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Dynamic Core Theory and Its Application in Traffic Management and Control

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Abstract: For the goal of realizing to control urban road traffic flow to reduce congestion and obstruction, and perfecting the theories of the dynamic core and its application in the traffic management and control, this paper proposes the concepts of the dynamic core and coritivity of the dynamic flow network system, and the method of calculating the core and coritivity of the directed connected network system. According to this graph theory, this paper further proposes the simple methods of calculating the dynamic core and coritivity of the traffic network system. Finally, this paper uses numerical examples to illustrate the application of calculating the dynamic core in the traffic management and control.

Key Words: System Engineering, Core and Coritivity, Dynamic Core, Traffic Control, Traffic Network System

1 Introduction

Whether in natural or in social reality network there is always occupied an important position or more nodes or parts, these parts of these nodes if the blockage occurs, will lead to such stability, operational efficiency, even in the runs of them to be affected. To determine the importance of these nodes or parts, the early 20th century 90Dec, XU Jin Et al proposed the concept of core and coritivity [1]. Coritivity is a measure of the connectivity level of core in network , is usually not variables, it reflects the role of core in the system or related degree.

Urban traffic network system is very important to human life, touch upon not only the lives of people living and working, but also closely linke with the social economy. With road traffic increases year by year, road traffic network system are bound to be crowded or blocked. Because of traffic congestion or obstruction often occurred on the road intersection, the road junction points of the road traffic network are very important, however these road junction points usually are the core we called of the road traffic network. If we know these nodes which are the core of the road traffic system, then install the detectors on that, to monitor and control road traffic flows, then always maintain a smooth traffic flow on control, whether would be able to greatly reduce the possibility of road traffic congestion or blocked occurred? Answer is clearly yes.

In large and medium cities in China the intersections where installed detectors are less, the account installed the detector in some cities is less than one-tenth of all the intersections. Therefore, to find places in which to need install the detector or the core detection method is necessary. When amounts of core and coritivity of network systems are not change on time, we can call them the static core and coritivity, can not be used to solve the urban road traffic system management and control issues. Because urban road traffic system is a dynamic system, the calculation of core and coritivity of the road network it is not only associated with the connectivity degree of the road network, but also to the dynamic changes in traffic flow on time, so it is necessary to put forward the concept of dynamic core. Urban traffic network system is typically a dynamic flow directed network system, its dynamic calculation of the core and coritivity is representative.

Based on the static core and coritivity theory, in this paper a dynamic core and coritivity calculation methods for the directed network system and their algorithms are given, used them could find places in which to need install the detector, and achieved the purpose to control dynamic flow in urban traffic network system.

This paper is organized as follows. In section 2, the definition and calculation method of the static core and static coritivity of directed flow network are proposed, include the problem of the calculation for the coritivity of the directed connected graph can be transferred to the issues calculated the rank of the full correlation matrix of the sub-graph. In section 3, the dynamic coritivity calculation method of dynamic directed flow network system is present. In section 4, Coritivity calculation method of road network system is present. In section 5, Algorithm for calculation of dynamic core and dynamic coritivity is given. In section 6, an example is given to show the practicability of the dynamic core theory application in the traffic management and control. Conclusions are presented in section 7.

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2 Definition and Calculation of the Static Core and Coritivity Of Directed Network

Definition 1: Suppose $\overline{G}=\left(\overline{V},\overline{E}\right)$ is the connected directed graph of a network system $\overline{\Sigma}$. Where $\overline{V}=\{v_1,v_2,\cdots,v_n\}$ is the node set, which includes three sub-sets: the starting point set $\overline{A}=\{v_i \mid$ the starting point v_i of the existence edge, but v_i is not end point of any edges$\}$, the end point set $\overline{B}=\{v_j \mid$, the end point v_j of the existence edge, but v_j is not starting point of any edges $\}$, and the intermediate point set $\overline{D}=\{v_j \mid$ the end point v_j of the existence edge, but v_j is also starting point of some edges $\}$, where the 3 point sets have not intersects. Where $\overline{E}=\{e_{ij} \mid e_{ij}$ is the edge of the starting point v_i and the end point v_j $\}$.The coritivity $h\left(\overline{G}\right)$ of a network system $\overline{\Sigma}$ is calculated as follows,

$$h\left(\overline{G}\right)=\max\left\{\omega\left(\overline{G}-\overline{S}\right)-\left|\overline{S}\right| \,\middle|\, \overline{S}\in C\left(\overline{G}\right)\right\} \quad (1)$$

Where $C\left(\overline{G}\right)$ is a semi-cut point set of all sets for the separation between $\overline{A}$ and $\overline{B}$, $\omega\left(\overline{G}-\overline{S}\right)$ is the number of branches of the maximal connected sub-graph $\overline{G}-\overline{S}$, $\left|\overline{S}\right|$ is the number of the nodes of $\overline{S}$. If there is a semi-cut point set $\overline{S^*}$, meet $\omega\left(\overline{G}-\overline{S^*}\right)-\left|\overline{S^*}\right|=h\left(\overline{G}\right)$, then $\overline{S^*}$ is called the system's core.

2.1 The Calculation of Rank of Directed Graph [3]

Definition 2: For a directed graph $\overline{G}$, the nodes number, the edges number, and the number of maximal connected sub-graph are denoted by respectively n 、 m 、 r. If an order $n\times m$ matrix is denoted by $A_e=\left\lfloor a_{ij}\right\rfloor$,where

$$a_{ij}=\begin{cases}1 & e_{ij}\in\overline{E}\\ -1 & e_{ji}\in\overline{E}\\ 0 & e_{ij}\notin\overline{E}\ and\ e_{ji}\notin\overline{E}\end{cases}$$

,then $A_e=\left\lfloor a_{ij}\right\rfloor$ is called the full correlation matrix of directed graph $\overline{G}$.

Definition 3: The difference between the number of nodes in a directed graph and the number of its maximal connected sub-graph is called the rank of the directed graph, denoted by $\gamma\left(\overline{G}\right)=n-r$.

Theorem 1: The rank of the full correlation matrix A_e equal to the difference between the number of nodes and the number of maximal connected sub-graph, that is $\gamma\left(A_e\right)=n-r$.

Proof (see [3]).

According to Theorem 1 can be concluded: the rank of the directed graph equal to the rank of the full correlation matrix.

2.2 Simple Calculation of Static Coritivity of Directed Connected Graph

Theorem 2: If $C\left(\overline{G}\right)$ is a semi-cut point set of all sets for the separation between $\overline{A}$ and $\overline{B}$ of the connected directed graph $\overline{G}$, and $\overline{S}\in C\left(\overline{G}\right)$, and $\left|\overline{S}\right|$ is the number of elements of the set $\overline{S}$,and n is the number of nodes of the connected directed graph, then the coritivity of the directed connected graph is simply calculated as follows,

$$h\left(\overline{G}\right)=n-\min\left\lfloor\gamma\left(\overline{G}-\overline{S}\right)+2\left|\overline{S}\right|\right\rfloor \quad (2)$$

Proof:
By Theorem 1,

$$\omega\left(\overline{G}-\overline{S}\right)=\left|\overline{G}-\overline{S}\right|-\gamma\left(\overline{G}-\overline{S}\right)=n-\left|\overline{S}\right|-\gamma\left(\overline{G}-\overline{S}\right),$$

So, $h\left(\overline{G}\right)=\max\left(n-\left|\overline{S}\right|-\gamma\left(\overline{G}-\overline{S}\right)-\left|\overline{S}\right|\right)$

$$=n-\min\left(\gamma\left(\overline{G}-\overline{S}\right)+2\left|\overline{S}\right|\right).$$

By Theorem 1, Theorem 2 can be known that the problem of the calculation for the coritivity of the directed connected graph can be transferred to the issues calculated the rank of the full correlation matrix of the sub-graph $\overline{G}-\overline{S}$.

3 Directed Dynamic Flow Network and Its Dynamic Coritivity

The so-called dynamic flow network system, is such a system as each edge has the direction of flow, and flow rate change with time.

Let Σ^* be a dynamic flow network system, $G^*(t)=\left(V^*(t),E^*(t)\right)$ is the connected directed diagram of the system Σ^*, and its edge set $E^*(t)=\{e_{ij} \mid$ The edge e_{ij} connected between a_i and a_j, and the positive flow on edge e_{ij} is greater than parameter α at time $t\}$ is a variable set, and its node set $V^*(t)=\{v_i \mid$ At least one edge e_{ij} connected to v_i at time $t\}$.

Definition 4: For any node $v_i\in V^*(t)$, $f^+(v_i)$ 、 $f^-(v_i)$ respectively expressed as the outflow and inflow of traffic nodes v_i; If the outflow $f^+(v_i)$ is more than parameter α, and the inflow $f^-(v_i)$ is less than parameter α, then v_i is called the source point; If the outflow $f^+(v_i)$ is less than parameter α, and the inflow $f^-(v_i)$ is more than parameter α, then v_i is called the

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convergence point; If the outflow $f^+(v_i)$ is more than parameter α , and the inflow $f^-(v_i)$ is more than parameter α , then v_i is called intermediate point; Obviously, they are non-negative integer-valued functions. Denoted the set of all source points by $K(t)$, the set of all the convergence points by $\overline{K}(t)$.

Definition 5: Suppose $G^*(t)=(V^*(t),E^*(t))$ to the n-order dynamics connected directed graph of the system Σ^* , the coritivity of this graph is calculated as follows,

$$h(G^*(t))=\max\{\omega(G^*(t)-S(t))-|S(t)||S(t)\in C(G^*(t))\} \quad (3)$$

Where $C(G^*(t))$ is a semi-cut point set of all sets for the separation between $K(t)$ and $\overline{K}(t)$, $\omega(G^*(t)-S(t))$ is the number of branches of the maximal connected sub-graph $G^*(t)-S(t)$, $|S(t)|$ is the number of the nodes of $S(t)$. If there is a semi-cut point set $S^*(t)$, meet $\omega(G^*(t)-S^*(t))-|S^*(t)|=h(G^*(t))$, then $S^*(t)$ is called the core of the system Σ^* .

4 Coritivity Calculation Method of Road Network System [4-9]

The road intersections and sections of the urban traffic network can be abstracted to the nodes and edges of the road network graph, and according to the relationship between the intersection and the section, the dynamics connected directed graphs of the system have been formed. In this directed graph, the urban traffic network system is a dynamic flow network system. Because the road network system, traffic is directional, and change on time, especially on early, middle and late time sections the vehicle flow rates vary widely, the running line which vehicles selected satisfy the user equilibrium conditions with saturation changes at different time change. Therefore, the urban traffic network system is a dynamic flow network system. It corresponds with the time-varying network graph to be a dynamic graph, the coriivity of the graph corresponding to the dynamic coriivity, its core corresponding to the dynamic core.

Let Σ^* be a road network system, it corresponds to a network diagram directed graph $G^*(t)=(V^*(t),E^*(t))$, and its edge set $E^*(t)=\{e_{ij}|$ The edge e_{ij} connected between a_i and a_j , and the positive flow on edge e_{ij} is greater than parameter α at time $t\}$ is a variable set, and its node set $V^*(t)=\{v_i|$ At least one edge e_{ij} connected to v_i at time $t\}$,and the set of all source points is $K(t)$, the set of all the convergence points is $\overline{K}(t)$,and $C(G^*(t))$ is a semi-cut point set of all sets for the separation between $K(t)$ and $\overline{K}(t)$, then the coritivity of this graph is calculated as follows,

$$h(G^*(t))=n(t)-\min[\gamma(G^*(t)-S(t))+2|S(t)|] \quad (4)$$

It can be known from above formula (4) that the problem of the calculation for the dynamic coritivity of the directed connected graph of the urban traffic network can be transferred to the issues calculated the rank of the full correlation matrix of the sub-graph.

5 Algorithm for Calculation of Dynamic Core and Coritivity

Here is the computational algorithm of dynamic core and dynamic coritivity of the traffic network.

(I) Initialization: one day divides into H periods, and let $t=1$, and define parameter α .

(II) If the one-way traffic flows of some edges are greater than parameter α , then let these edges into the set of directed edges $E^*(t)$, and let the nodes associated with these edges into the nodes set $V^*(t)$, after that the dynamic network graph at time t can be determined to $G^*(t)=(V^*(t),E^*(t))$. Thus determine the source points set $K(t)$, the convergence points set $\overline{K}(t)$, the semi-cut point set $C(G^*(t))$of all sets for the separation between $K(t)$ and $\overline{K}(t)$, and let $M(t)=|C(G^*(t))|$, the semi-cut point subset $S_k\in C(G^*(t)), k=1,2,\cdots,M$.

（III）Let

$$MI1(t)=\min\{\gamma(G^*(t)-S_k(t))+2|S_k(t)||k=1,2,\cdots,M(t)\}$$

,and the semi-cut point subset $S_k(t)$ satisfied this condition is marked as $S_k^*(t)$.

（IV）Let $MI2(t)=|S_k^*(t)|$,and the $MI3(t)=|V^*(t)|-MI1(t)$ is called the coritivity of the dynamic network graph $G^*(t)$ at time t.

（V）Let

$$MA(t)=\max\Big\{\sum_{v_i\in S_k(t)}f^+(v_i)\Big||S_k(t)|=MI2(t),$$
$$\gamma(G^*(t)-S_k(t))+2|S_k(t)|=MI1(t),\quad k=1,2,\cdots,M\Big\}$$

and the semi-cut point subset $S_k(t)$ satisfied this condition is called the core of the dynamic network graph $G^*(t)$ at time t.

(VI) Let $t=t+1$; If $t\le H$, then the switch (II); otherwise transfer (VII).

(VII) End.

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6 Application

For a city road network (see Figure 1) of one city, we illustrate application the dynamic core and the dynamic coritivity in urban road traffic network management and control. Nodes following data into and out of traffic of cases, because they do not affect the description, it is omitted.

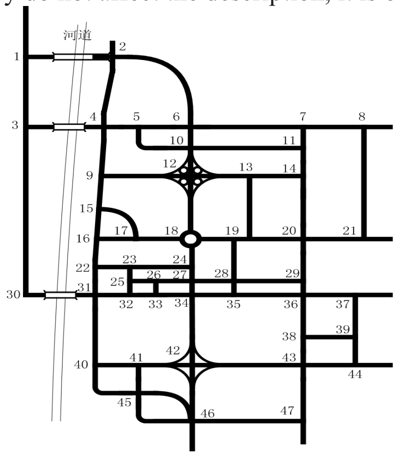

Fig 1: a city road network

One day divides into 3 periods. At early morning, the source point set includes nodes: 12,18,31,34, the convergence point set includes nodes: 1,3,8,14,21,30,37,44,46,47, by calculating from the computational algorithm of dynamic core and dynamic coritivity of the traffic network , we obtain : its coritivity is 4, and its core is these nodes 6,9,12,20,31,34,42. So intersection nodes need to monitor in real time should be included 4,9,12,20,31,34,42. At noon, can determine the source point set and the convergence point set of dynamic network graph, will be regarded as two-way map, determine the set of partitioning points set, and from the algorithm gain its coritivity 2, real-time monitoring the intersection at noon should be included 4,12,18,28,32,34.

At dusk the source point set includes nodes: 1,2,8,24,30,37,44,46,47, the convergence point set includes nodes: 12,18,31,34, determine the set of partitioning points set, and from the algorithm gain its coritivity 4, and its core is these nodes 4,6,12,14,16,19,20,24,25,28,31,34, 36,42,should be real-time monitored for the core intersection.

Seen from the above examples, at different times the dynamic flow network has the different dynamic core, and its coritivity is also different. The traffic at dusk is maximum, and its coritivity maximum; noon traffic is minimum, also the smallest of its coritivity; the traffic is less the morning time than dusk, but the coritivity not less. Examples show: the change of coritivity of traffic network not only associated with silent network structure and dynamic flow, but also with time-varying traffic network, so their relationship is nonlinear.

At early morning and dusk, these road traffic junctions in the cores are nuclear parts in the road traffic network. and they have higher coritivity; but at noon the important of the central core junctions is not obvious, and its coritivity smaller; indicating that its role of core in the dynamic flow networks do have a response to dynamic coritivity at different times, and the importance from the cores can be seen different at different times, and the critical intersection of the road transport network is different at different times. Therefore, in real-time monitoring in the daily urban traffic network, control key is not immutable, should be changeable day, and the important of the core and coritivity calculation in different periods shows very obvious. The core and coritivity calculation method of the dynamic flow network can solve the nuclear intersection calculation in the urban traffic network, which can be used to monitor road traffic in order to control traffic flow to reduce congestion and obstruction.

7 Conclusion

Based on the static calculation of core and coritivity for non-directed connected network diagram, a calculation method of dynamic core and coritivity of a dynamic flow connected network graph is proposed, which can be used to solve core intersection computing problem for the urban traffic network, which can be used to monitor road traffic congestion and obstruction. In real life, the dynamic flow network systems like this urban road traffic network have a lot of , such as data communications network, electricity supply network, logistics networks and so on, which are related to the calculation issues of the dynamic key nodes, indicating that the calculation of dynamic core and coritivity for a dynamic flow network is very importance. In this paper, the dynamic core and coritivity calculation method of the dynamic flow network system is playing a certain reference role.

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