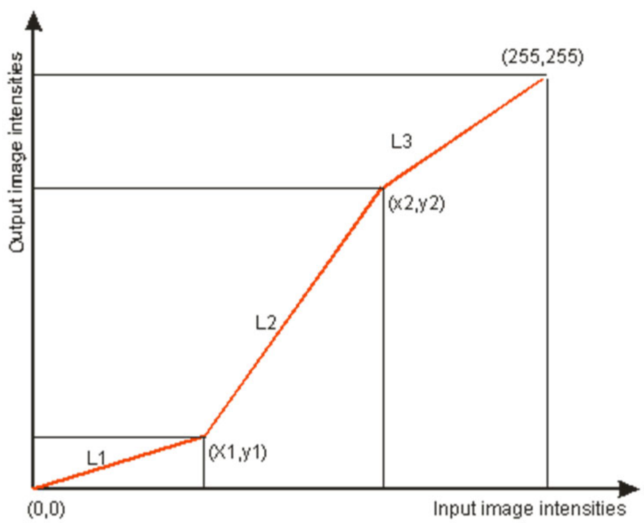
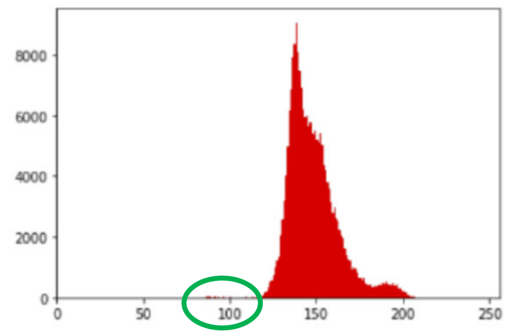
Image Data Adjustment & Equalisation (Week 9)

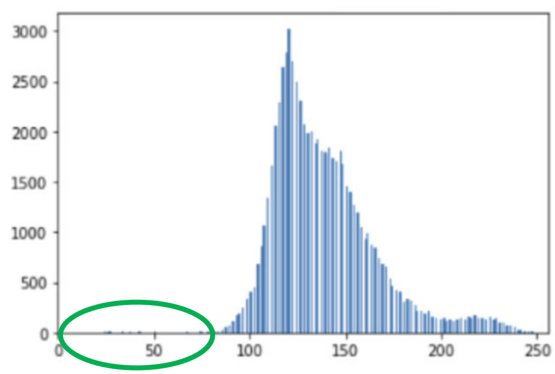
Piecewise Linear Adjustment

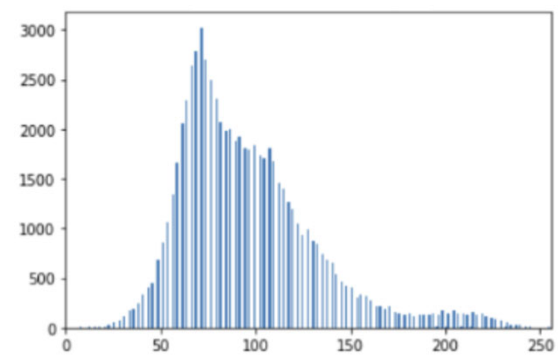
- Contrast-Stretching transformation is the one that uses Piecewise Linear Functions for mapping the pixels.
- Write mathematical expression for L_1, L_2 and L_3:
  - For L_1 $(0 \leq x \leq x_1)$: $y = \frac{y_1}{x_1} x$
  - For L_2 $(x_1 < x \leq x_2)$: $y = \frac{y_2 - y_1}{x_2 - x_1} x + y_1$
  - For L_3 $(x_2 < x \leq L - 1)$: $y = \frac{(L-1) - y_2}{(L-1) - x_2} x + y_2$



Linear Contrast VS Piecewise Linear Adjustment

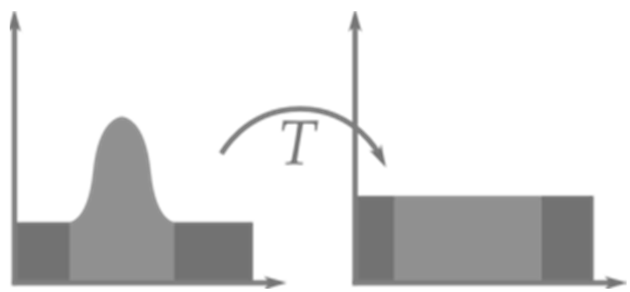






Histogram Equalisation

- Histogram Equalization is a computer image processing technique used to improve the quality of the images.
- It accomplishes this by effectively spreading out the most frequent intensity values, i.e. stretching out the intensity range of the image.
- This method usually increases the global contrast of images when its usable data is represented by close contrast values.
- This allows for areas of lower local contrast to gain a higher contrast.



Conventional Histogram Equalisation

- It maps the input intensity levels to new levels using the cumulative distribution function (cdf) as a transformation function.
- Consider the input image $I(i, j)$ of dimension $u \times v$ having L discrete intensity levels $X_0, X_1, .., X_{L-1}$ (for 8-bit image, L =256).
- For simplicity k^{th} intensity level X_k is denoted by k. The probability distribution function (pdf) for k^{th} intensity level is defined as:

$$p(\mathrm{X_k}) = \aleph(\mathrm{X_k}) \Bigg/ \left(\sum_{k=0}^{L-1} \aleph(\mathrm{X_k}) \right) = \aleph(\mathrm{X_k})/N$$

where, k is an integer in the range 0 to $L-1$ i.e. $k \in [0, L-1]$, $\aleph(X_k)$ is number of pixels with k^{th} intensity level, and N $(= u \times v)$ is total number of pixels in the image.

Conventional Histogram Equalisation

- The histogram of an image, $H[X_l, Xu]$ is simply the plot of $\aleph(X_k)$ vs X_k, where X_l and X_u are lower and upper intensity levels respectively.
- The cumulative distribution function (cdf) is defined as:

$$c(\mathrm{X_k}) = \sum_{q=0}^{\mathrm{k}} p(\mathrm{X_q}) \qquad \forall\, k \in [0, L-1]$$

where, $c(X_k)$ is the cdf at the k^{th} intensity level.

- The CHE method maps the input image I (having dynamic range (X_l, X_u) into the entire range $(X_0, X_{L-1}\ )$ by using the cdf as a transformation function.
- Let T be the transformation function which maps the input intensity level X_k into output intensity level $T(X_k)$ is defined as:

$$T(\mathrm{X_k}) = X_0 + \lfloor\, (X_{L-1} - X_0) \times c(\mathrm{X_k}) \rceil$$

where, $\lfloor x \rceil$ is the integer nearest to x.

Conventional Histogram Equalisation: DIY

- Consider a 6-level image.

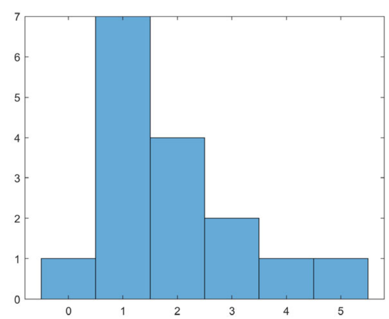
$$p(\mathrm{X_k}) = \aleph(\mathrm{X_k}) \Big/ \left(\sum_{k=0}^{L-1} \aleph(\mathrm{X_k}) \right) = \aleph(\mathrm{X_k})/N$$

$$c(\mathrm{X_k}) = \sum_{q=0}^{\mathrm{k}} p(\mathrm{X_q}) \qquad \forall\, k \in [0, L-1]$$

$$T(\mathrm{X_k}) = X_0 + \lfloor (X_{L-1} - X_0) \times c(\mathrm{X_k}) \rceil$$

4	1	3	2
3	1	1	1
0	1	5	2
1	1	2	2

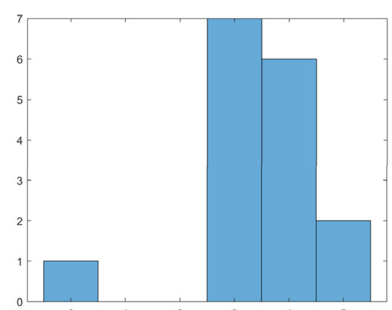
input image



5	3	4	4
4	3	3	3
0	3	5	4
3	3	4	4

output image

Intensity (old)	sum	normalized sum	Intensity (new)
0	1	(1/16)×5=0.31255	0
1	8	2.5	3
2	12	3.75	4
3	14	4.375	4
4	15	4.6875	5
5	16	5.0	5



Conventional Histogram Equalisation

Bi-/Multi-Histogram Equalisation

- The brightness preservation is commonly achieved by applying HE over multiple segmented histograms (sub-histograms), instead of global histogram.
- Let $H[X_l, X_u]$ be the global histogram of the input image $I(i,j)$, where X_l and X_u are lower and uppermost intensities of the image. Let $H[X_l, X_u]$ be segmented into 'n' sub-histograms i.e.

$$H[\mathrm{X}_l, \mathrm{X}_u] = \bigcup_{r=1}^{n} H_r\left[X_l^{r,n}, X_u^{r,n}\right]$$

where, H_r represents r^{th} segment of H. $X_l^{r,n}$ and $X_u^{r,n}$ represents lower and uppermost boundaries of r^{th} (out of n) segment respectively.

- It should be noted that $X_l^{1,n} = X_l$ and $X_u^{n,n} = X_u$.

Bi-Histogram Equalisation: Mean Based Segmentation

- For the image $I(i,j)$ having histogram $H[X_l, X_u]$ or its pdf $p(X_k)$, the mean intensity value (μ) of the image is defined as:

$$\mu = \left\lfloor \sum_{k=\mathrm{X}_l}^{\mathrm{X}_u} \mathrm{X_k} \,.\, p(\mathrm{X_k}) \right\rfloor$$

- The mean intensity value μ is then selected as the threshold for decomposing the image histogram into two segments i.e. $H_1\ [X_l^{1,2}, X_u^{1,2}]$ and$H_2\ [X_l^{2,2}, X_u^{2,2}]$, where each segment represents a sub-image.
- Note that $H_1\ [X_l^{1,2}, X_u^{1,2}]$ consists of $\{X_l, X_l + 1, \dots, \mu\}$ intensity levels, and $H_2\ [X_l^{2,2}, X_u^{2,2}]$ consists of $\{\mu + 1, \mu + 2, \dots, X_u\}$ intensity levels.
- The corresponding sub-images may be represented as:

$$I^{1,2} = \{I(i,j) | I(i,j) \leq \mu, \qquad \forall (i,j) \in I\}$$
$$I^{2,2} = \{I(i,j) | I(i,j) > \mu, \qquad \forall (i,j) \in I\}$$

Bi-Histogram Equalisation: Median Based Segmentation

- Assume that A is an array consisting of image pixels in the ascending or descending order, and $N(A)$ is the number of elements in that array. If X_k is the middle index of A, then median intensity 'm' is defined as:

$$m = \begin{cases} A(\mathrm{X_k}) & if\ N(A)\ is\ odd \\ (A(\mathrm{X_k}) + A(\mathrm{X_k}+1))/_2 & if\ N(A)\ is\ even \end{cases}$$

- The corresponding sub-images may be represented as:

$$I^{1,2} = \{I(i,j) | I(i,j) \leq m, \qquad \forall (i,j) \in I\}$$
$$I^{2,2} = \{I(i,j) | I(i,j) > m \qquad \forall (i,j) \in I\}$$

where, $I^{r,n}$ is the r^{th} sub-image out of total n sub-images.

- The mapping process is similar to that of CHE. The mapping of r^{th} segment is determined as:

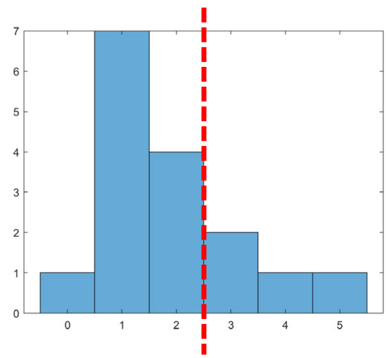
$$T(k) = T(\mathrm{X_k}) = X_l^{r,n} + \lfloor (X_u^{r,n} - X_l^{r,n}) \times c(k) \rfloor$$

Bi-Histogram Equalisation: DIY

- Consider a 6-level image. Threshold via mean value.

4	1	3	2
3	1	1	1
0	1	5	2
1	1	2	2

input image

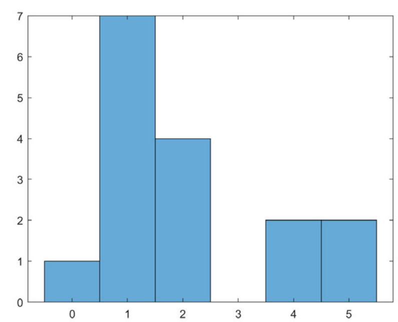


$$p\left(\mathrm{X}_k^{r,n}\right) = \aleph\left(\mathrm{X}_k^{r,n}\right) \Bigg/ \left(\sum_{\mathrm{X}_k^{r,n}=\mathrm{X}_l^{r,n}}^{\mathrm{X}_u^{r,n}} \aleph\left(\mathrm{X}_k^{r,n}\right) \right) = \aleph\left(\mathrm{X}_k^{r,n}\right) / N^{r,n}$$

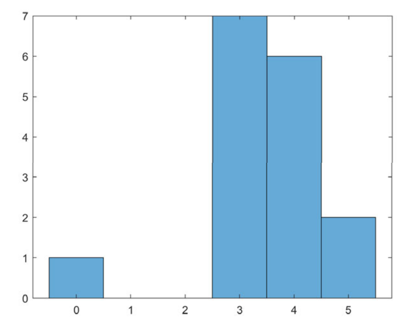
$$c\left(\mathrm{X}_k^{r,n}\right) = \sum_{q=\mathrm{X}_l^{r,n}}^{k} p\left(\mathrm{X}_q^{r,n}\right) \qquad \forall\, k \in \left[\mathrm{X}_l^{r,n}, \mathrm{X}_u^{r,n}\right]$$

$$T\left(\mathrm{X}_k^{r,n}\right) = \mathrm{X}_l^{r,n} + \left[\left(\mathrm{X}_u^{r,n} - \mathrm{X}_l^{r,n}\right) \times c\left(\mathrm{X}_k^{r,n}\right)\right]$$

Intensity (old)	sum	normalized sum	Intensity (new)
0	1	(1/6)×2=0.16667	0
1	8	1.33	1
2	12	2.0	2
3	14	[(2/4)×2]+3=4.0	4
4	15	[(3/4)×2]+3=4.5	5
5	16	[(4/4)×2]+3=5.0	5



Bi-HE



CHE