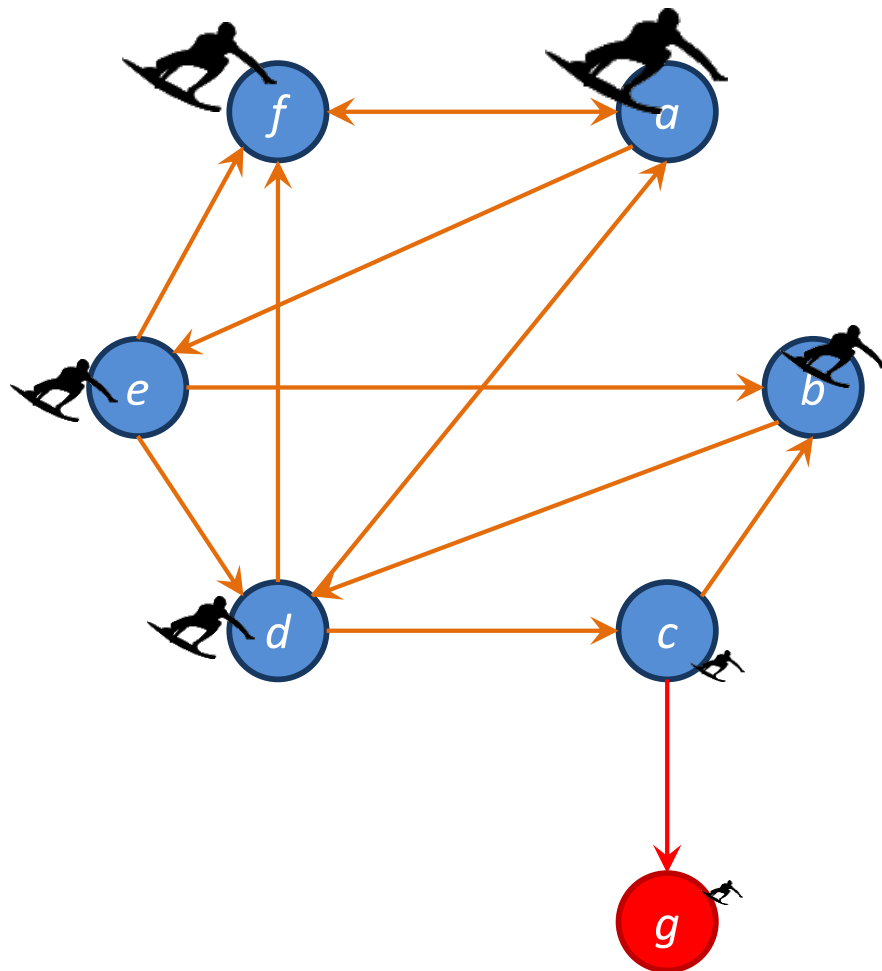
What would happen with **g**  
over time?

- What is the probability of being at page  $x$  after  $n$  hops?
- *Initial state*: surfer equally likely to start at any node
- PageRank applied iteratively for each hop: score indicates probability of being at that page after that many hops

# PageRank: Random Surfer Model



= someone surfing the web,  
clicking links randomly

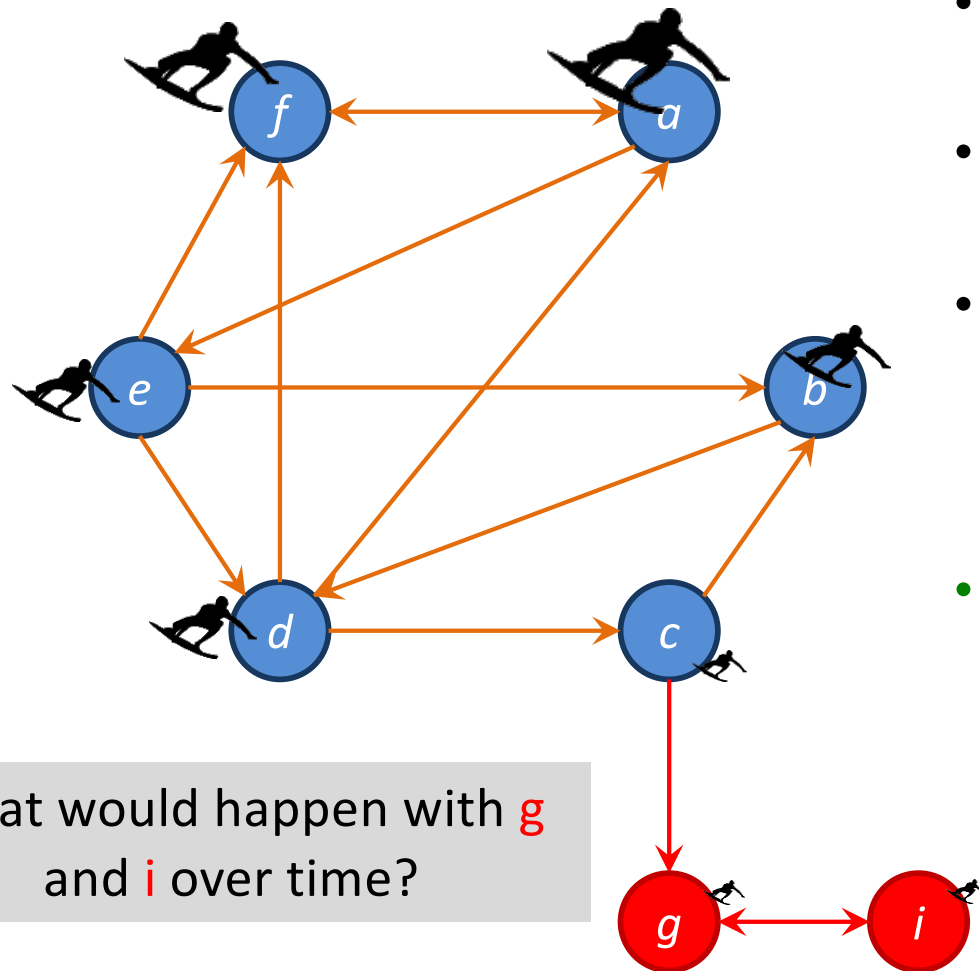


- What is the probability of being at page  $x$  after  $n$  hops?
- *Initial state*: surfer equally likely to start at any node
- PageRank applied iteratively for each hop: score indicates probability of being at that page after than many hops
- If the surfer reaches a page without links, the surfer randomly jumps to another page

# PageRank: Random Surfer Model



= someone surfing the web,  
clicking links randomly



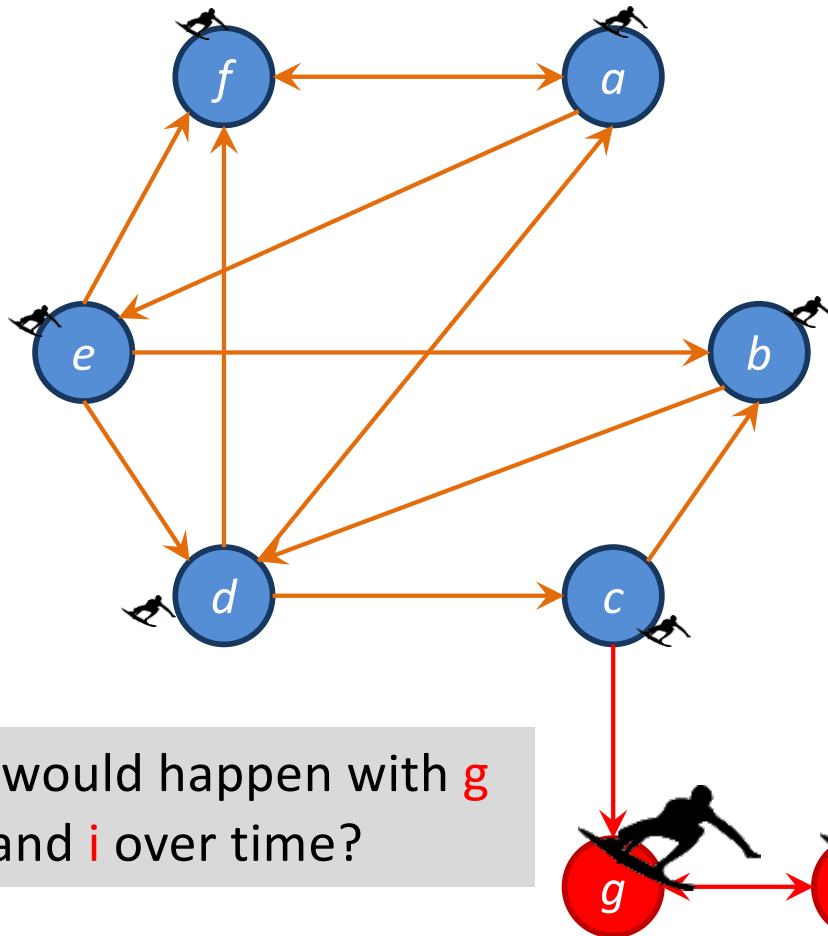
- What is the probability of being at page  $x$  after  $n$  hops?
- *Initial state*: surfer equally likely to start at any node
- PageRank applied iteratively for each hop: score indicates probability of being at that page after than many hops
- If the surfer reaches a page without links, the surfer randomly jumps to another page



# PageRank: Random Surfer Model



= someone surfing the web,  
clicking links randomly



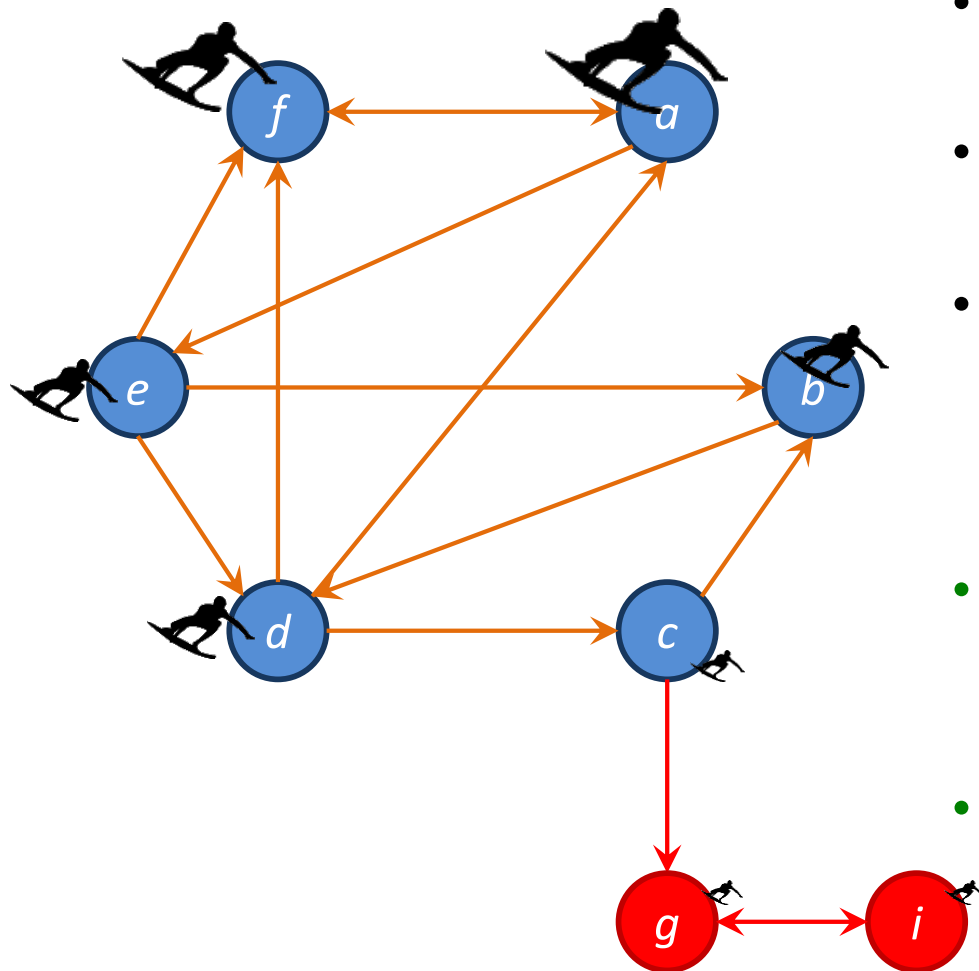
What would happen with **g**  
and **i** over time?

- What is the probability of being at page  $x$  after  $n$  hops?
- *Initial state*: surfer equally likely to start at any node
- PageRank applied iteratively for each hop: score indicates probability of being at that page after than many hops
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# PageRank: Random Surfer Model



= someone surfing the web,  
clicking links randomly



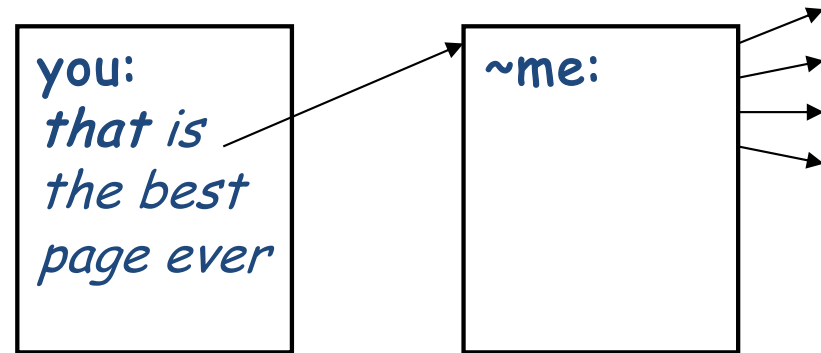
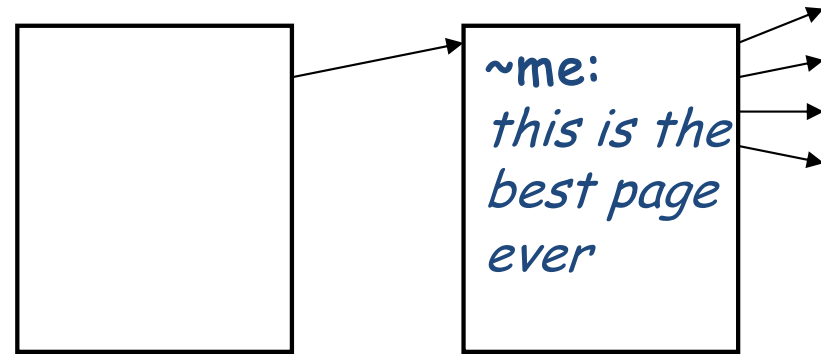
- What is the probability of being at page  $x$  after  $n$  hops?
- *Initial state*: surfer equally likely to start at any node
- PageRank applied iteratively for each hop: score indicates probability of being at that page after than many hops
- If the surfer reaches a page without links, the surfer randomly jumps to another page
- The surfer will jump to a random page at any time with a probability  $1 - d$  ... *this avoids traps and ensures convergence!*

# Google search: anchor text

- ❖ Pagerank
- ❖ Anchor text

## Google uses:

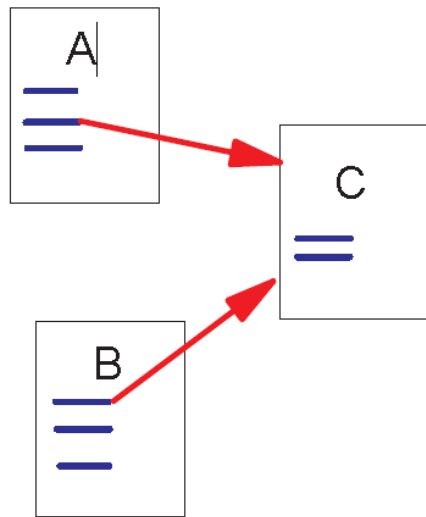
- ❖ In anchor text?
- ❖ In URL?
- ❖ Title
- ❖ Meta tags
- ❖ <h> level
- ❖ Rel font size
- ❖ Capitalization
- ❖ Word pos in doc
- ❖ Secret ingredients



... and weighs them according to a secret recipe

# Link Structure of the Web

- 150 million web pages → 1.7 billion links



Backlinks and Forward links:

- A and B are C's backlinks
- C is A and B's forward link

Intuitively, a webpage is important if it has a lot of backlinks.

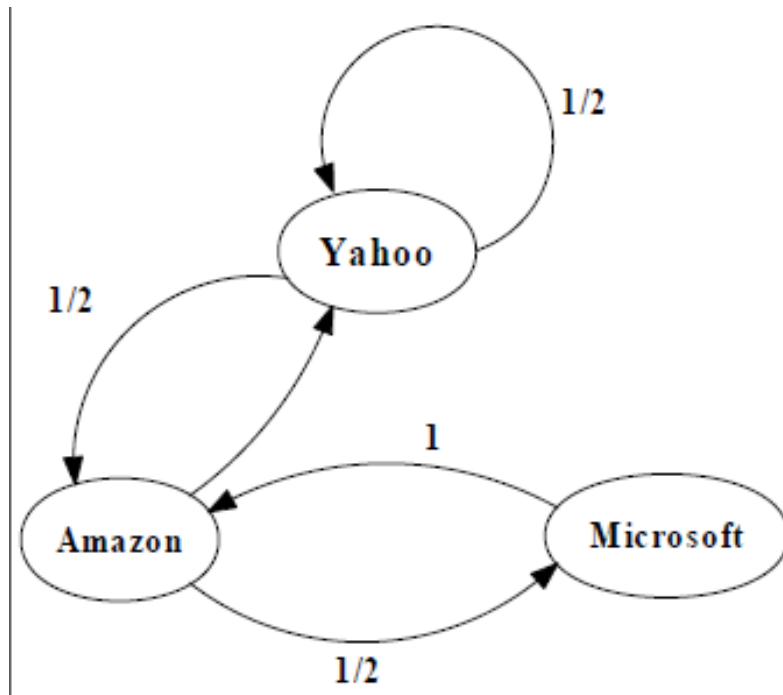


# A Simple Version of PageRank

$$R(u) = c \sum_{v \in B_u} \frac{R(v)}{N_v}$$

- $u$ : a web page
- $B_u$ : the set of  $u$ 's backlinks
- $N_v$ : the number of forward links of page  $v$
- $c$ : the normalization factor to make  $R(1) + \dots + R(T) = 1$  where there are  $T$  pages in total

# An example of Simplified PageRank



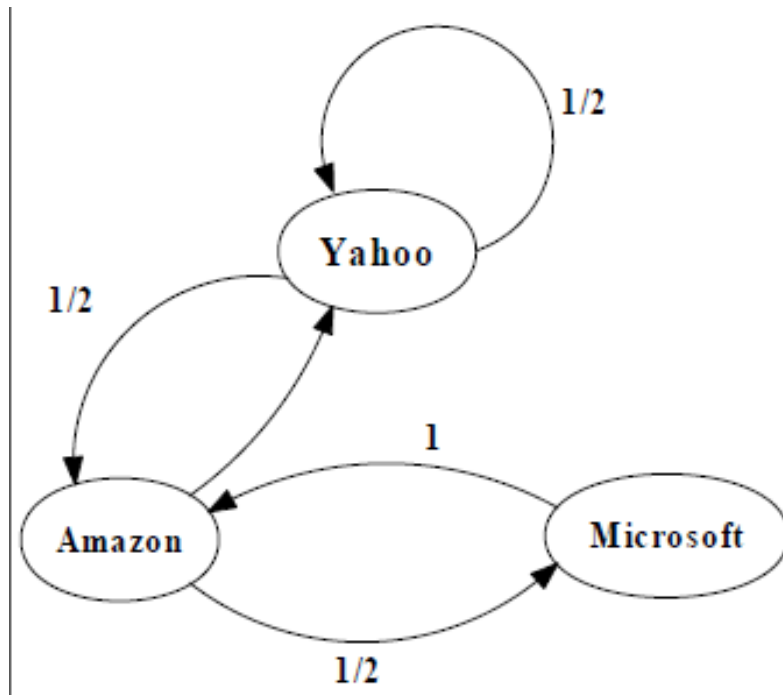
$$M = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1 \\ 0 & 1/2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 1/3 \\ 1/2 \\ 1/6 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1 \\ 0 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

PageRank Calculation: first iteration

# An example of Simplified PageRank



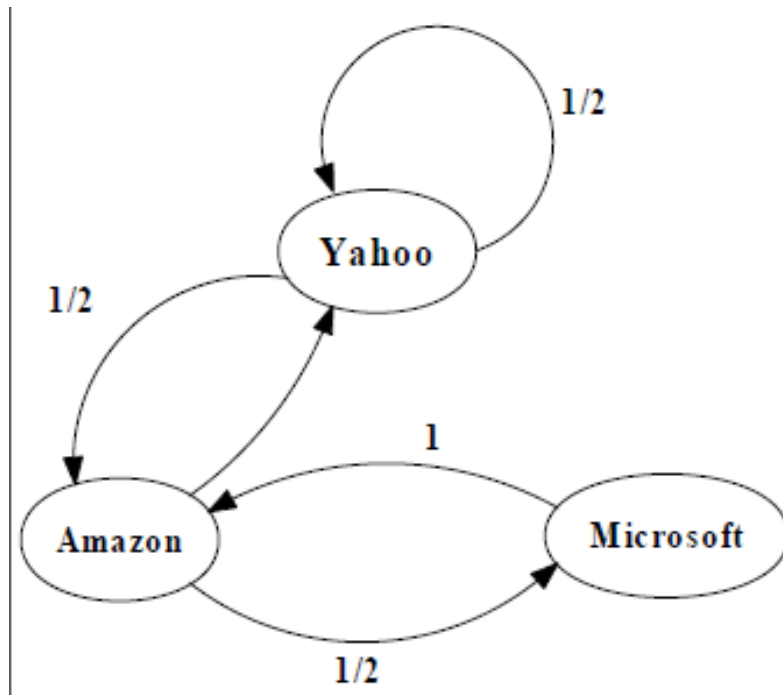
$$M = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1 \\ 0 & 1/2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 5/12 \\ 1/3 \\ 1/4 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1 \\ 0 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/2 \\ 1/6 \end{bmatrix}$$

PageRank Calculation: second iteration

# An example of Simplified PageRank



$$M = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1 \\ 0 & 1/2 & 0 \end{bmatrix}$$

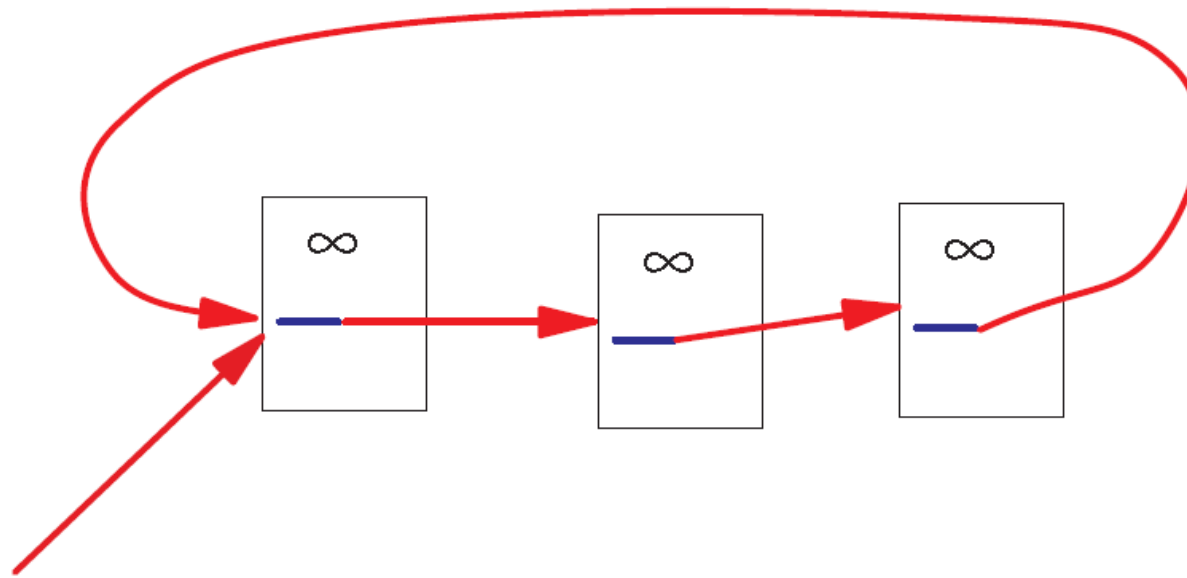
$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 3/8 \\ 11/24 \\ 1/6 \end{bmatrix} \quad \begin{bmatrix} 5/12 \\ 17/48 \\ 11/48 \end{bmatrix} \quad \dots \quad \begin{bmatrix} 2/5 \\ 2/5 \\ 1/5 \end{bmatrix}$$

Convergence after some iterations

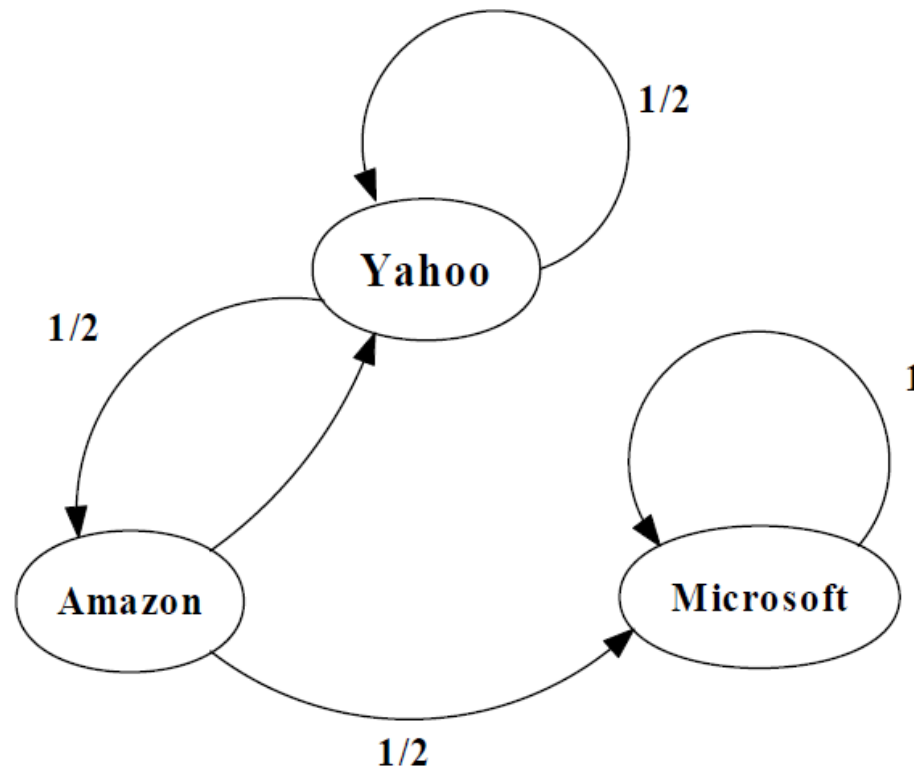
# A Problem with Simplified PageRank

A loop:



During each iteration, the loop accumulates rank but never distributes rank to other pages!

# An example of the Problem



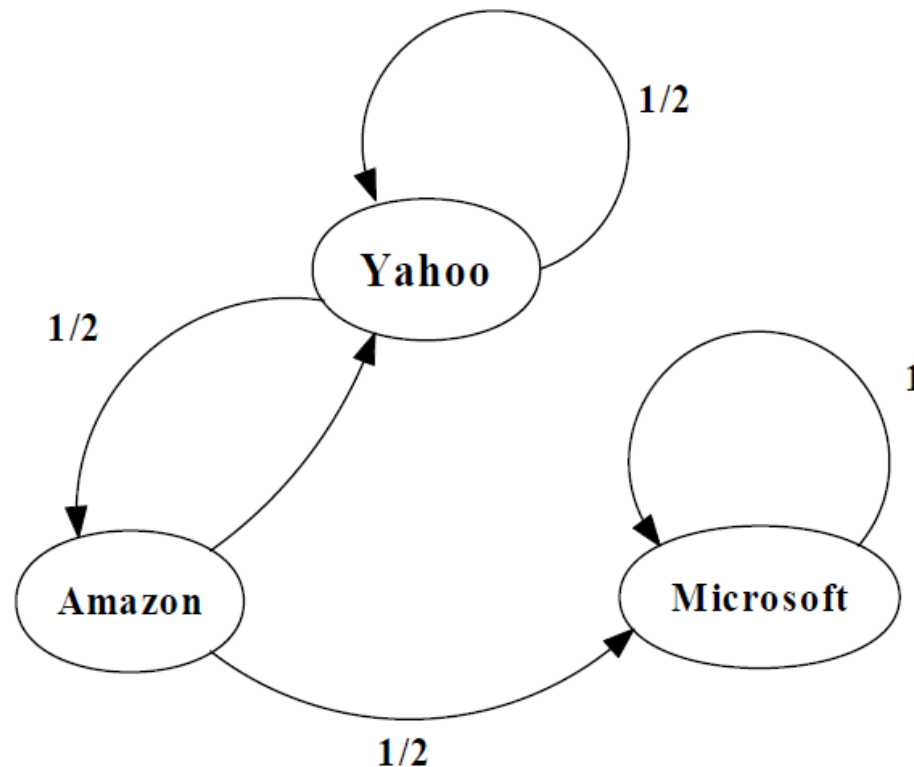
$$M = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 1/3 \\ 1/6 \\ 1/2 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$



# An example of the Problem

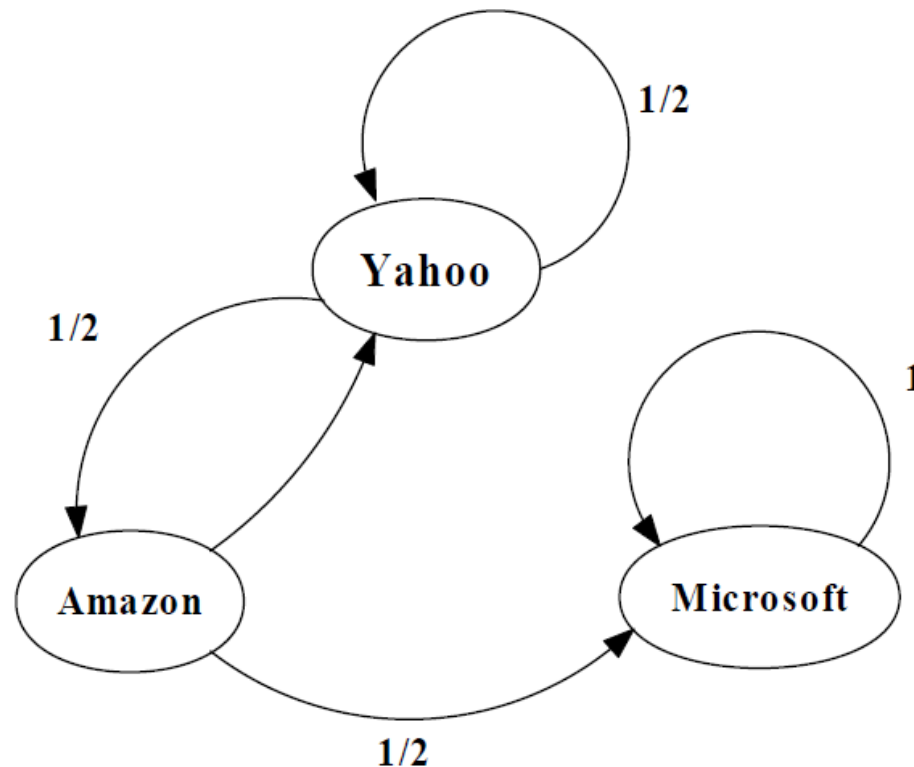


$$M = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix}^*$$

$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 1/4 \\ 1/6 \\ 7/12 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/6 \\ 1/2 \end{bmatrix}^*$$

# An example of the Problem



$$M = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 5/24 \\ 1/8 \\ 2/3 \end{bmatrix} \begin{bmatrix} 1/6 \\ 5/48 \\ 35/48 \end{bmatrix} \dots \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

# Random Walks in Graphs

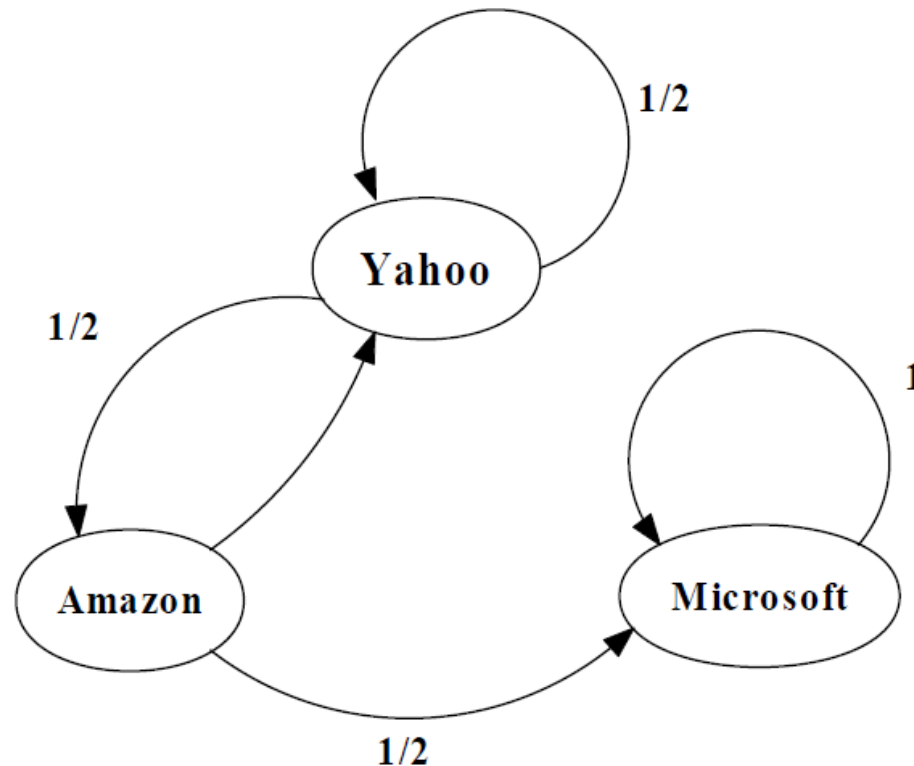
- The Random Surfer Model
  - The simplified model: the standing probability distribution of a random walk on the graph of the web. simply keeps clicking successive links at random
- The Modified Model
  - The modified model: the “random surfer” simply keeps clicking successive links at random, but periodically “gets bored” and jumps to a random page based on the distribution of  $E$

# Modified Version of PageRank

$$R'(u) = c_1 \sum_{v \in B_u} \frac{R'(v)}{N_v} + c_2 E(u)$$

$E(u)$ : a distribution of ranks of web pages that “users” jump to when they “gets bored” after successive links at random.

# An example of Modified PageRank



$$M = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$C_1 = 0.8 \quad C_2 = 0.2$$

$$\begin{bmatrix} 0.333 \\ 0.333 \\ 0.333 \end{bmatrix} \quad \begin{bmatrix} 0.333 \\ 0.200 \\ 0.467 \end{bmatrix} \quad \begin{bmatrix} 0.280 \\ 0.200 \\ 0.520 \end{bmatrix} \quad \begin{bmatrix} 0.259 \\ 0.179 \\ 0.563 \end{bmatrix} \quad \dots \quad \begin{bmatrix} 7/33 \\ 5/33 \\ 21/33 \end{bmatrix}$$

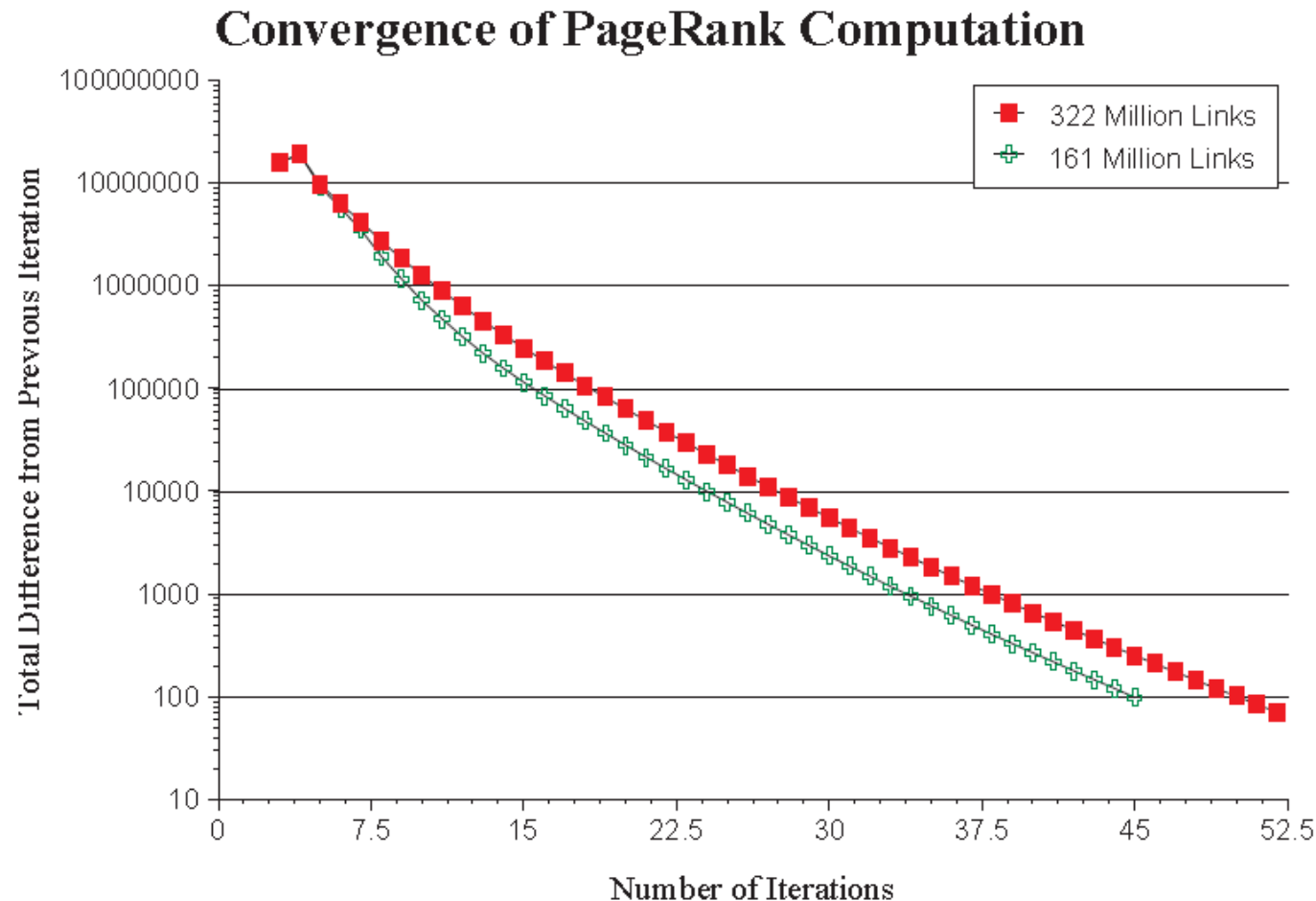
# Dangling Links

- Links that point to any page with no outgoing links
- Most are pages that have not been downloaded yet
- Affect the model since it is not clear where their weight should be distributed
- Do not affect the ranking of any other page directly
- Can be simply removed before pagerank calculation and added back afterwards



# Convergence Property

- PR (322 Million Links): 52 iterations
- PR (161 Million Links): 45 iterations
- Scaling factor is roughly linear in  $\log n$



# Convergence Property

- The Web is an expander-like graph
  - Theory of random walk: a random walk on a graph is said to be rapidly-mixing if it quickly converges to a limiting distribution on the set of nodes in the graph. A random walk is rapidly-mixing on a graph if and only if the graph is an expander graph.
  - Expander graph: every subset of nodes  $S$  has a neighborhood (set of vertices accessible via outedges emanating from nodes in  $S$ ) that is larger than some factor  $\alpha$  times of  $|S|$ . A graph has a good expansion factor if and only if the largest eigenvalue is sufficiently larger than the second-largest eigenvalue.

# PageRank vs. Web Traffic

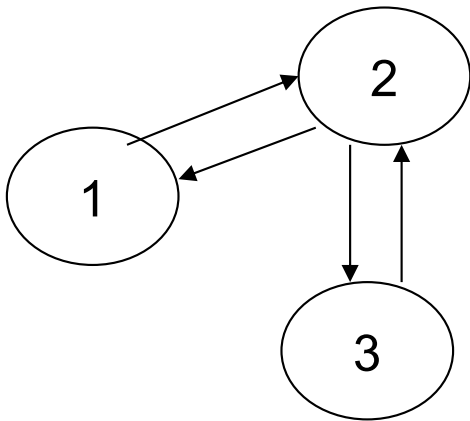
- Important component of PageRank calculation is  $E$ 
  - A vector over the web pages (used as source of rank)
  - Powerful parameter to adjust the page ranks
- The vector  $E$  corresponds to the distribution of web pages that a random surfer periodically jumps to from the search engine
- Some highly accessed web pages have low page rank possibly because
  - People do not want to link to these pages from their own web pages (the example in 1998 PageRank paper is pornographic sites...)
  - Some important backlinks are omitted
  - Use web usage data as a start vector for PageRank.

# Web Spamming by Gaming Pagerank

- Since 2000, Google Search has become the default gateway to the web
- Very high premium to appear on the first few pages of search results
  - E-commerce sites
  - Advertising-driven sites
- **Spamming**: Manipulating the text of web pages in order to appear relevant to queries
- Approximately 10-15% of web pages are spam
- **Spammers' goal**: Maximize the page rank of a target page  $t$
- **Spammers' technique**: Manipulating the text of web pages so as to appear relevant to queries and get as many links from accessible pages as possible to target page  $t$

# Exercise on PageRank

- Consider a Web graph with three nodes 1, 2, and 3. The links are as follows: 1->2, 3->2, 2->1, 2->3. Write down the transition probability matrices P for the surfer's walk with teleporting, with the value of teleport probability  $\alpha=0.5$ .

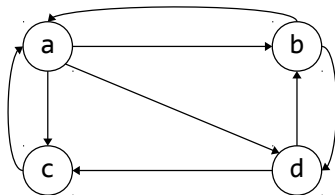


$$\begin{aligned}
 A = & \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \\
 (1 - \alpha)^* & \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix} \\
 + & \\
 \alpha^* & \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \\ \frac{5}{12} & \frac{1}{6} & \frac{5}{12} \\ \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \end{bmatrix}
 \end{aligned}$$

Each 1 divided by the number of ones in this row

# PageRank example

$$r_i = \sum_{j:j \rightarrow i \in \mathcal{E}} \frac{r_j}{d_j}$$



## Equations:

- $r_a = \frac{r_b}{2} + r_c.$
- $r_b = \frac{r_a}{3} + \frac{r_d}{2}.$
- $r_c = \frac{r_a}{3} + \frac{r_d}{2}.$
- $r_d = \frac{r_a}{3} + \frac{r_b}{2}.$

- 4 equations, 4 unknowns, no constants.

No **unique solution**: all solutions are equivalent modulo a scale factor.

- Additional **constraint** for uniqueness:

$$\sum_i r_i = 1.$$

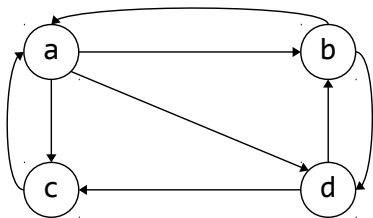
- **Solution** by **Gaussian elimination**:

- $r_a = \frac{1}{3}.$
- $r_b = r_c = r_d = \frac{2}{9}.$



# Random walkers

- For **large graphs**, solving linear systems of equations is intractable.
- **Random surfers**: Where do you end if you follow links at random?

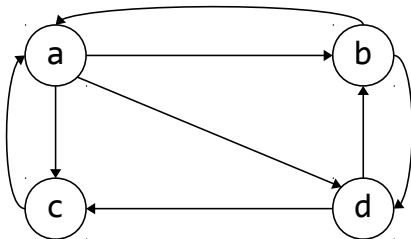


Start at node a: after one step, end up in b, c, or d with probability  $\frac{1}{3}$ .

- **Transition matrix**:  $M_{ij} = \frac{1}{d_j}$  if  $j \rightarrow i \in \mathcal{E}$  and 0 otherwise.

The transition matrix is **column-stochastic**: columns sum to 1.

# Random walkers: Transition matrix example



- **Transition matrix:**

$$\begin{bmatrix} 0 & 1/2 & 1 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix}$$

# PageRank with random walkers

- Start random surfers **at all pages** with **equal probability**  $\frac{1}{n}$

$$\vec{v}_0 = [1/n, 1/n, \dots, 1/n].$$

- **After one step**, the distribution will be

$$\vec{v}_1 = M\vec{v}_0.$$

- **After k steps**:

$$\vec{v}_k = M^k \vec{v}_0.$$

- **Markov process**: The distribution approaches a limiting distribution  $\vec{v}$  such that  $\vec{v} = M\vec{v}$  if
  - The graph is **strongly connected**: can get from a node to any other node.
  - No **dead ends**: nodes that have no out-links.

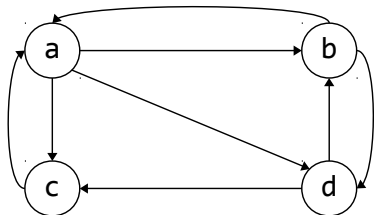
# PageRank with random walkers

$$\vec{v} = M\vec{v}.$$

- Surfers are **stationary**.
- The more important a page, and the more likely it is to have a surfer.
- $\vec{v}$  is ... **the principal eigenvector** of  $M$ . ( $M$  stochastic has largest eigenval 1.)
- **Power iteration**: compute  $\vec{v}$  by iterative **matrix-vector multiplications**.
  - Stop when  $||\vec{v}_t - \vec{v}_{t-1}|| \leq \epsilon$ .
  - How eigenvectors are computed in large dimensions (eg. Lanczos method.)
  - Amenable to **MapReduce** parallelization.
- Equivalent to previous PageRank formulation:

$$v_i = \sum_{j: i \rightarrow j \in \mathcal{E}} \frac{v_j}{d_j}$$

# Example



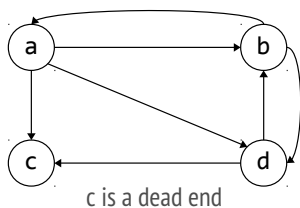
Transition matrix:

$$\begin{bmatrix} 0 & 1/2 & 1 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix}$$

- **Initialization:**  $\vec{v}_0 = [1/4, 1/4, 1/4, 1/4]$ .
- **After one step:**  $\vec{v}_1 = [9/24, 5/24, 5/24, 5/24]$ .
- **After two steps:**  $\vec{v}_2 = [15/48, 11/48, 11/48, 11/48]$ .
- ...
- **Converges to:**  $\vec{v} = [1/3, 2/9, 2/9, 2/9]$ .

# Dead ends

- **Dead ends:** nodes that have no out-links.

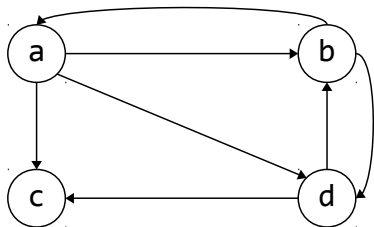


Transition matrix:

$$\begin{bmatrix} 0 & 1/2 & 0 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix}$$

- The **transition matrix** does not have full rank.
- It cannot be **inverted**, i.e. our linear system of equations has **no solution**.
- The **power method** converges to  $\vec{v} = \vec{0}$ .
- **Solutions:**
  - Recursively **remove** dead ends and their incoming links.
  - When at a dead end, **teleport** (with equal probability) to another node.

# Example



Transition matrix:

$$\begin{bmatrix} 0 & 1/2 & \mathbf{0} & 0 \\ 1/3 & 0 & \mathbf{0} & 1/2 \\ 1/3 & 0 & \mathbf{0} & 1/2 \\ 1/3 & 1/2 & \mathbf{0} & 0 \end{bmatrix}$$

- New **transition matrix**:

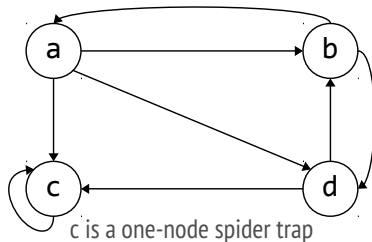
$$\begin{bmatrix} 0 & 1/2 & \mathbf{1/4} & 0 \\ 1/3 & 0 & \mathbf{1/4} & 1/2 \\ 1/3 & 0 & \mathbf{1/4} & 1/2 \\ 1/3 & 1/2 & \mathbf{1/4} & 0 \end{bmatrix}$$

- Eventually,  $\vec{v} = [1/5, 4/15, 4/15, 4/15]$ .



# Spider traps

- **Spider trap:** set of nodes with no dead ends but no links out.
- **Problem:**
  - All random surfers end up in the spider trap.



- **Transition matrix:**

$$\begin{bmatrix} 0 & 1/2 & \mathbf{0} & 0 \\ 1/3 & 0 & \mathbf{0} & 1/2 \\ 1/3 & 0 & \mathbf{1} & 1/2 \\ 1/3 & 1/2 & \mathbf{0} & 0 \end{bmatrix}$$

- $\vec{v}$  **converges to**  $\vec{v} = [0, 0, 1, 0]$ .

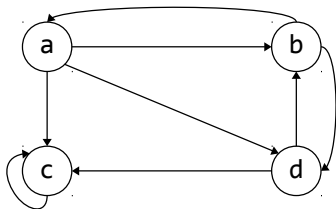
# Taxation

- How to get out of **spider traps**?
  - A random surfer can **leave the graph** at any moment.
  - **New surfers** can be started at any page at any moment.
- **Taxation**: Allow each random surfer a probability  $1 - \beta$  of **teleporting** to a random page

$$\vec{v} = \beta M \vec{v} + \frac{(1 - \beta)}{n} \vec{1}.$$

Typically,  $\beta \in [0.8 - 0.9]$ .

# Example



Transition matrix:

$$\begin{bmatrix} 0 & 1/2 & \mathbf{0} & 0 \\ 1/3 & 0 & \mathbf{0} & 1/2 \\ 1/3 & 0 & \mathbf{1} & 1/2 \\ 1/3 & 1/2 & \mathbf{0} & 0 \end{bmatrix}$$

$$\vec{v} = \beta \mathbf{M} \vec{v} + \frac{(1 - \beta)}{n} \vec{1}$$

–  $\beta = 0.8 = 4/5$

$$\vec{v} = \begin{bmatrix} 0 & 2/5 & 0 & 0 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 0 & 4/5 & 2/5 \\ 4/15 & 2/5 & 0 & 0 \end{bmatrix} \vec{v} + \begin{bmatrix} 1/20 \\ 1/20 \\ 1/20 \\ 1/20 \end{bmatrix}, \quad \vec{v}_0 = \left[ \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4} \right].$$

– **Solution:**  $\vec{v} = \left[ \frac{15}{148}, \frac{19}{148}, \frac{95}{148}, \frac{19}{148} \right]$ .

# Summary

- **Large-scale data** poses new technical problems for:
  - **storage**  $\Rightarrow$  distributed file systems.
  - **computations**  $\Rightarrow$  MapReduce programming model.
    - Split the data in chunks.
    - Map workers all execute the same operation on a chunk and return a key-val pair.
    - Reduce workers process all key-val pairs with the same key at once.
- **Algorithmic costs** of MapReduce:
  - **Communication costs** vs. **computation costs**.
  - **Reducer size** and **replication rate**.
- Extensions of MapReduce: **Spark** and **TensorFlow**.
- MapReduce for **machine learning**.
- **Link analysis** with **PageRank**.

# PageRank Summary



- Robust and scalable algorithm with proven convergence guarantees
- Distributed algorithm in Google's data center-drive breakthroughs in compute (Google MapReduce) and storage (Google File System)
- Amenable to distributed computation via parallel computation (MapReduce in next Lecture)
- MapReduce Code walkthrough:
- <http://web.archive.org/web/20221216071408/https://michaelnielsen.org/blog/using-mapreduce-to-compute-pagerank>