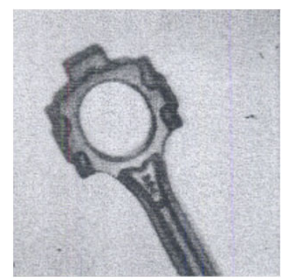
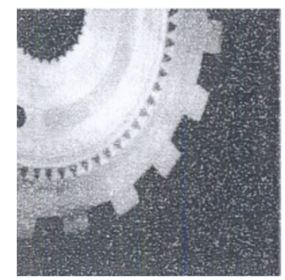
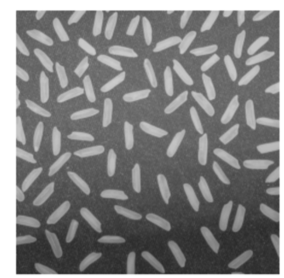
Mathematical Morphology (Week 2)

Remote Sensing

- Remote sensing is the acquisition of information about an object or phenomenon without making physical contact with the object, in contrast to on-site observation.
- Due to the advancement of drones, now remote sensing is not limited to satellite-based earth observation, it is now frequently used in various domains such as agriculture, military, humanitarian applications etc.
- One of the limitations while working with a consumer-grade drone is the amount of noise one can encounter in data.
- Due to the small size of the drone's sensor, the captured image looks noisy with non-linear illumination, especially when observing shadow regions.
- One of the possible solutions for such noisy and non-linearly illuminated data is image filters.

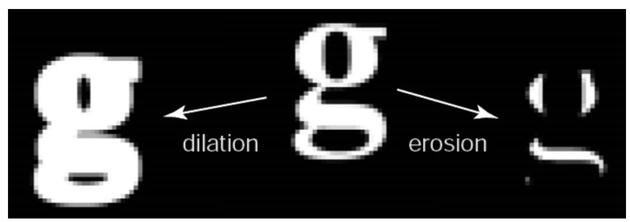
Image Filters

- The common image related artifacts during image acquisition are noise caused due to external interference and imbalance in illumination.
  - Salt and pepper noise contains random occurrences of both black and white intensity values.
  - Impulse noise contains only random occurrences of white intensity values.
  - Gaussian noise contains variations in intensity that are drawn from a Gaussian or normal distribution and is a very good model for many kinds of camera sensor noise.
  - Uneven illumination is one of the most unavoidable issues that make images look imperfect.

Mathematical Morphology

- Mathematical Morphology is a tool for extracting image components that are useful for representation and description.
- The shapes of objects in a binary image are represented by object membership sets. This theory can be extended to grayscale images.
- Morphological operations can simplify image data, preserve the objects' essential shape characteristics, and can eliminate irrelevant objects.
- The two basic morphological set transformations are:
  - Dilation
  - Erosion



Mathematical Morphology

- Dilation (represented by $\bigoplus$) operation usually uses a structuring element (S) for probing and expanding the shapes contained in the binary input image (I).
  - Suppose, S is centred at reference pixel (i, j) on I, which is denoted as $S_{(i,j)}$, then dilated pixel $D_{(i,j)}$ can be defined as:

$$D_{(i,j)} = I \bigoplus S_{(i,j)} = \max\left(\bigcup I \otimes S_{(i,j)} \right)_{(i,j)\in I}$$

where, $\otimes$ represents element by element multiplication of metrices.

- Erosion (represented by $\ominus$) operation usually uses inverse logic.
  - Suppose, S is placed with its reference pixel at (i, j) on I, which is denoted as $S_{(i,j)}$, then eroded pixel $E_{(i,j)}$ can be defined as:

$$E_{(i,j)} = I \ominus S_{(i,j)} = \min\left(\bigcup I \otimes S_{(i,j)} \right)_{(i,j)\in I}$$

Mathematical Morphology

- Consider a matrix:

$$I = \begin{bmatrix} 2 & 4 & 8 \\ 3 & 5 & 9 \\ 1 & 0 & 4 \end{bmatrix}, \text{and } \; S = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Note, centre of $S_{(i,j)}$ is $S_{(2,2)}$.

$$D_{(2,2)} = I \bigoplus S_{(2,2)} = \begin{bmatrix} 2 & 4 & 8 \\ 3 & 9 & 9 \\ 1 & 0 & 4 \end{bmatrix}$$

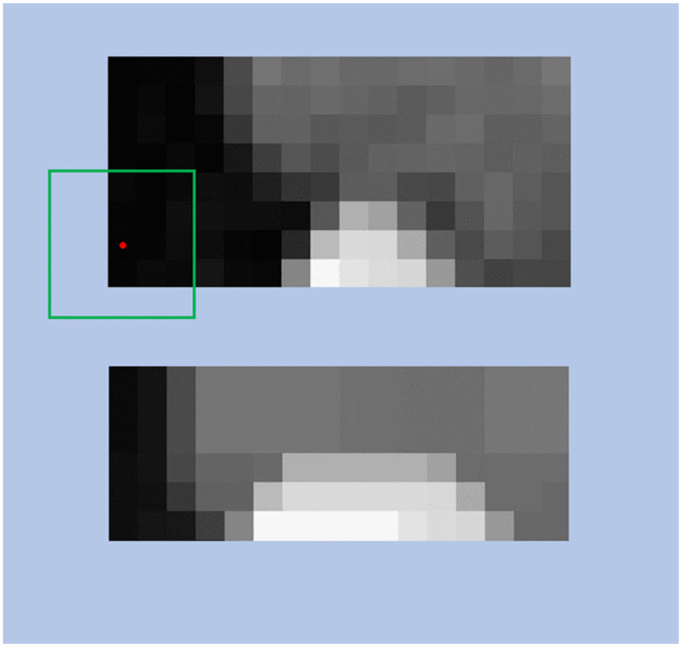
$$E_{(2,2)} = I \ominus S_{(2,2)} = \begin{bmatrix} 2 & 4 & 8 \\ 3 & 0 & 9 \\ 1 & 0 & 4 \end{bmatrix}$$

Calculate $D_{(2,2)}$ and $E_{(2,2)}$

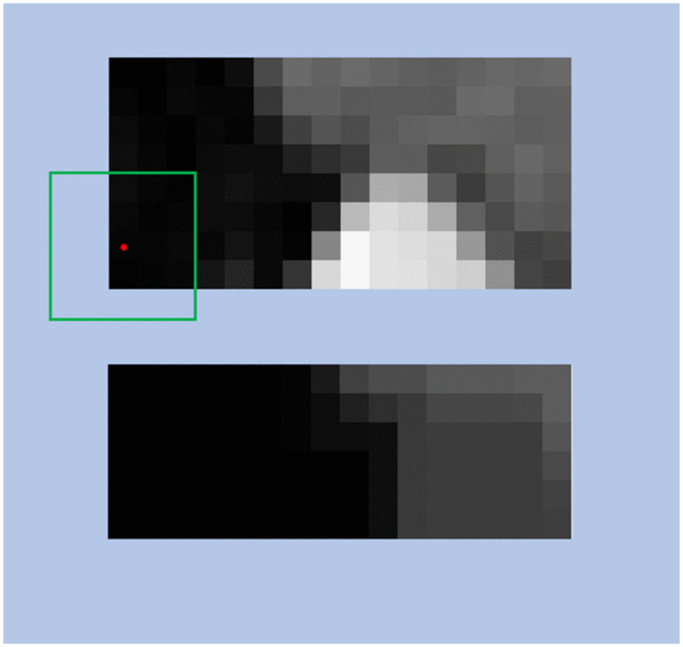
Mathematical Morphology

- Structuring element (S) is a 5x5 matrix.

Note the missing edges.



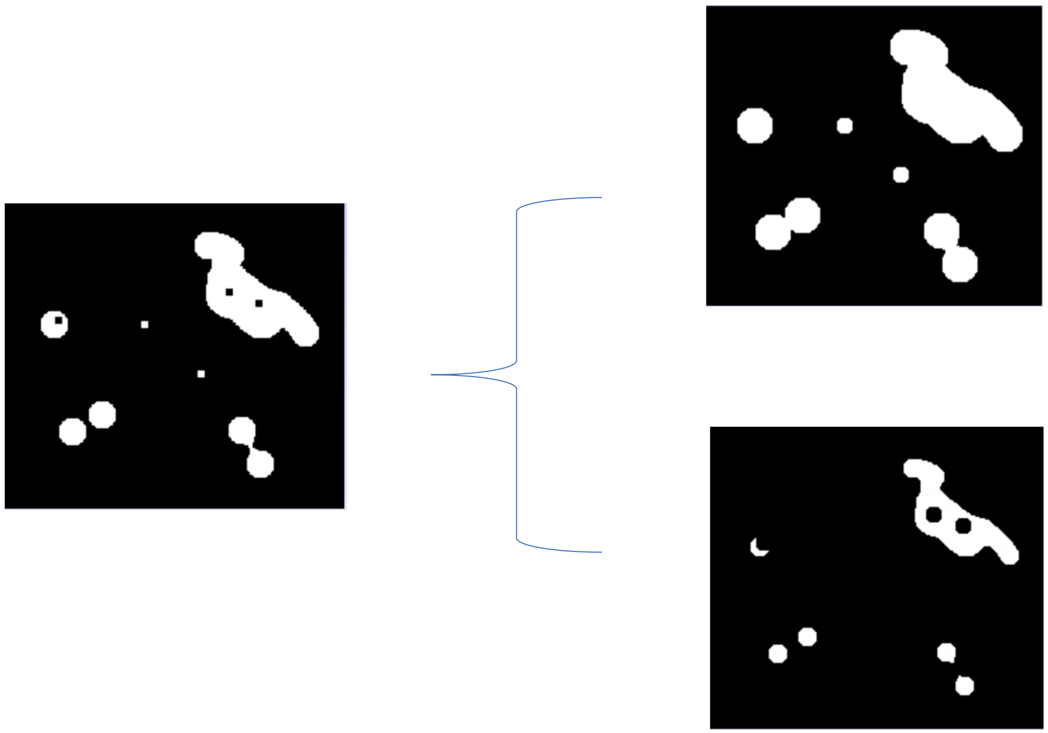
Dilation



Erosion

Mathematical Morphology: Binary Image

Image (I)



Dilation
Erosion

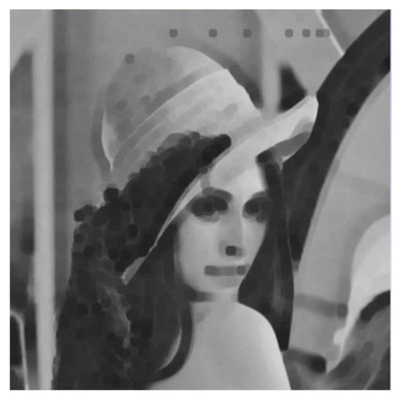
Erosion
Dilation

Able to understand Mathematical Morphology?

Mathematical Morphology: Gray Image

Image (I)

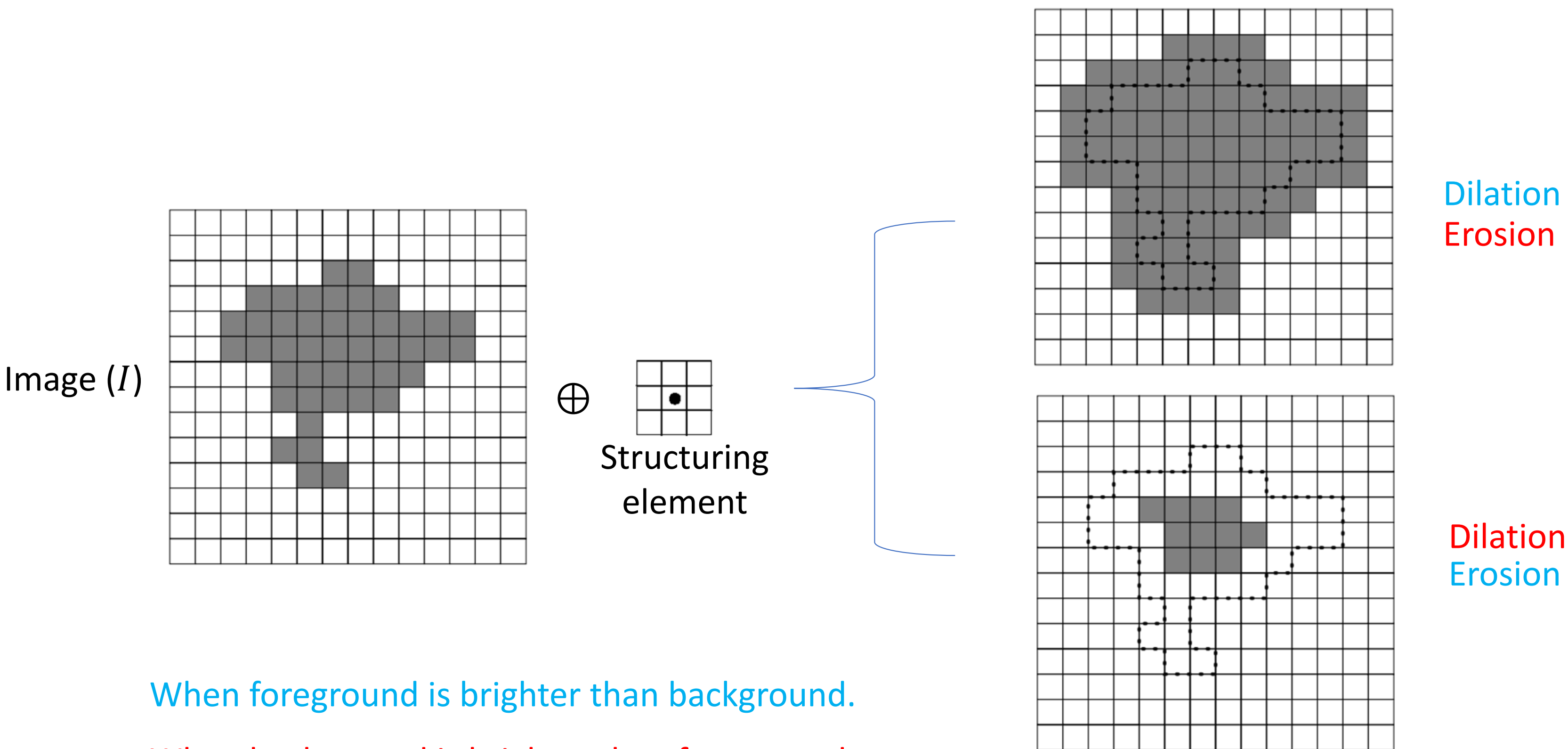
Dilation



Erosion

Why dilatated image is brighter than eroded image?

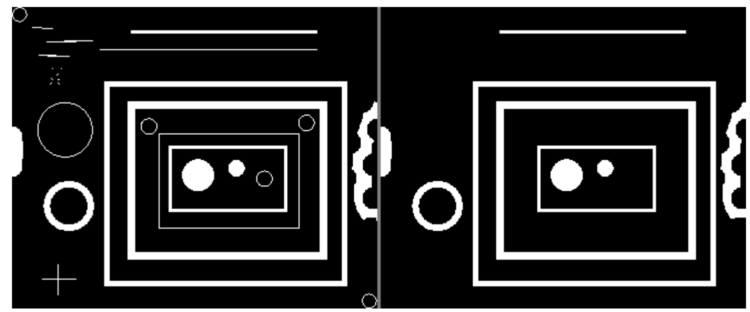
Mathematical Morphology: Background Dependency



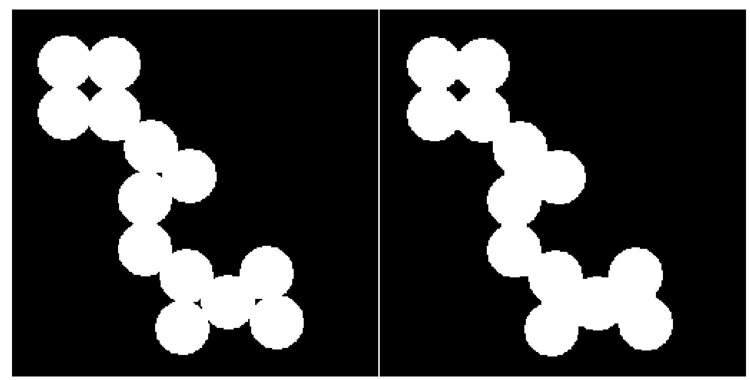
When foreground is brighter than background.

When background is brighter than foreground.

Mathematical Morphology



- Morphological opening (○): First erode, then dilate, using the same structuring element for both operations.
  - Morphological opening is useful for removing small objects and thin lines from an image while preserving the shape and size of larger objects in the image.



- Morphological closing (●): First dilate, then erode, using the same structuring element for both operations.
  - Morphological closing is useful for filling small holes in an image while preserving the shape and size of large holes and objects in the image.

Mathematical Morphology

- Morphological opening

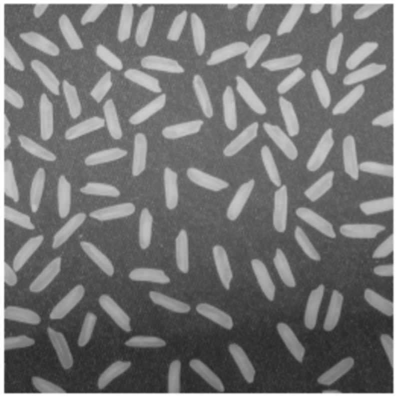
- Morphological closing

Mathematical Morphology

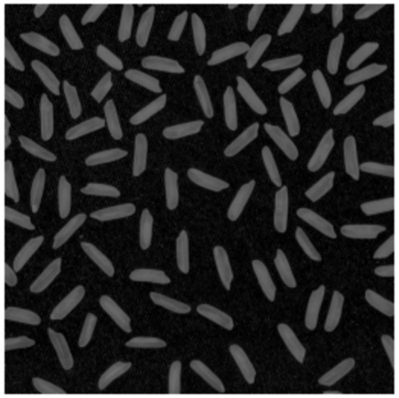
- White top-hat: It transforms I using following equation:
$$T^{w}{}_{(i,j)} = I - (I \circ S_{(i,j)})$$
where, $\circ$ denotes the opening operation.
- Black top-hat: It transforms I using following equation:
$$T^{B}{}_{(i,j)} = \left(I \bullet S_{(i,j)}\right) - I$$
where, $\bullet$ denotes the closing operation.
- Top-hat transforms are used for various image processing tasks, such as feature extraction, background equalization, image enhancement, etc.

Mathematical Morphology

- Input image
  - Sts2zsnktwr mqzr nsfyrts.



- White top-hat transformation



DIY

- Consider a matrix I and structuring element S.

10	20	85	97	55
40	60	70	66	52
9	70	90	87	12
15	54	33	60	11
6	26	73	59	9

1	1	1
1	1	1
1	1	1

- Apply white top-hat or black top-hat through any two corners of the matrix, and observe if there is any difference or not.
- Work has to be done by forming a group.