University of Roehampton London

Mathematics for Data Science	2022/23

Week-2
Introduction to Probability

Answer the following Multiple-Choice Questions

1. A collection of all elementary results, or **outcomes** of an experiment, is called

a. Sample space
b. Event
c. Probability

Answer : (a) Sample Space

2. What will be the probability of getting odd numbers if a dice is thrown?

a. 1/2

b. 2

c. 4/2

d. 5/2

Answer : (a) 1/2

Explanation: The sample space when a dice is rolled, S = (1, 2, 3, 4, 5, and 6)

So, n (S) = 6

E is the event of getting an odd number.

So, n (E) = 3

Probability of getting an odd number P (E) = Total number of favourable outcomes / Total number of outcomes

n(E) / n(S) = 3/6 = ½

3. The probability of getting two tails when two coins are tossed is -

a. 1/6
b. 1/2
c. 1/3
d. 1/4

Answer: (d) 1/4

Explanation: The sample space when two coins are tossed = (H, H), (H, T), (T, H), (T, T)

So, n(S) = 4

The event "E" of getting two tails (T, T) = 1

So, n(E) = 1

So, the probability of getting two tails, P (E) = n(E) / n(S) = 1/4

4. What will be the probability of losing a game if the winning probability is 0.3?

a. 0.5
b. 0.6
c. 0.7
d. 0.8

Answer: (c) 0.7

Explanation: Let P(E) is the probability of winning the game, and P(not E) be the probability of not winning the game.

P(E) + P(not E) = 1

So, P(not E) = 1 - P(E)

Since P(E) = 0.3

P(not E) = 1 - 0.3

P(not E) = 0.7

Solve the following problems

1. If a system appears protected against a new computer virus with probability 0.7, then what is the probability that it will be exposed to it?

Answer: The probability that it will be exposed to it is:

$$1 - 0.7 = 0.3.$$

2. Suppose that a computer code has no errors, has the probability of 0.45, then what is the probability that the computer code has atleast 1 error?

Answer : The probability that the computer code has atleast 1 errors is

$$1 - 0.45 = 0.55$$

3. Ninety percent of flights depart on time. Eighty percent of flights arrive on time. Seventy-five percent of flights depart on time and arrive on time.

- (a) You are meeting a flight that departed on time. What is the probability that it will arrive on time?
- (b) You have met a flight, and it arrived on time. What is the probability that it departed on time?
- (c) Are the events, departing on time and arriving on time, independent?

Answer: Denote the events,

$$A=\{\text{arriving on time}\},D=\{\text{departing on time}\}.$$

We have:

$$P\{A\}=0.8,P\{D\}=0.9,P\{A\cap D\}=0.75.$$

(a) $P\{A|D\}=P\{A\cap D\}P\{D\}=0.750.9=0.8333\_.$

(b) $P\{D|A\}=P\{A\cap D\}P\{A\}=0.750.8=0.9375\_.$

(c) Events are not independent

4. Given two fair dices, what is the probability that two dices sum to 8? What is the probability that two dices sum to 8 when the first dice is 3?

Answer: Bayes theorem and conditional probability

The difference between P(AB) and P(A|B) is that:

- P(AB) is 1/36: out of 36 outcomes, only (3,5) both satisfy event A and event B;
- P(A|B) is 1/6: out of 6 outcomes from event B, (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), only one outcome sums to 8 at (3,5), so that P(A|B) is 1/6. (also can be calculated by 1/36 / 1/6 = 1/6)

5. 50% of all people who receive a first interview receive a second interview; 95% of your friends that got a second interview felt they had a good first interview; 75% of your friends that DID NOT get a second interview felt they had a good first interview. If you feel that you had a good first interview, what is the probability you will receive a second interview?

Answer: The key to solving problems like this is to define the events carefully. Suppose your friends are a good representation of the entire population:

- Let's define feel good about the first interview as event A and define receive the second interview event B;
- "50% of all people who receive a first interview receive a second interview" means that P(B)=0.5, thus P(not B) is one minus P(B), which is 0.5 as well;
- "95% of your friends that got a second interview felt they had a good first interview" means P(A|B) =0.95;
- "75% of your friends that DID NOT get a second interview felt they had a good first interview" means P(A|not B) = 0.75.

- The question is asking given P(B), P(A|B), P(A|not B), what is P(B|A)? (If you feel that you had a good first interview, what is the probability you will receive a second interview?)

According to Bayes' theorem:

$$P(B|A) = \frac{P(A|B) * P(B)}{P(A)} = \frac{P(A|B) * P(B)}{P(A|B) * P(B) + P(A|notB) * P(notB)}$$

thus:

$$P(B|A) = \frac{0.95 * 0.5}{0.95 * 0.5 + 0.75 * 0.5} = \frac{19}{34}$$