Time-Dependent Data (Week 11)

Time-Dependent Data

- In data processing, a signal is a function that conveys information about a phenomenon. Any quantity that can vary over space or time can be used as a signal to share information between observers.
- In nature, signals can be seen as actions done by an organism to alert other organisms, ranging from cell signaling (within the human body) to sounds, motions made by animals (to alert other animals of food or danger), etc.
- In man-made scenarios, variation of demand in a company, machine sounds, etc.
- There are two main types of signals:
  - Continuous signal is generally bound to a range, but there is an infinite number of values within that continuous range.
  - Discrete signal represents data as a sequence of discrete values.

Heaviside step function

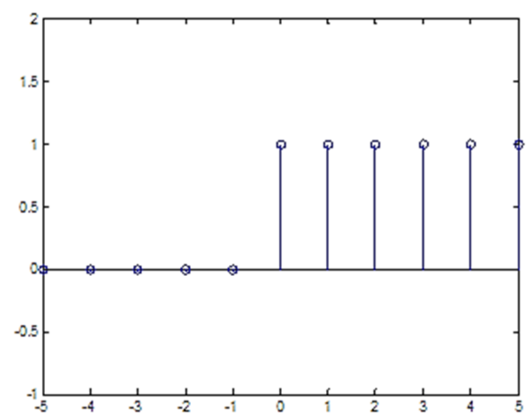
- In the continuous form, Heaviside function can also be considered as the integral of the Dirac delta function:

$$H(t) = \int_{s=t}^{+\infty} \delta(s)\, ds$$

- In the discrete form, Heaviside step function, can be defined as:

$$H[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$

where n is an integer.



Ramp function

- The ramp function is a unary real function, whose graph is shaped like a ramp.
- It can be expressed as, "0 for negative inputs, output equals input for non-negative inputs".

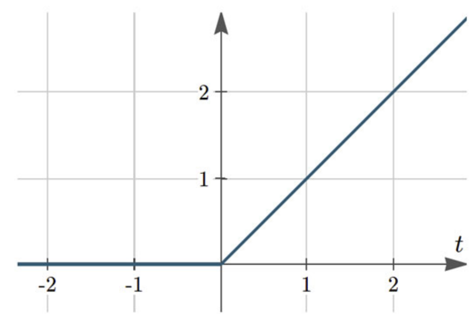
$$R(t) = \begin{cases} 0, & t < 0 \\ t, & t \geq 0 \end{cases}$$

- Alternatively, the Heaviside step function multiplied by a straight line with unity gradient is ramp function:

$$R(t) = t \cdot H(t)$$

- Alternatively, integral of Heaviside is ramp function,

$$R(t) = \int_{-\infty}^{t} H(\tau)\, d\tau$$



Fourier Transform

- The Fourier Transform is a tool that breaks a waveform (a function or audio data) into an alternate representation, characterized by the sine and cosine functions of varying frequencies.
- The Fourier Transform shows that any waveform can be rewritten as the sum of sinusoidal waves.
- Also, Fourier Transform is the mathematical procedure connecting $x(t)$ and $X(\omega)$.
  - If $x(t)$ is specified, $X(\omega)$ may be computed, and vice versa.
  - The equations require some knowledge of complex numbers and calculus to make sense.

Fourier Transform: Applications

- Performing non-invasive remote patient monitoring to detect events.
  - Detection and classification of cardiovascular abnormalities.
  - Analysis of coughing, sneezing, snoring, and other sounds can facilitate pre-screening, identifying a patient's status, assessing the infection level in public spaces, and so on.
- Data compression for IoT communication.
  - Achieving image, sound, and video compression by keeping few coefficients and neglecting others.
- Environmental sound recognition
  - Detection and classification of vibrations/machine sound and facilitate predictive maintenance to monitor equipment health and prevent costly failures.
- Data cleaning.
  - Elimination of noise from the data.
- Speech and voice recognition.
  - Bird species recognition using template matching.
- Demand forecasting in a company.

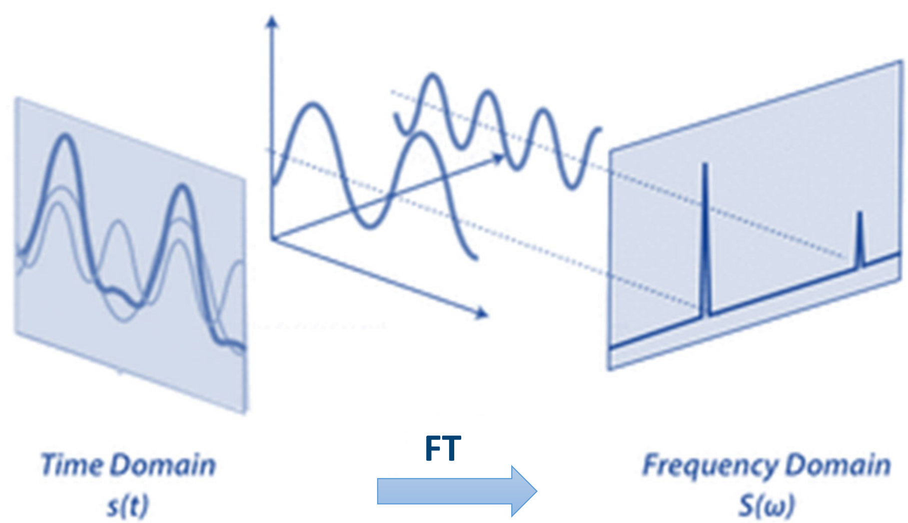
Fourier Transform

- Mapping time to frequency is Fourier Transform.

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

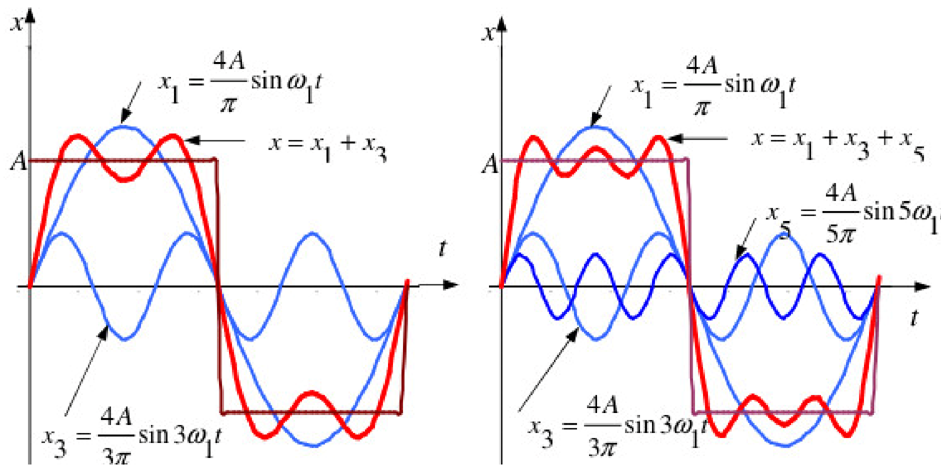
- Mapping frequency to time is Inverse Fourier Transform.

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

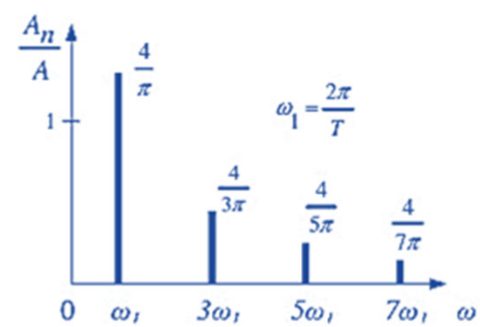


Fourier Transform: Square Wave

- For instance, a periodic signal that is in the form of a square waveform can be represented as an infinite trigonometric sequence that includes odd harmonics (1 3, 5, 7, …) with decreasing magnitudes.

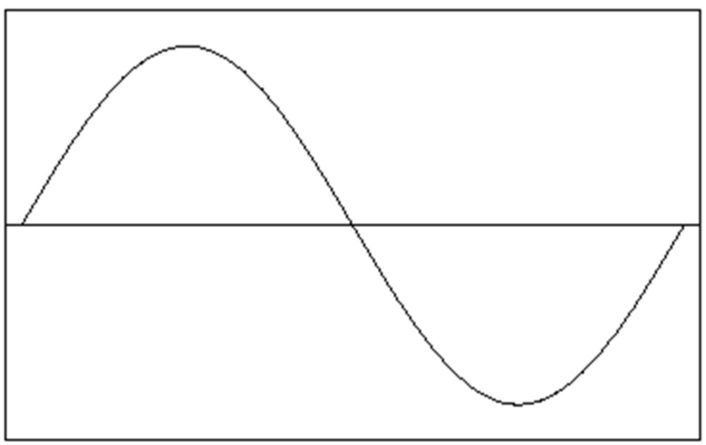


$$x(t) = \frac{4A}{\pi}(\sin\omega_1 t + \frac{1}{3}\sin\omega_3 t + \frac{1}{5}\sin\omega_5 t + \cdots)$$



Fourier Transform: Square Wave

- Visualising square wave: The Fourier Transform decomposes a waveform - basically any real-world waveform, into sinusoids. That is, the Fourier Transform gives us another way to represent a waveform.



Fourier Transform

- Calculate the Fourier transform of the function:

$$x(t) = \begin{cases} 1 - 2t & 0 \leq t \leq 1/2 \\ 0 & \text{otherwise} \end{cases}$$

- Solution: Use $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

$$X(\omega) = \int_{-\infty}^{\infty} (1 - 2t) e^{-j\omega t} dt$$

$$X(\omega) = \int_{0}^{1/2} (1 - 2t) e^{-j\omega t} dt = \frac{j\omega(2t - 1) + 2}{(j\omega)^2} e^{-j\omega t} \Bigg|_{t=0}^{1/2}$$

$$= \frac{2 - j\omega - 2e^{-j\omega/2}}{\omega^2}$$

Euler's formula

- Euler's formula in complex analysis establishes the fundamental relationship between the trigonometric functions and the complex exponential function.
- Euler's formula states that for any real number φ:

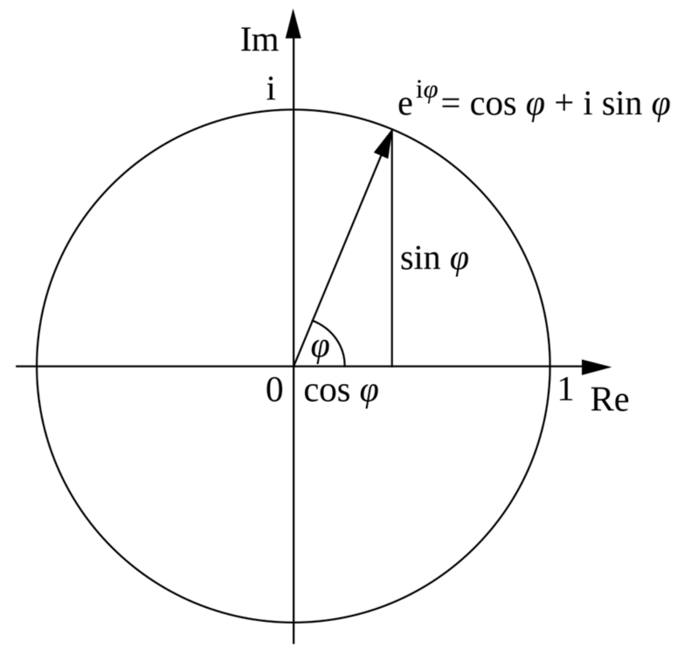
$$e^{i\varphi} = \cos(\varphi) + i\ sin(\varphi)$$

- Relations between cosine, sine and exponential functions:

$$e^{\pm i\varphi} = \cos(\varphi) \pm i\ sin(\varphi)$$

$$\cos(\varphi) = \frac{1}{2}\left(e^{i\varphi} + e^{-i\varphi}\right)$$

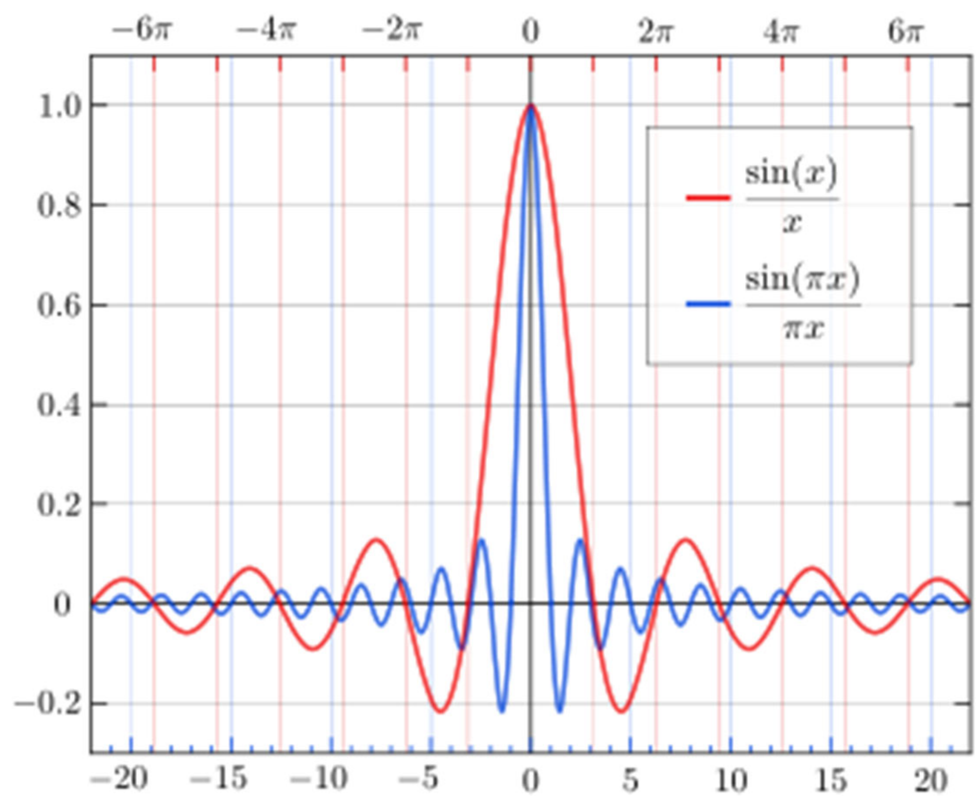
$$\sin(\varphi) = \frac{1}{2i}\left(e^{i\varphi} - e^{-i\varphi}\right)$$



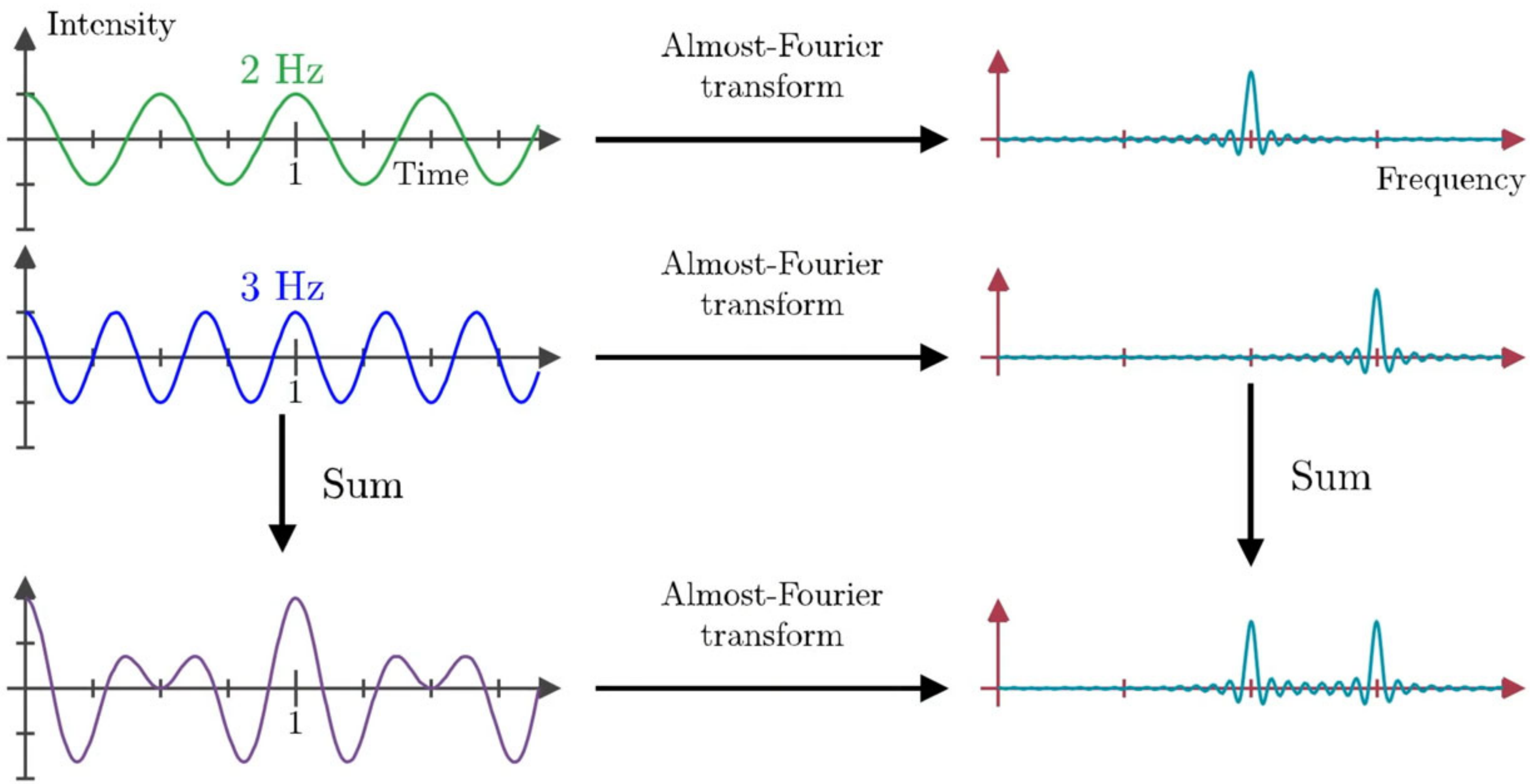
Sinc Function

- $\text{sinc}(\pi x) = \dfrac{\sin(\pi x)}{\pi x}$

- $\text{sinc}(x) = \dfrac{\sin(x)}{x}$



Fourier Transform



Fourier Transform: Sound Editing