Integration

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What is integration?

- Quite simply , it is the opposite of Differentiation. If differentiation calculates the slope or the rate of change at any given point, then integration calculates the area under a curve or a line.

Rules for Integrals

Power Rule

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad n \neq -1$$

$$\int x^{-1} dx = \ln|x| + C$$

Exponential

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

Constant Multiples

$$\int kf(x)dx = k\int f(x)dx$$

Absolute Value

$$\int |x| dx = \frac{x|x|}{2} + C$$

Sums and Differences

$$\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$$

$$\int [f(x) - g(x)]dx = \int f(x)dx - \int g(x)dx$$

Integrals of Polynomials

- Given a polynomial term of the form ax^b (where a is a constant coefficient and b is a constant exponent), to perform an indefinite integral over the variable x, increase the exponent to b+1 and then divide by b+1 according to the following formulas, where c is a constant of integration:

$$\int ax\hat{}b \quad \mathrm{dx} = \frac{\mathrm{ax}^{\mathrm{b+1}}}{b+1} \quad + \mathrm{c}$$

$$\int ax^{-1}\, dx = \int \frac{a}{x}\, \mathrm{dx} = \mathrm{a} \ln \mathrm{x} + \mathrm{c}$$

Example

How do you evaluate the integral $\int x^3 + 4x^2 + 5dx$?

Because this equation only consists of terms added together, you can integrate them separately and add the results, giving us:

$$\int x^3 + 4x^2 + 5dx = \int x^3 dx + \int 4x^2 dx + \int 5dx$$

Each of these terms can be integrated using the Power Rule for integration, which is:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

Plugging our 3 terms into this formula, we have:

$$\int x^3 dx = \frac{x^{3+1}}{3+1} = \frac{x^4}{4}$$

$$\int 4x^2 dx = \frac{4x^{2+1}}{2+1} = \frac{4x^3}{3}$$

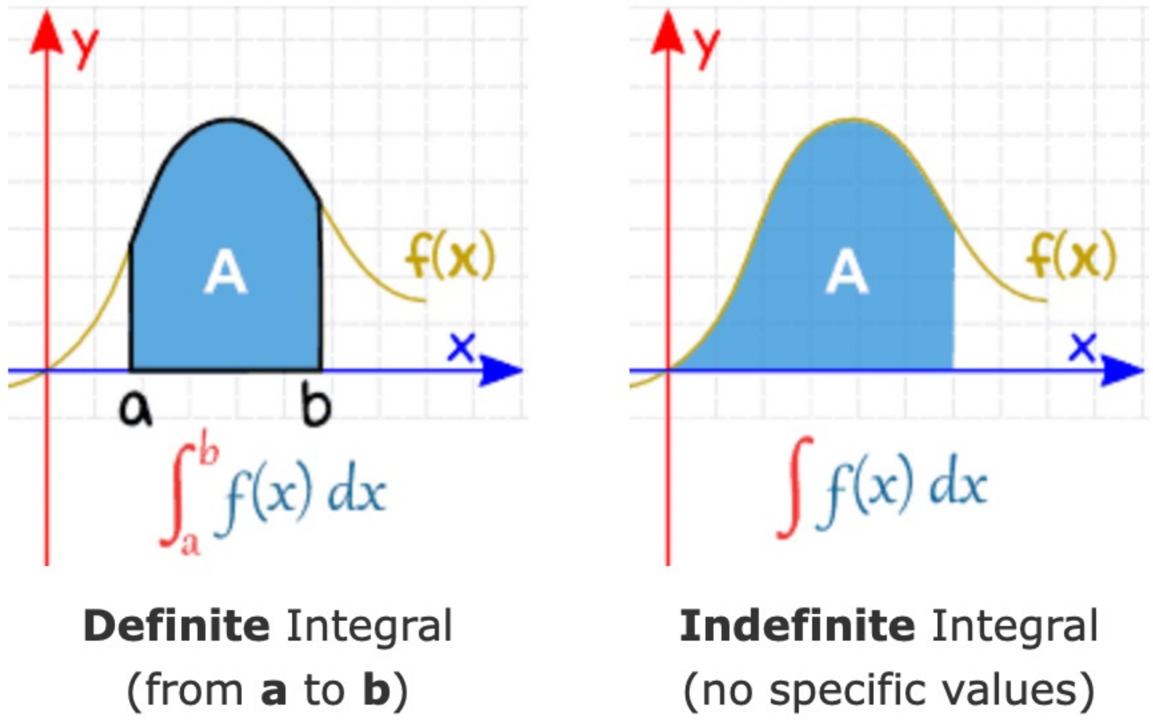
$$\int 5dx = \int 5x^0 dx = \frac{5x^{0+1}}{0+1} = \frac{5x^1}{1} = 5x$$

Now we arrive at our final answer by adding these together, remembering to add our constant (C) on the end:

$$\int x^3 + 4x^2 + 5dx = \frac{x^4}{4} + \frac{4x^3}{3} + 5x + C$$

Definite Integrals

- Definite Integral includes limits, specifying an interval (a,b).



Rules for definite Integrals

(a) $\int_a^a f(x)dx = 0$

(b) $\int_a^b kf(x)dx = k\int_a^b f(x)dx,\ k =$ *constant*

(c) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

(d) $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$

(e) $\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

How do you evaluate the integral $\int_0^4 x^3 + 2x^2 - 8x - 1$?

First you integrate the function:

$$\int x^3 + 2x^2 - 8x - 1 = \frac{1}{4}x^4 + \frac{2}{3}x^3 - 4x^2 - x$$

Then you substitute in your values for the upper and lower bounds. Start with 4:

$$\frac{1}{4}(4)^4 + \frac{2}{3}(4)^3 - 4(4)^2 - 4 = \frac{1}{4}(256) + \frac{2}{3}(64) - 4(16) - 4$$

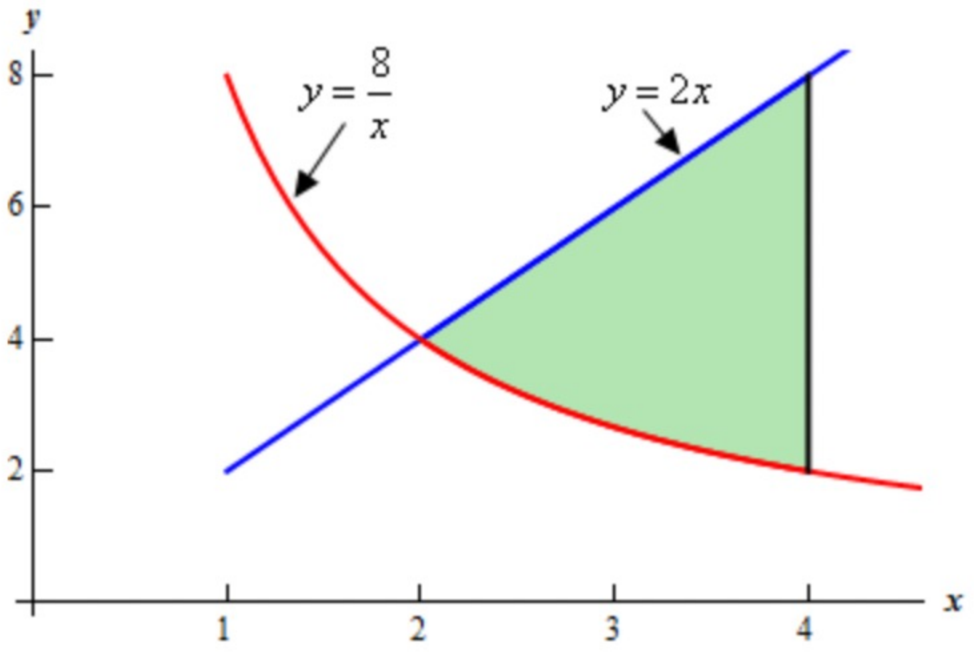
Solving that out yields:

$$64 + \frac{128}{3} - 64 - 4 = \frac{116}{3}(\text{ or } 38.66666)$$

Next you would substitute in 0, but looking at the equation, you can see that subbing 0 in will just yield zero. So last you do $\frac{116}{3} - 0$, which of course is just $\frac{116}{3}$, and that's your answer.

Area bounded by a line and curve

- Determine the area of the region bounded by $y = \frac{8}{x}$, y=2x and x=4



- The area is given by

$$A = \int_{2}^{4} 2x - \frac{8}{x}\, dx = \left(x^2 - 8\ln|x|\right)\Big|_2^4 = \boxed{12 - 8\ln(4) + 8\ln(2) = 6.4548}$$