

Winter Semester 2016/2017

Introduction to Cooperative Game Theory (50168 – 2SWS)

1. Description and objectives

Like political philosophy, one can broadly think of cooperative game theory (CGT) as providing answers to two fundamental questions: (1) Who ought to get what? and (2) Who ought to say who ought to get what? Given a group of agents who wish to reap the benefits of cooperation, question one outlines an *allocation problem*: that is to say, how should these benefits be distributed among some or all of the agents. Question two, on the other hand, points to a *coalition formation problem*: in other words, which groups of agents will end up cooperating or not, thus determining the allocation of benefits.

Unlike political philosophy, the normativity of the ‘ought’ in these questions is of rational, rather than moral, nature. Different conceptions of ‘rational play’ supply different criteria for solving the two problems. Should the allocation of benefits map onto the distribution of the agents’ power, as conceived in some way? Or should it follow a pattern of fairness, such as merit? If rational agents join a coalition together so as to determine an allocation, when will it be and in what sense can we say that this coalition will be ‘stable’, i.e. that *this* (or *these*) will be the coalition(s) that will solve the allocation problem?

In this course we will get acquainted with what CGT has to offer towards answering these questions. We will open and close the course with two prominent solutions – the core and the Shapley value, respectively – which address question one above. In-between, we will explore three alternative concepts – the bargaining set, the kernel and the nucleolus – tackling both the problem of allocation and of coalition formation.

The aim of the course is to **not** just teach you a bag of scattered tools but to help you understand what the purpose of these tools is, what they are made of, how they are related and how they differ. The only way of achieving this is by repeatedly applying the tools. Hence, the course has a heavily applied focus which means that, consistently throughout the semester, you will be asked to solve exercises both in and outside class.

Completing the course successfully implies being able to demonstrate that you know how to answer the following two questions:

1. COMPREHENSION: What conceptions of ‘rational play’ drive the normative intuitions behind each of the five solutions to the two problems?
2. APPLICATION: What are these five solution concepts and how can you use them to solve specific problems?

2. Structure

Given these objectives, the course will consist of two types of sessions: lecture and exercise sessions. Lecture sessions introduce new material: in the first half we will get acquainted with a solution concept, while in the second half we will solve together some simple exercises that illustrate how it functions. Every lecture session is followed by an exercise session where we will work on more complex applications of the solution concept to a particular context. During both lecture and exercise sessions you will be asked to work for a few minutes in groups on some problem, after which one or more of you will try to work through it on the blackboard. All these in-class exercises are *ungraded* and *not* part of the assessment so don’t be afraid or shy to take advantage of them.

3. Assessment

Philosophy & Economics (BA) students can take the course as a P3 or P9 seminar for 2CP, 4CP (GdE II) or 6CP. Economics (BA) students can take the course under ‘Individueller Schwerpunkt’ for 5CP. The following table summarises the elements corresponding to each of these options:

	1: Attendance	2: Homework	3: Exam
2CP:	Attendance (30%)	A weekly homework (one exercise) (70%)	—
4CP:	Attendance (20%)	A weekly homework (one exercise) (40%)	A take-home exam (40%)
5CP/ 6CP:	Attendance (20%)	A weekly homework (two exercises) (40%)	A (longer) take-home exam (40%)

To pass the course, you need to pass *each* of the respective CP elements. Failing to pass any of them would result in getting no credits for the course.

ATTENDANCE

Attendance is compulsory. You have the right to be absent at most twice. That is to say, passing the ‘Attendance’ element implies being present for at least 13 out of all 15 sessions. During every session, I will keep attendance by asking you – at some point – to write down in two-three minutes a short answer to a pass/fail surprise question. You are not expected to give an elaborate or even correct answer, if there is any. These questions test attendance and attention. **An answer can bring you either a pass (1.0) or a fail (5.0).** Failure results out of not submitting an answer or not addressing the question. **If you pass this element, you will get a grade of 1.0 for Attendance.**

To take an example from the more familiar non-cooperative game theory, a surprise question during a lecture on the Nash equilibrium illustrated by the Prisoner’s Dilemma game could be: ‘What is the dilemma in the Prisoner’s

Dilemma?’ or ‘Why exclude dominated strategies?’ A pass answer would be *anything* that relates to the Prisoner’s Dilemma which shows that you were listening, if not understanding everything, in class (e.g. if you write ‘The Prisoner’s Dilemma is a two-person game’, that’s a pass answer). A fail answer would be a blank sheet or a submission saying ‘I want to go home’.

HOMEWORK

To pass the ‘Homework’ element, you need to submit *all* 13 weekly homeworks. This does not mean that your solutions need to be entirely correct or even complete – handing in an attempt to solve an exercise you had difficulties with counts as a submission. Homeworks are due a week after the respective session (see the schedule for deadlines). **Each homework is graded independently and can bring you a grade of anything between 1.0 and 5.0. The final grade for the Homework element is just a simple average of the separate grades.**

For more on what kind of exercises you can expect as well as some guidelines, see the front matter in the Book of exercises.

EXAM

The take-home exam will consist of both typical exercises that ask you for explicit solutions and open questions where you would need to reason about the material and come up with some informed argument. **Each exercise will be assessed independently and your final grade for the Exam element will be a simple average of the separate grades.**

The exam questions will be similar to the weekly homework questions – we will discuss it at more length later in the course.

Your final grade for the course will be a weighted average (according to the table above) of the grades for each respective element. If you are taking the course for 2CP (pass/fail), then all you need for a pass is to ensure that you get a final passing grade.

4. Deadlines

To ensure that we progress through the course at a good pace, deadlines will be enforced in the following manner. No penalty for being up to 24 hours late. Every extra 24 hours afterwards reduce your points with 10%. So if you are 11 days late, you will automatically get 0 points.

5. Language

The language of the course is English.

6. Prerequisites

There are no prerequisites for the course. All of you have some familiarity with non-cooperative game theory so the course will be introduced in reference to that. Furthermore, we will try to make some connections between conceptions of rationality in CGT and some properties you would have encountered in classes on decision theory.

7. Suggested literature

You will receive lecture notes on each solution concept on the day of the respec-

tive lecture session. Hence, there is no single textbook that you need to consult. Nevertheless, if you wish to read up more on some subject or try out additional exercises, have a look at the week-by-week break-down where you will find further page sources for each topic.

Additionally, the following list contains – in an increasing order of difficulty – some books or chapters in books you might want to pick from:

ELEMENTARY

Gura and Maschler (2008): This is a very accessible book written for a general audience and requiring nothing but some elementary high school mathematics. It consists of four long, and somewhat simplified, applications of CGT concepts to specific problems. It is good bed-side reading material, particularly after getting acquainted with the respective topic in a more rigorous manner.

UNDERGRADUATE

Rapoport (1970): This is perhaps the best book to start from, particularly for P&E students – it discusses the core, the bargaining set, the kernel and the Shapley value in a way that matches precision with reflection. It is also rich in illustrating examples.

Maschler, Solan and Zamir (2013): Chapters 16–20 and 22 of this book cover the core, the bargaining set, the nucleolus and the Shapley value in an accessible but precise manner and are chock-full of valuable examples. One of your best strategies in the course would be to read these chapters alongside Rapoport (1970), perhaps supplemented by the relevant applications in Gura and Maschler (2008).

Osborne (2004): Chapter 8 of this book is a nice introduction to the core with a lot of examples. Unfortunately, it does not address the other solution concepts we will study in the course.

Kahan and Rapoport (1984): This is a nice book that covers all solution concepts in this course at a level that is accessible but still rigorous enough. It is slightly more technical than Rapoport (1970) but progresses slowly and has extensive critical discussions of the material. Another optimal strategy for you would be to substitute this book for Maschler, Solan and Zamir (2013) in the suggestion above.

Gilles (2010): The first half of this book is a very lucid presentation of the core and the Shapley value – it presents a lot of material that can be found in more advanced texts but at a level that is accessible and richly illustrated.

INTERMEDIATE

Binmore (2007): Chapter 18 of this book is an entertaining but precise discussion of the core and the Shapley value. Don't worry if you find some of the examples a bit too quick and not completely clear – try to work them out on your own before returning to Binmore's solutions.

Luce and Raiffa (1989): Chapters 8–12 include presentations of the core and the Shapley value. The book is rather outdated but its extensive critical discussion still offers invaluable food for thought. Again, following the examples may not at times be completely straight-forward.

Moulin (1988): This book covers the core, the nucleolus and the Shapley value. It is very rich in examples but could be a bit more technically demand-

ing at times.

Chakravarty, Mitra and Sarkar (2015): This book is a more technical exposition of all solution concepts in this course together with extensive applications.

ADVANCED

Owen (1995): Chapters 10–13 cover all solution concepts in this course at a graduate level.

Osborne and Rubinstein (1994): This is a standard graduate-level reference that covers all solution concepts in the course.

Peleg and Sudhölter (2007): This is another standard graduate-level reference for all solution concepts in the course and CGT, in general. Nevertheless, it is perhaps the most technically demanding book of all those listed above.

8. Questions

Feel free to approach me with any questions in class, by e-mail, during office hours or at other times (send me a note to arrange a time). Also, if you have any problems around the material, the exercises, the deadlines or anything to do with the course, just speak to me or send me a note. E-mail: marina1.uzunova@uni-bayreuth.de.

9. Class conduct and honesty

A word about electronic devices: you are free to use your laptops in class. However, you are heartily discouraged from doing so. There are at least two reasons for this. First, taking notes is done much more effectively – in terms of the cognitive process involved in this – with a simple pen and paper. Second, and more importantly, if you use your electronic devices on or off purpose, you decide to run the risk of being (perhaps constantly) distracted. At best, you won't be able to answer a surprise question. At worst, you won't grasp the respective material.

And a word about academic integrity: passing off someone else's work as your own (plagiarism) is wrong and a serious offence. You are evaluated on the basis of the knowledge and thinking you demonstrate through the work you submit. You are highly encouraged to discuss and work on the exercises in groups. However, simply copying an answer is never an optimal (and usually easily spotted) strategy. In short, if you solve an exercise or come up with an answer collectively, then each of you should write it down in his or her own words (see the Book of exercises for guidelines).

Schedule

Week	Date			Room	L	E	HW	LANGUAGE	OF CGT
01	19 Oct	Wed	16:00 – 18:00	S 106 (FAN C)	Introduction				
02	26 Oct	Wed	16:00 – 18:00	S 106 (FAN C)	L 01				
	02 Nov	Wed	16:00				HW 01	THREATS:	THE CORE
03	02 Nov	Wed	16:00 – 18:00	S 106 (FAN C)		E 01			
	09 Nov	Wed	16:00				HW 02		
04	09 Nov	Wed	16:00 – 18:00	S 106 (FAN C)	L 02				
	16 Nov	Wed	16:00				HW 03	CREDIBLE THREATS:	BARGAINING SET, KERNEL, NUCLEOLUS
05	16 Nov	Wed	16:00 – 18:00	S 106 (FAN C)		E 02			
	23 Nov	Wed	16:00				HW 04		
06	23 Nov	Wed	16:00 – 18:00	S 106 (FAN C)		E 03		FAIRNESS:	SHAPLEY VALUE
	30 Nov	Wed	16:00				HW 05		
07	30 Nov	Wed	16:00 – 18:00	S 106 (FAN C)	L 03				
	07 Dec	Wed	16:00				HW 06	CREDIBLE THREATS:	BARGAINING SET, KERNEL, NUCLEOLUS
08	07 Dec	Wed	16:00 – 18:00	S 106 (FAN C)		E 04			
	14 Dec	Wed	16:00				HW 07		
09	14 Dec	Wed	16:00 – 18:00	S 106 (FAN C)	L 04				
	14 Jan	Sat	midnight				HW 08	CREDIBLE THREATS:	BARGAINING SET, KERNEL, NUCLEOLUS
10	11 Jan	Wed	16:00 – 18:00	S 106 (FAN C)		E 05			
	21 Jan	Sat	midnight				HW 09		
11	18 Jan	Wed	16:00 – 18:00	S 106 (FAN C)	L 05				
	28 Jan	Sat	midnight				HW 10	FAIRNESS:	SHAPLEY VALUE
12	25 Jan	Wed	16:00 – 18:00	S 106 (FAN C)		E 06			
13	01 Feb	Wed	16:00 – 18:00	S 106 (FAN C)	L 06				
	08 Feb	Wed	midnight				HW 11	FAIRNESS:	SHAPLEY VALUE
14	08 Feb	Wed	16:00 <u>s.t.</u> – 18:00	S 106 (FAN C)		E 07			
	15 Feb	Wed	midnight				HW 12		
	08 Feb	Wed	18:00 – 19:30	S 106 (FAN C)	Q&A and Feedback				
	31 March	Fri	midnight				Bonus HW 13	FAIRNESS:	SHAPLEY VALUE
	31 March	Fri	midnight				Final exam		

L = Lecture session

E = Exercise session

HW = Homework due for submission

Week 01

Introduction

19 Oct (Wed), 16:00 – 18:00

S 106 (FAN C)

Our first meeting will be a general introduction to CGT centred around two questions: What is CGT? and What is CGT for? We will go through an outline of the course motivating each solution concept and sketching some of the problems they address. Finally, we will discuss the organisational details around the course and clarify any questions.

SUGGESTED LITERATURE:

Rapoport (1970: 45–86 [Chapters 1 and 2]).

Serrano (2005: 219–224).

Week 02

The language of CGT

26 Oct (Wed), 16:00 – 18:00

S 106 (FAN C)

In week 03, we will get acquainted with the language of CGT as well as some important properties which we will use or take as starting points for the material later on. The aim of this session is to make you comfortable with the way CGT is presented and help you appreciate the kind of problems it can solve.

SUGGESTED LITERATURE:

Kahan and Rapoport (1984: 19–55 [Chapter 2]).

Maschler, Solan and Zamir (2013: 659–685 [Chapter 16]).

Osborne (2004: 239–243).

Rapoport (1970: 68–92 [Chapters 2 and 3]).

HW 01 due:

02 Nov (Wed), 16:00

Week 03

The language of CGT: Exercises

02 Nov (Wed), 16:00 – 18:00

S 106 (FAN C)

In the exercise session, we will practice modelling different situations as cooperative games. The ultimate aim is to understand and learn to make the kind of choices and restrictions that go into expressing a certain context in the language of CGT.

SUGGESTED LITERATURE:

The exercises in Maschler, Solan and Zamir (2013) and Osborne (2004) above.

HW 02 due:

09 Nov (Wed), 16:00

Week 04

The core

09 Nov (Wed), 16:00 – 18:00

S 106 (FAN C)

The core is the most famous and perhaps widely applied solution concept in CGT. That is why, it will be the first solution we will look into. The core solves the allocation problem by proposing the following: find the allocation which is immune to any threats or complaints and objections from any possible group of players. As such, the core allocation reflects the power of coalitional members in the game (we will see an alternative approach based on fairness considerations when we discuss the Shapley value). In order to appreciate the versatility of the core, we will spend two exercise sessions on exploring some of its prominent applications.

SUGGESTED LITERATURE:

Osborne (2004: 243–270).

Kahan and Rapoport (1984: 56–71).

Gilles (2010: 29–70 [Chapter 2]).

HW 03 due:

16 Nov (Wed), 16:00

Week 05

The core: Rawlsian justice

16 Nov (Wed), 16:00 – 18:00

S 106 (FAN C)

In the first exercise session, we will tackle a problem from political philosophy, i.e. the idea of a social contract. Specifically, we will see how to present Rawls' contractarian theory of justice as a 'justice game' that yields as its core exactly the maximin distributions. In the second half, you will work on applying the core to cost-sharing games.

SUGGESTED LITERATURE:

Howe and Roemer (1981).

Osborne (2004: 247–251).

Moulin (1988: 87–106 [Chapter 4]).

HW 04 due:

23 Nov (Wed), 16:00

Week 06

The core: Matching games

23 Nov (Wed), 16:00 – 18:00

S 106 (FAN C)

In the second exercise session, we will deal with matching games in some detail. These are situations in which, for example, doctors have to be matched with hospitals or patients with donors, or universities with students, etc. Specifically, we will explore a procedure – the so called ‘deferred acceptance algorithm’ – for finding stable matchings in the core of such games. In the second half, you will work on solving simple market games.

SUGGESTED LITERATURE:

Roth and Sotomayor (1990: 15–39).

Gura and Maschler (2008: 1–58 [Chapter 1]).

Osborne (2004: 263–269).

Chakravarty, Mitra and Sarkar (2015: 150–166 [Chapter 8]).

HW 05 due:

30 Nov (Wed), 16:00

Week 07

The bargaining set

30 Nov (Wed), 16:00 – 18:00

S 106 (FAN C)

We have seen that the core solves the allocation problem by singling out those allocations, *if* there are any, that are immune to the threats of other players. The major problem with the core is that in some situations this requirement is too strong, i.e. there might be no such allocations. The next three solution concepts weaken this condition by taking into account not just any possible threat but only those which are *credible*. Additionally, they also help us answer the coalition formation question by appealing to different kinds of *stability*.

We will first look at the conceptions of credibility and stability behind the bargaining set. Roughly put, the bargaining set singles out as stable those coalitions and allocations to which no player has a *justified* objection against any other player, i.e. where every possible threat by a player is neutralised by a counter-threat from a different player.

SUGGESTED LITERATURE:

Rapoport (1970: 114–124 [Chapter 6]).

Osborne and Rubinstein (1994: 281–283).

Chakravarty, Mitra and Sarkar (2015: 58–61).

Maschler, Solan and Zamir (2013: 782–800 [Chapter 19]).

Maschler (1992: 591–602).

Aumann and Maschler (1964).¹

HW 06 due:

07 Dec (Wed), 16:00

¹ The original paper that introduced the bargaining set.

Week 08

The bargaining set: Market games

07 Dec (Wed), 16:00 – 18:00

S 106 (FAN C)

In this exercise session, we will apply the bargaining set to a market game, i.e. a situation where we have to determine how wages and profits should be distributed among firm owners and their workers. In other words, how much should a ‘fair’ wage be?

SUGGESTED LITERATURE:

Maschler (1976).

Baton and Lemaire (1981: 110–113).

HW 07 due:

14 Dec (Wed), 16:00

Week 09

The kernel

14 Dec (Wed), 16:00 – 18:00

S 106 (FAN C)

One of the issues with the bargaining set is that although it solves the allocation problem for *each possible* answer to the coalition formation question, it gives us little guidance as to which of these possible answers should be picked. The next two solution concepts focus on successively smaller parts of the bargaining set so as to help us tackle this issue.

Roughly put, the first of these solutions – the kernel – includes only those coalitions and allocations where no player can complain about another player’s payoff by threatening to exploit a more lucrative alternative that leaves the second player worse off.

SUGGESTED LITERATURE:

Rapoport (1970: 125–136 [Chapter 7]).

Osborne and Rubinstein (1994: 283–285).

Kahan and Rapoport (1984: 126–136).

Maschler (1992: 603–610).

Davis and Maschler (1965).²

HW 08 due:

14 Jan (Sat), midnight

² The original paper that introduced the kernel.

Week 10

The kernel: Apex games

11 Jan (Wed), 16:00 – 18:00

S 106 (FAN C)

In the exercise session, we will explore a class of so called apex games – these are situations with a number of ‘small’ players and one dominant ‘large’ player.

SUGGESTED LITERATURE:

Davis and Maschler (1965: 235–424).

Kahan and Rapoport (1984: 52–54).

Horowitz (1973).

HW 09 due:

21 Jan (Sat), midnight

Week 11

The nucleolus

18 Jan (Wed), 16:00 – 18:00

S 106 (FAN C)

While the kernel excludes allocations where players have certain ‘justified’ complaints, the nucleolus is based on a different kind of stability. To wit, it admits the existence of complaints from coalitions but at the same time proposes that allocation which *minimises* them in a certain way. During week 11, we will unpack and clarify what this certain way consists of.

SUGGESTED LITERATURE:

Kahan and Rapoport (1984: 136–142).

Maschler, Solan and Zamir (2013: 801–852 [Chapter 20]).

Maschler (1992: 610–616).

Schmeidler (1969).³

HW 10 due:

28 Jan (Sat), midnight

Week 12

The nucleolus: Bankruptcy problems

25 Jan (Wed), 16:00 – 18:00

S 106 (FAN C)

During the exercise session in week 12, we will apply the nucleolus to the task of solving bankruptcy problems, i.e. situations where an estate has to be divided among a group of creditors, claiming in total more than the value of the estate.

SUGGESTED LITERATURE:

Maschler, Solan and Zamir (2013: 831–844).

Gura and Maschler (2008: 166–204 [Chapter 4]).

Aumann and Maschler (1985).

HW 11 due:

08 Feb (Wed), midnight

³ The original paper that introduced the nucleolus.

Week 13

The Shapley value

01 Feb (Wed), 16:00 – 18:00

S 106 (FAN C)

The solution concepts we have studied so far prescribe allocations and coalitions which reflect – in various senses – the power or strength of individual players or coalitions of players. Very roughly put, they suggest that what a player ought to get depends on what a player can secure by threats, objections or complaints.

The final solution we will consider is based on a different intuition. Specifically, the Shapley value solves the allocation problem by proposing the following: distribute the benefits of cooperation so that each player receives what they contribute to the game. Crudely put, to each according to what they deserve.

SUGGESTED LITERATURE:

Rapoport (1970: 106–113 [Chapter 5]).

Maschler, Solan and Zamir (2013: 749–781 [Chapter 18]).

Gura and Maschler (2008: 97–165 [Chapter 3]).

Shapley (1953).⁴

HW 12 due:

15 Feb (Wed), midnight

Week 14

The Shapley value: Simple games

08 Feb (Wed),

16:00 s.t. – 18:00

S 106 (FAN C)

In our last exercise session, we will apply the Shapley value to a class of so called simple games. These are natural models of voting situations and we will see how to use the Shapley value as a measure of the voting power of the members of a decision-making committee.

SUGGESTED LITERATURE:

Straffin (1983).

Taylor and Pacelli (2009: 90ff).

Bonus HW 13 due:

31 March (Fri), midnight

Q & A and feedback

08 Feb (Wed), 18:00 – 19:30

S 106 (FAN C)

In our last session, we will clarify any outstanding questions you might have as well as talk about the take-home exams in more detail. If time permits, we can discuss some extra material, such as non-transferable utility games.

⁴ The original paper that introduced the Shapley value.

References

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