Time-Dependent Data (Week 12)

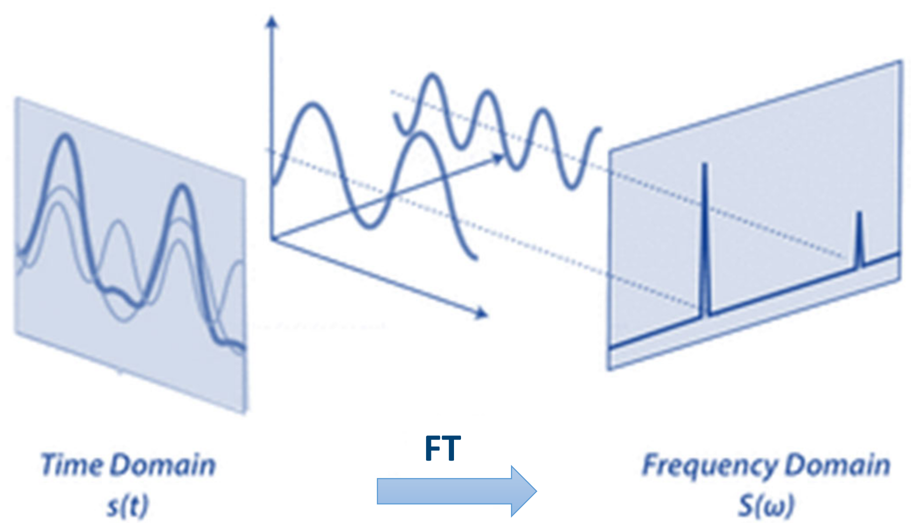
Fourier Transform

- Mapping time to frequency is Fourier Transform.

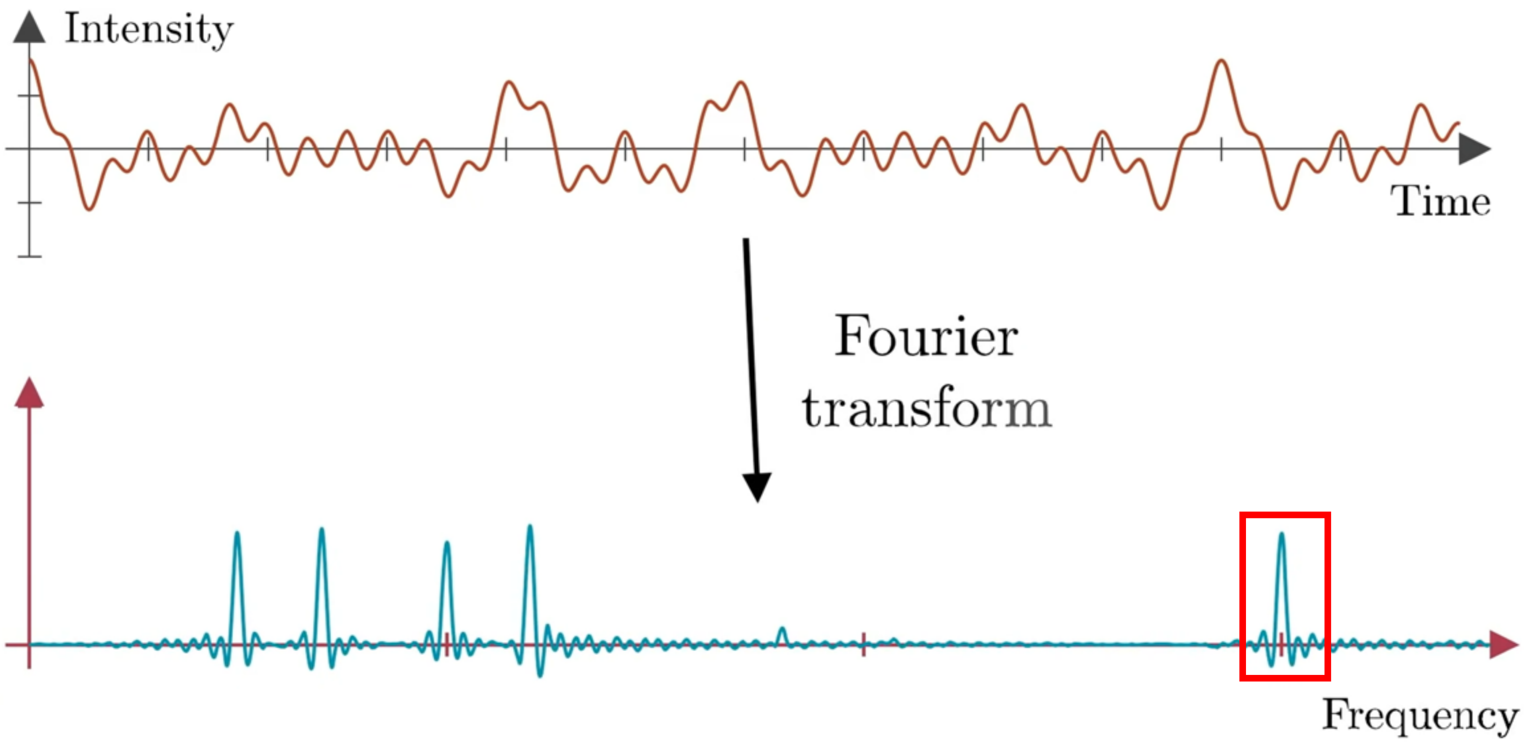
$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

- Mapping frequency to time is Inverse Fourier Transform.

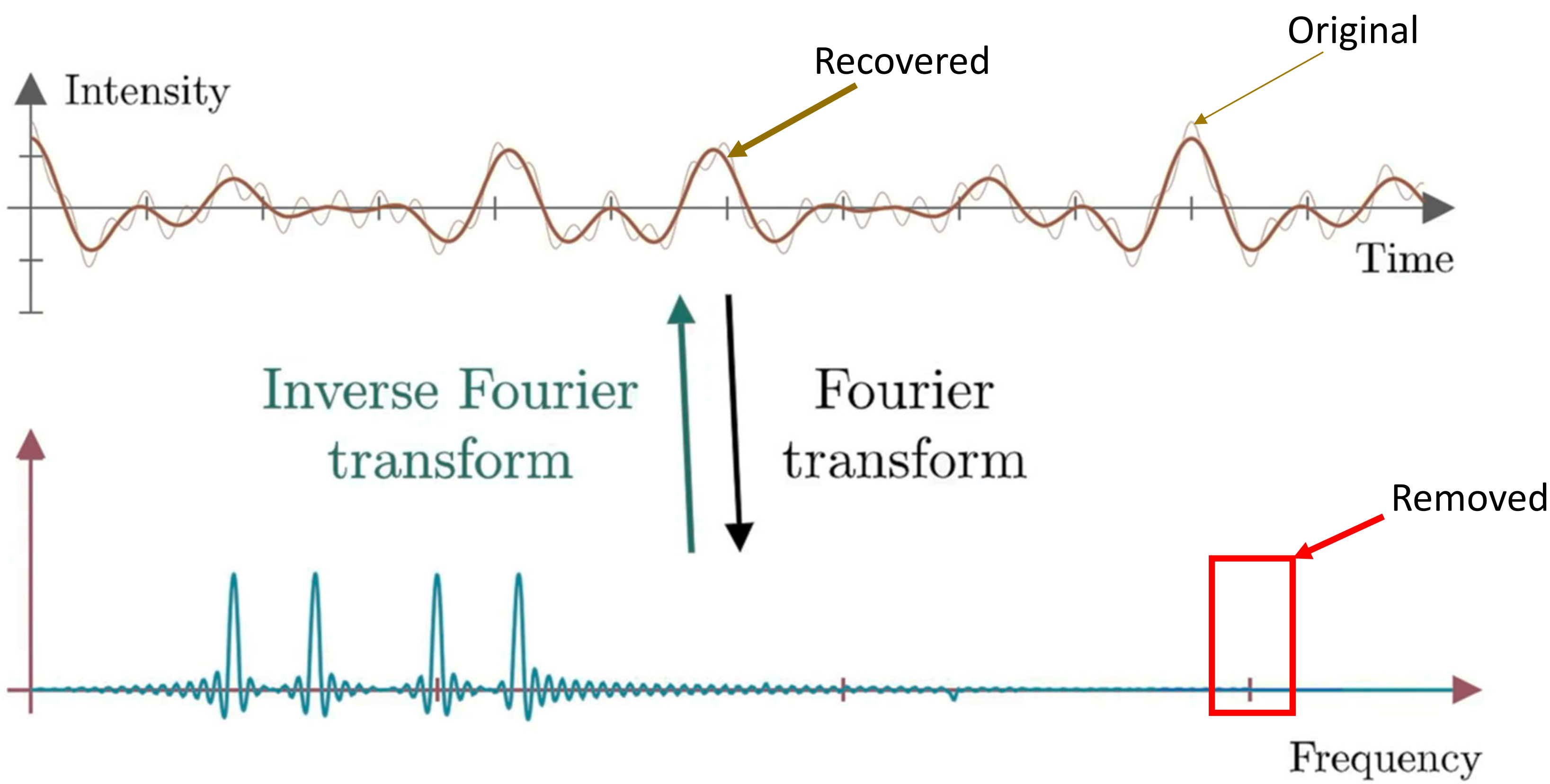
$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$



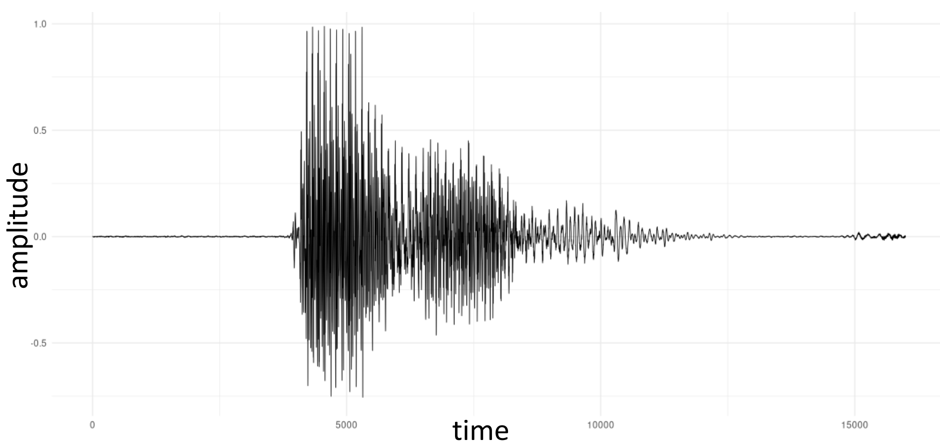
Fourier Transform: Sound Editing

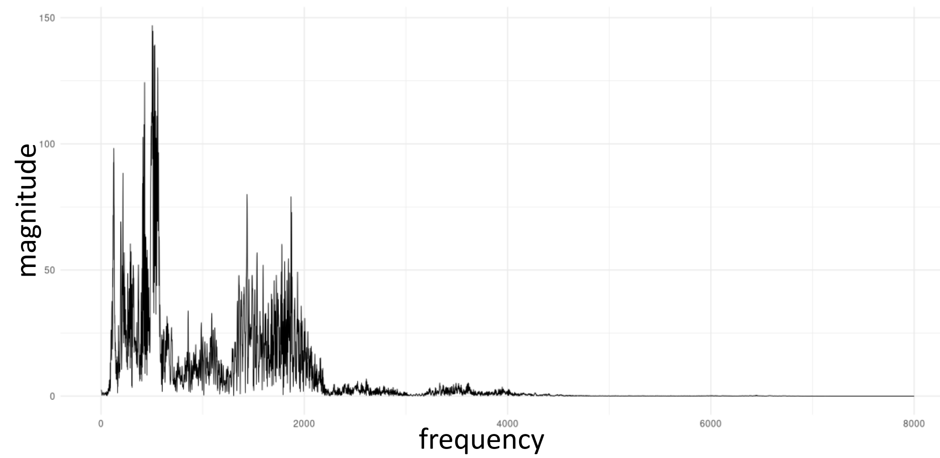


Inverse Fourier Transform



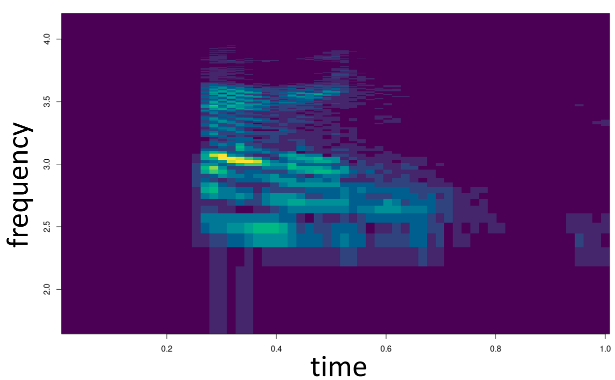
Audio classification: Bird Chirping





Spectrogram

Using
Gabor transform



Feed into Machine
Learning Models

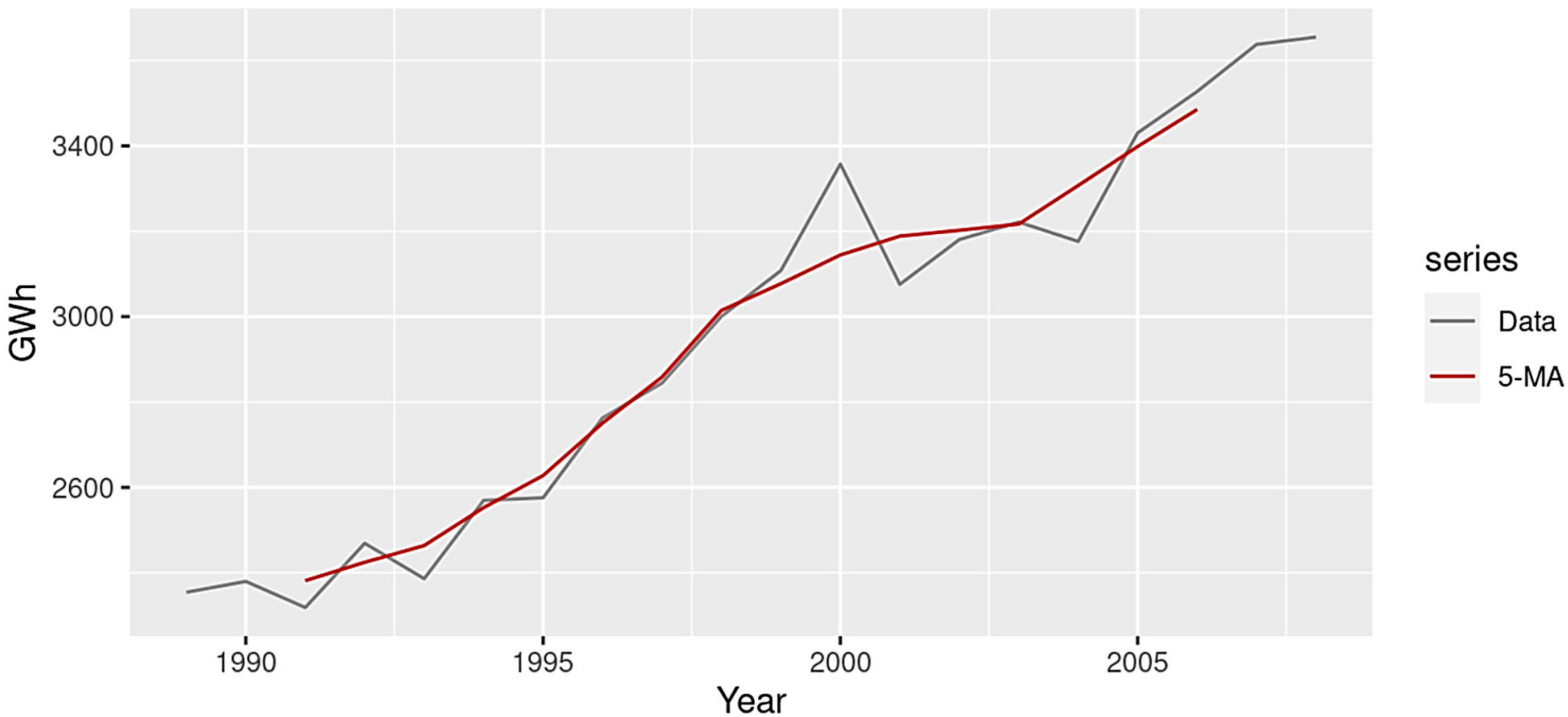
Moving Average: Removing Randomness/Noise

- This method establishes the underlying trend (smoothing out peaks and troughs) in a set of data.
- A moving average is a series of averages, calculated from historic data.
- Moving averages can be calculated for any number of time periods, for example a three-month moving average, a seven-day moving average, or a four-quarter moving average.
- A moving average of order m (known as, m-MA) can be written as:

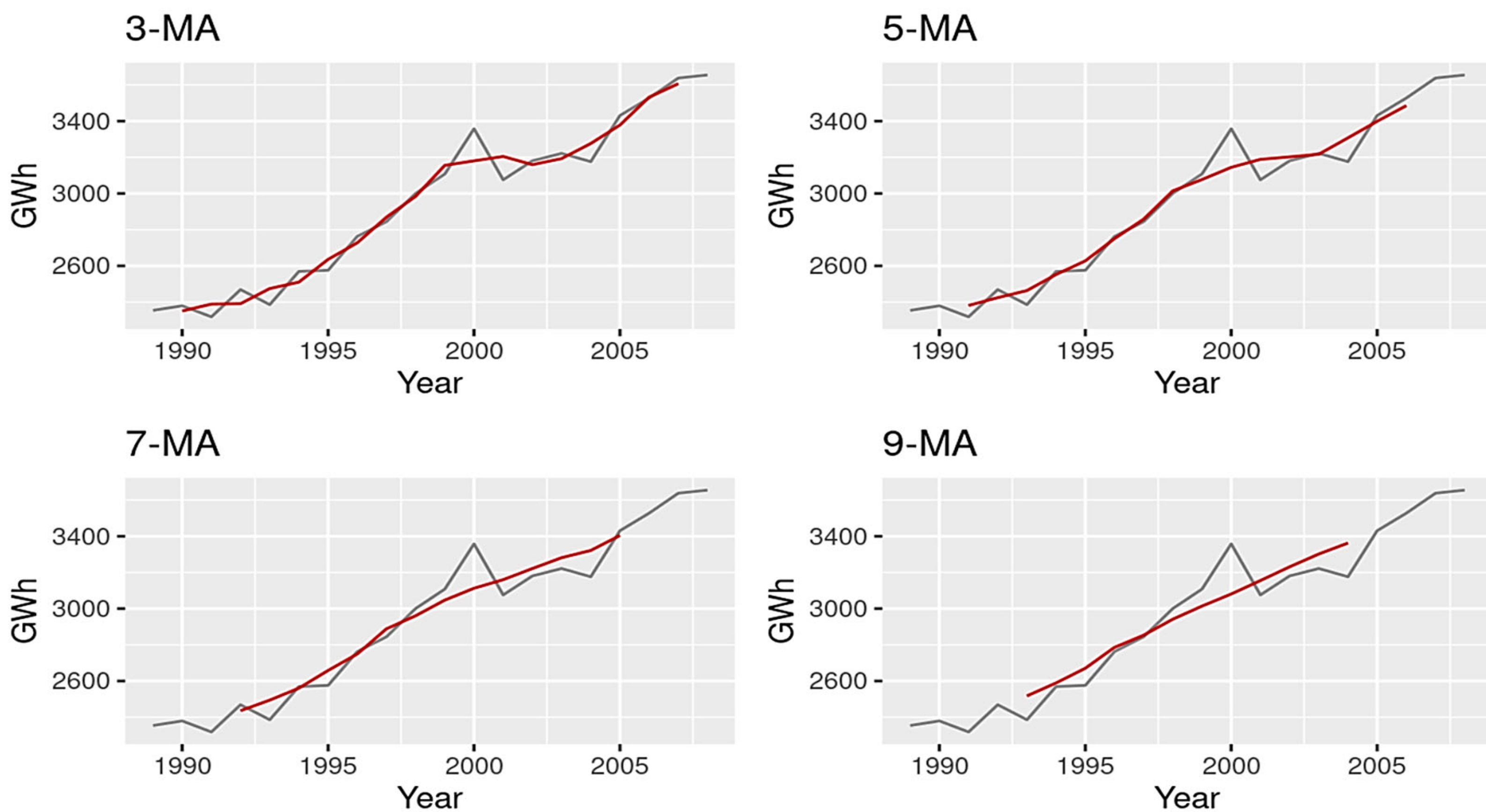
$$y_t = \frac{1}{m} \sum_{j=-k}^{k} y_{t+j}\,; \qquad \text{where } m = 2k + 1$$

- That is, the estimate of the trend-cycle at time t is obtained by averaging values of the time series within k periods of t.

Moving Average: Residential Electricity Sale



Moving Average: Residential Electricity Sale



Moving Average: Predicting Sales Revenue for Next June & July

- Increasing trend of $2,000 per month from 3-MA.
- November value is $170,000.
- Next June: Revenue will increase by $14,000 ($2,000 per month for seven months) i.e. $184,000 (= $14,000 + $170,000).
- Previous June sees positive variation of $32,000.
- Therefore, Next June will see $184,000 + $32,000 = $216,000 revenue.
- Similarly, for next July, revenue will be

  $186,000 - $25,000 = $161,000.

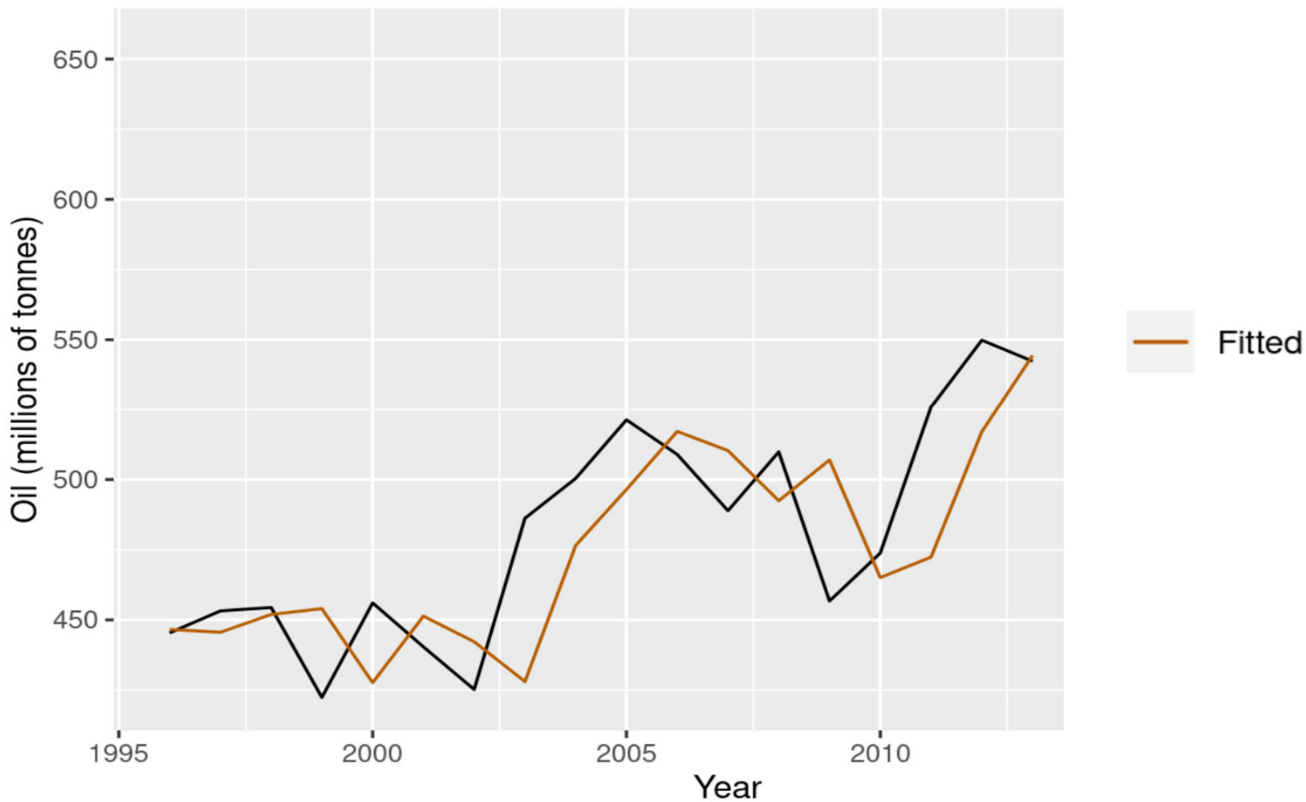
Month	Sales ($000)	Three-month moving total ($000)	Three-month moving average ($000)	Seasonal variation ($000)
January	125			
February	145	456	152	(145 – 152) = -7
March	186	462	154	(186 – 154) = 32
April	131	468	156	(131 – 156) = -25
May	151	474	158	(151 – 158) = -7
June	192	480	160	(192 – 160) = 32
July	137	486	162	(137 – 162) = -25
August	157	492	164	(157 – 164) = -7
September	198	498	166	(198 – 166) = 32
October	143	504	168	(143 – 168) = -25
November	163	510	170	(163 – 170) = -7
December	204			

Exponential Smoothing

- Forecasts produced using exponential smoothing methods are weighted averages of past observations, with the weights decaying exponentially as the observations get older.
- In other words, the more recent the observation the higher the associated weight.
- This framework generates more reliable forecasts than Moving Average.
- The exponential smoothing can be written as:

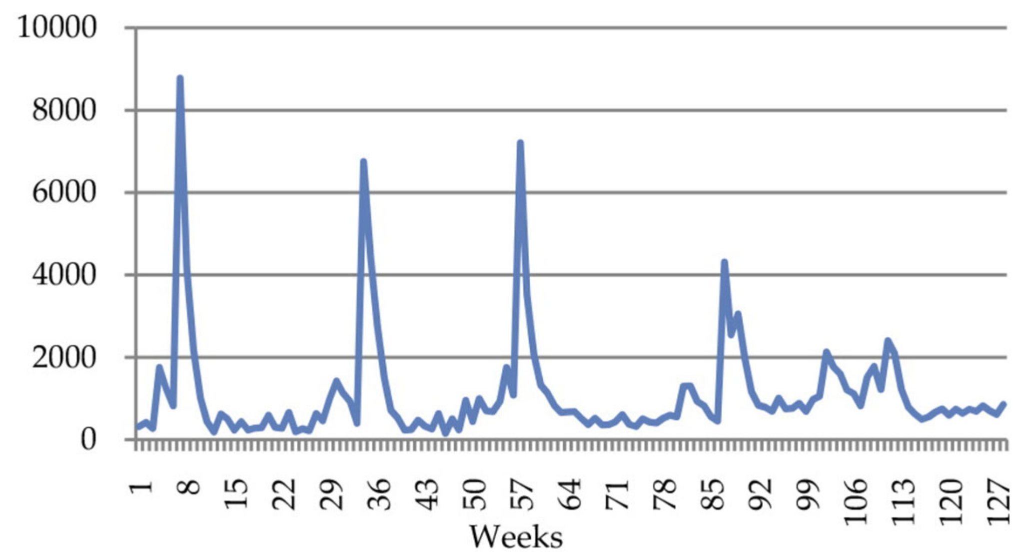
$$\hat{y}_{t+1} = \alpha y_t + \alpha(1-\alpha) y_{t-1} + \alpha(1-\alpha)^2 y_{t-2} + \cdots; \quad 0 \le \alpha \le 1$$

Exponential Smoothing



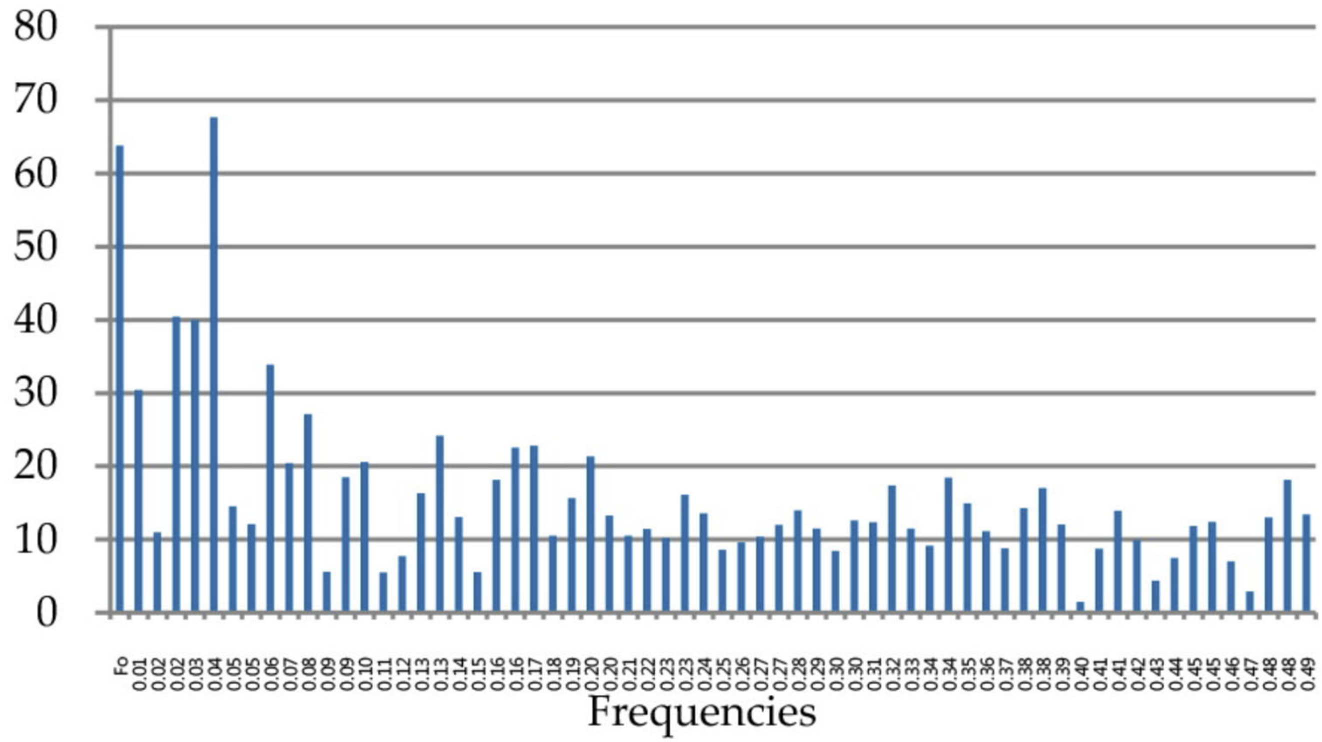
DFT vs MA vs ES: Trend Estimation of Demand: Training

- Fourier transform (FT), moving average (MA), and exponential smoothing (ES) can be used to calculate the sales forecasts of a company.
- DFT bases estimation:
  - The time series data has to be divided into training and testing datasets.
  - Transform time data to the frequency domain using DFT.
  - Eliminate low-rank frequencies.
  - Apply IDFT and get approximate general trend, free from noise and fluctuations.
  - Find to MA.
  - Predict future values.



DFT vs MA vs ES: Trend Estimation of Demand: Training

- DFT



DFT vs MA vs ES: Trend Estimation of Demand: Training

- Find dominating frequencies, perform IDFT, and sessional variation.

Component	Amplitude	Frequency	Phase
f_0	63.8	–	–
f_5	67.7	0.04	−1.04
f_3	40.5	0.02	−0.73
f_4	40.0	0.03	+2.72
f_8	33.9	0.06	−1.77
f_1	30.5	0.01	+2.31
f_{10}	27.2	0.08	−2.47

DFT vs MA vs ES: Trend Estimation of Demand: Testing