Image Compression using Singular Value Decompostion(SVD)

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***Abstract*—Image compression techniques are the most concerned topics in today is technological developments. Various methods have been proposed for image compression. The biggest challenge, however, is compressing the photos without losing information. Singular Value Decomposition (SVD) is one such image compression technique. This SVD performs its operations on matrices. In this study, we will look at how SVD is used on images, the approach for image compression with SVD, and the MATLAB for image compression.**

Keywords—Image compression, PSNR, MSE, Singular Value Decomposition, Image Processing, Image as a matrix

I. Introduction

With the advancement in technology, multimedia content of digital information is increasing day by day. Images, either photographs or video frames, make up the majority of the content. As a result, storing and transmitting these photos demands a lot of memory and bandwidth. The solution to this challenge is to lower the amount of storage space needed for these photographs by compressing them while maintaining acceptable image quality.

Compression is a common method for representing visual data in as few bits as possible by minimizing data redundancy while retaining an acceptable level of quality for the user. The SVD image compression technique minimises the image's storage size while maintaining image quality.

II. Computing SVD

A. Singular Value Decomposition

In linear algebra, the singular value decomposition (SVD) is a factorization of a real or complex matrix that extends the eigen decomposition, which is only valid for square normal matrices, to any m X n matrix.

SVD has many practical and theoretical values. A special feature of SVD is that it can be performed on any real (m, n) matrix. Suppose we consider a matrix A with m rows and n columns, with rank r and $r \leqslant n \leqslant m$. Then the A can be factorized into three matrices:

$$A = U \Sigma V^T$$

Calculating the SVD consists of finding the eigenvalues and eigenvectors of AA^T and A^TA. The eigenvectors of A^TA make up the columns of V , the eigenvectors of AAT make up the columns of U. Also, the singular values in Σ are square roots of eigenvalues from AA^T or A^TA. The singular values are the diagonal entries of the Σ matrix and are arranged in descending order. The singular values are always real numbers. If the matrix A is a real matrix, then U and V are also real.

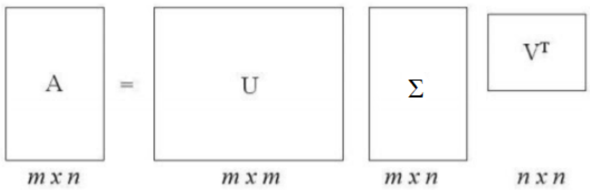


Fig. 1

B. Image Compression

In a variety of applications, photographs need to be sent and retained frequently. The lower the image size, the lower the cost of transmission and storage. As a result, data compression techniques are frequently used to reduce the amount of storage space taken up by the image. Applying Singular Value Decomposition (SVD) to the image matrix is one method. The digital image is delivered to SVD in this technique. The supplied digital image is refactored into three matrices by SVD. Singular values are used to restructure the image, and the image is represented with a smaller set of values at the end of the process, resulting in a reduction in the amount of storage space required by the image.

A picture can actually be represented as data in a matrix. To make an image grey, the values for red, green, and blue need to be the same. Therefore a pixel can be represented as having a value of 0 through 255 (in hexadecimal 00 through FF), and then repeating that value across the red, green, and blue saturation to get the corresponding shade of grey.

III. Literature Review

1. An Image of a bridge(night view) with dimensions 498 x 736 pixels is the dataset taken for this project. Its size is approximately 870 KB. It is a fairly clean and clear picture which is suitable for compression. The image format is png.
2. When SVD is applied to the image, the image matrix A is decomposed into three matrices U, Σ and V. However, just applying SVD to an image will not result in it being compressed.
3. Only a few singular values are kept after applying SVD to compress a picture, while other singular values are discarded. This is because singular values on the diagonal of D are ordered in descending order, with the first singular value comprising the most information and subsequent singular values containing decreasing quantities of picture information. Thus, the lower singular values containing negligible or less important information can be discarded without significant image distortion.

IV. Dataset

An Image of a bridge(night view) with dimensions 498 x 736 pixels is the dataset taken for this project. Its size is approximately 870 KB. It is a fairly clean and clear picture which is suitable for compression.

Fig. 2. Dataset for Compression

V. Methods

1. First we read the color image and convert it into grayscale
2. The image is decomposed using singular value decomposition.
3. Different number of singular values are used to compress and reconstruct the image.
4. Singular values not required for the compression are discarded.
5. Error and PSNR of the compressed images are calculated and stored for plotting the graph.

VI. Evaluation Metrics

Many performance measures are available to compare the results of different compressed images and to measure the degree to which an image is compressed, such as:

1. Compression Ratio(Cr):

   Compression Ratio is the ratio of the storage space required to store original image to that required to store a compressed image. It measures the degree to which an image is compressed.

$$Cr = \frac{Uncompressed\ image\ file\ size}{Compressed\ image\ file\ size}$$

2. Mean Square Error(MSE):

   MSE is the measure of deterioration of image quality as compared to the original image when an image is compressed. It is defined as square of the difference between pixel value of original image and the corresponding pixel value of the compressed image averaged over the entire image.

$$\mathrm{MSE} = \frac{1}{N}\sum\sum(E_{ij} - o_{ij})^2$$

3. Power Signal to Noise Ratio(PSNR):

   Peak Signal to Noise Ratio (PSNR) is the ratio of maximum signal power to the noise power that corrupts it. In Image compression maximum signal power refers to the original image and noise is introduced to compress it.

$$\begin{aligned} PSNR &= 10 \cdot \log_{10}\left(\frac{MAX_I^2}{MSE}\right) \\ &= 20 \cdot \log_{10}\left(\frac{MAX_I}{\sqrt{MSE}}\right) \\ &= 20 \cdot \log_{10}(MAX_I) - 10 \cdot \log_{10}(MSE) \end{aligned}$$

VII. Results and discussions

Images given below show the results of applying SVD on the dataset, taking different values of k. When the value of k is taken as 10, the image is very blurred as shown in fig. 3 which implies that the image is reconstructed considering only the first ten eigen values of the matrix. Whereas for k=40 the reconstructed image fig. 4 is less distorted as compared to the fig. 3.

By observing figures 3 to 8, it is clear that the compressed image approaches the original image as the value of k (the number of Eigen values used for reconstruction of the compressed image) increases. This means that as the value of k is increased, the image quality improves. When k equals the image matrix's rank, the reconstructed image is nearly identical to the original.

Fig. 3 Original

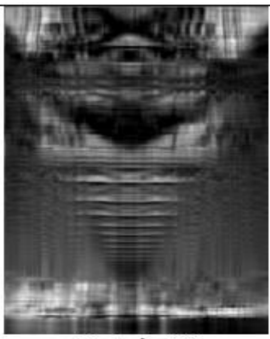


Fig. 4 k = 10

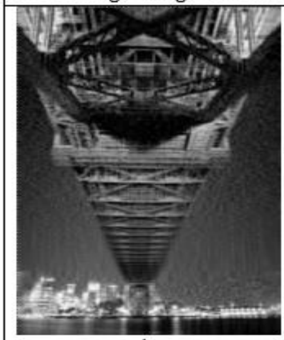


Fig 5 k = 40

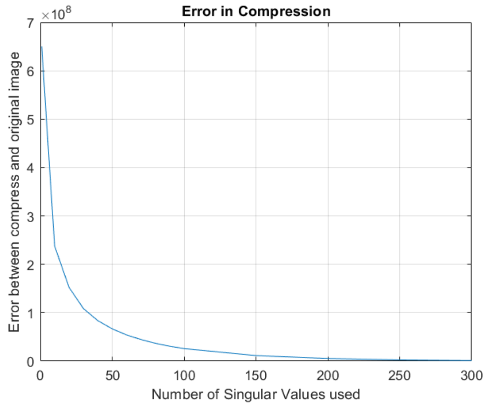
Fig. 6 k = 100

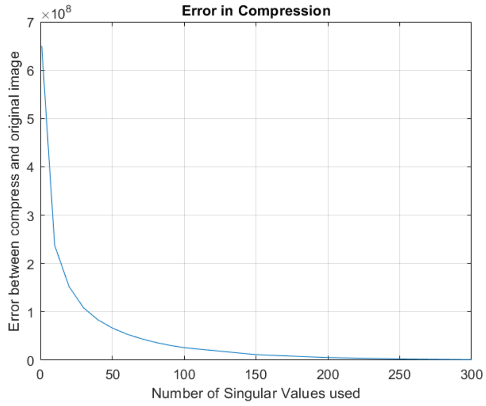
Fig. 7 k = 150

Fig. 8 k = 300

Singular Value	C R	PSNR	MSE
5	1.69	18.6148	0.0138
10	1.43	20.0112	0.0100
40	1.16	24.5488	0.0035
100	1.02	29.6547	0.0011
150	1.01	33.2038	4.7821e-04
300	0.99	47.0414	4.7104e-04
498	0.96	304.5442	3.5122e-31

The higher the PSNR, the better the quality of the compressed, or reconstructed image. Higher value of MSE designates a greater difference amid the original image and processed image





VIII. Conclusion

It is observed that SVD produces good compression results while requiring less computational effort. By selecting an acceptable value of k(i.e. the number of eigen values), the level of compression necessary by an application can be accomplished. However, in order to attain a high compression ratio, image quality must be compromised. As a result, the right value of k must be chosen to choose between compression ratio and image quality. The same benchmark can be utilised for all frames after the value of k is chosen for a specific application or movie. SVD is used in noise reduction, face identification, water marking, and other applications in addition to picture compression.

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