Victoria Allen

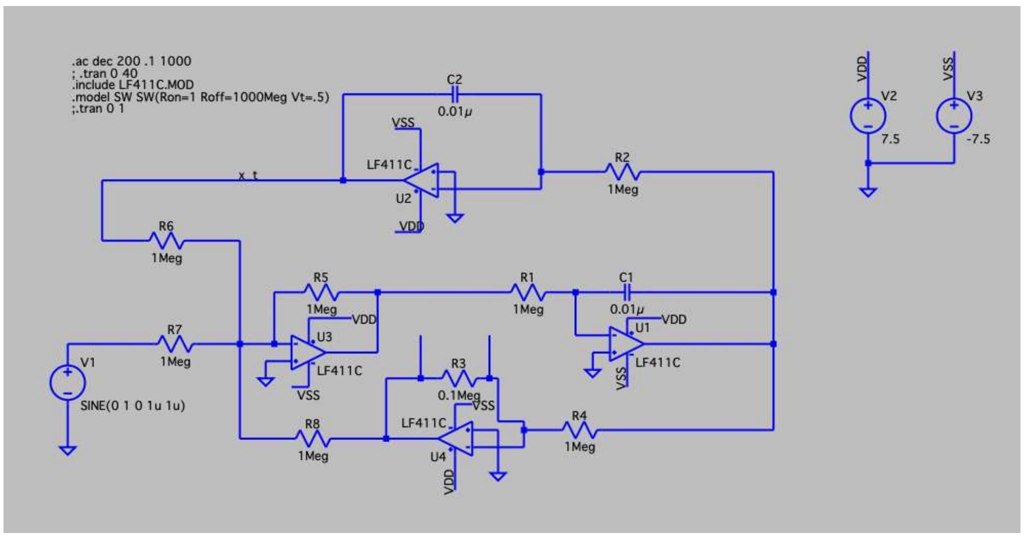
Circuit Analysis Laboratory

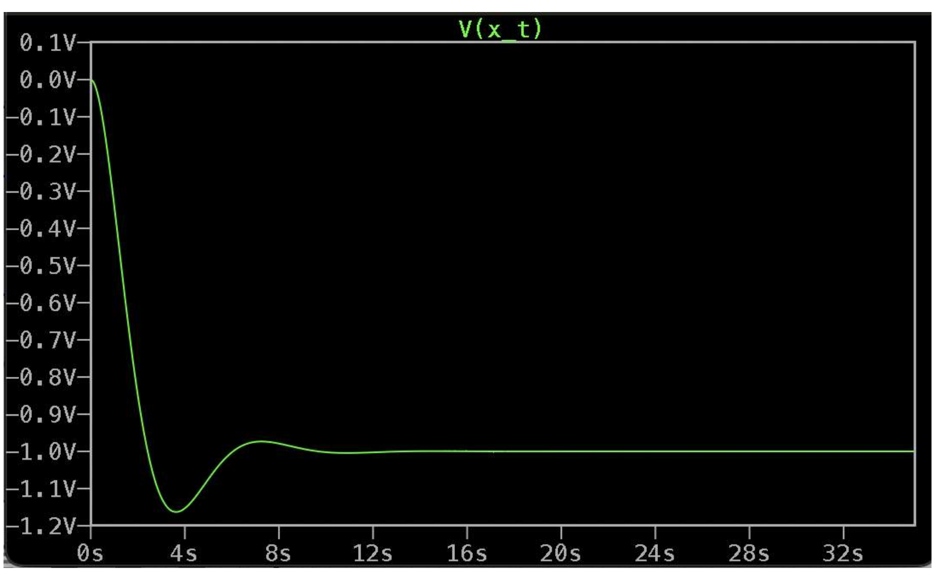
December 3rd, 2025

Experiment 4:

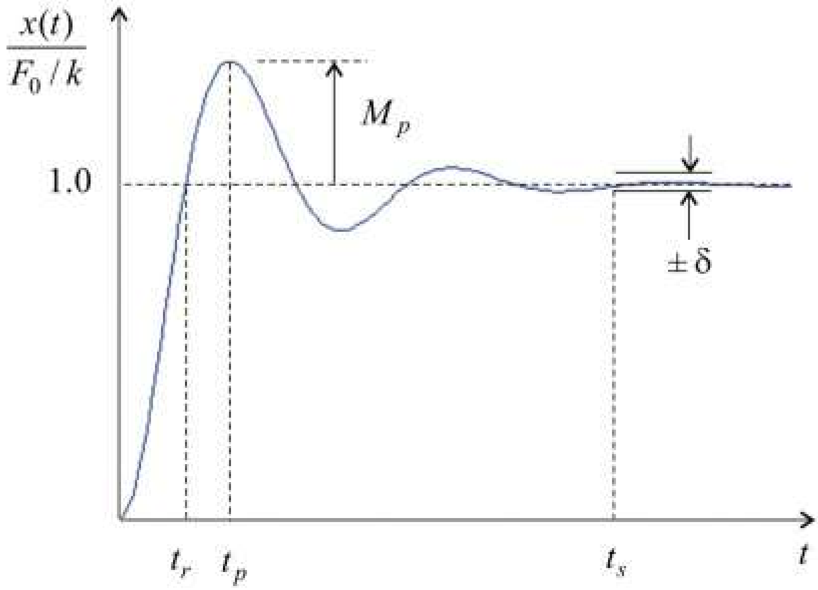
1. **Prelab:**

a)





- Damping factor = ½, Critical frequency = 2, Quality factor = 1
- Since the damping factor is less than 1, the system is underdamped. The ideal response for an underdamped system looks like:



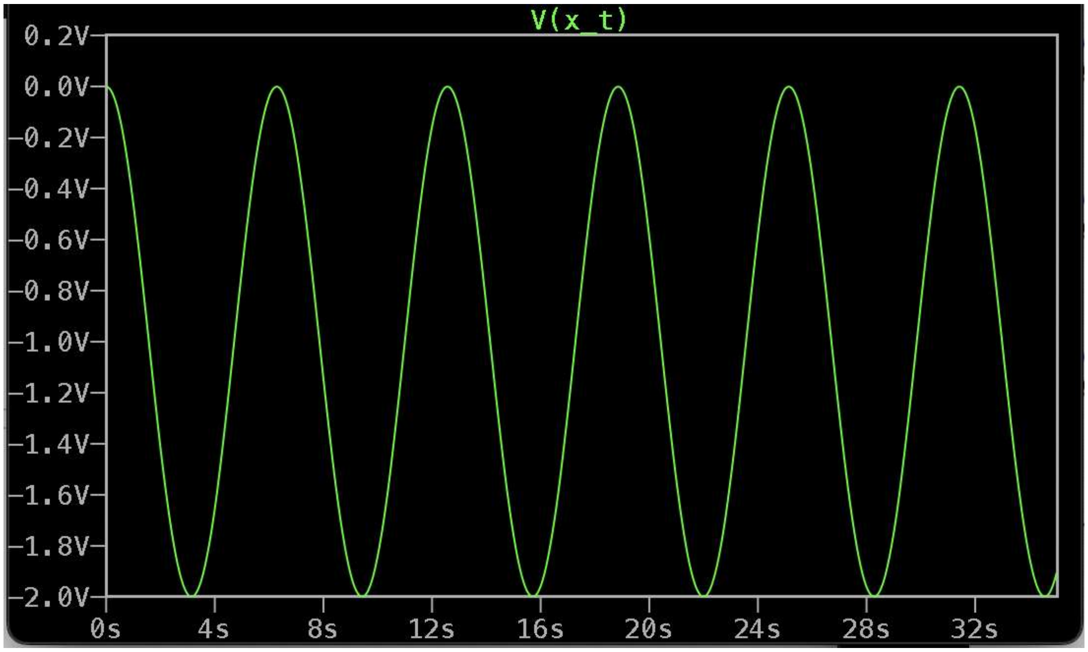
- They look very similar in shape; however, our response is inverted because of the op amp's inherent inversion.
- Underdamped step response:

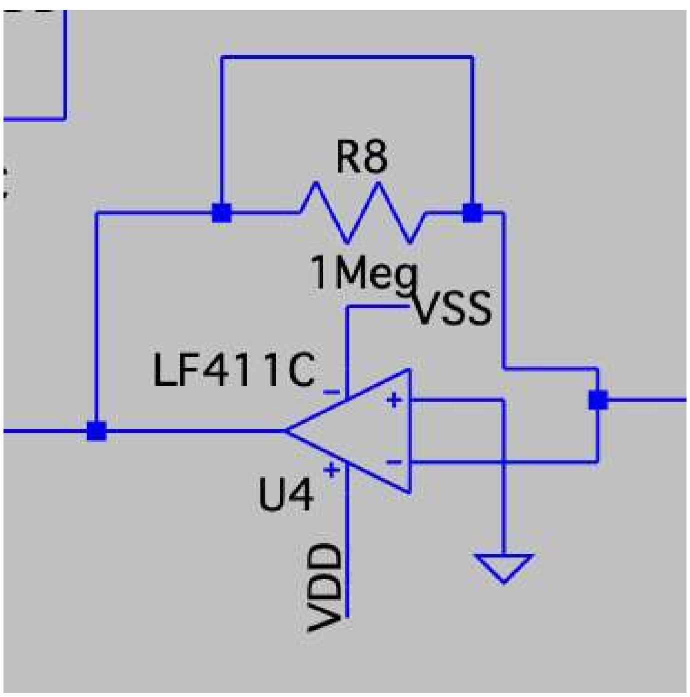
$$x(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta w_0 t} \sin(w_d t + \phi)$$

$$\phi = \arctan\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

b)

R3 shorted:



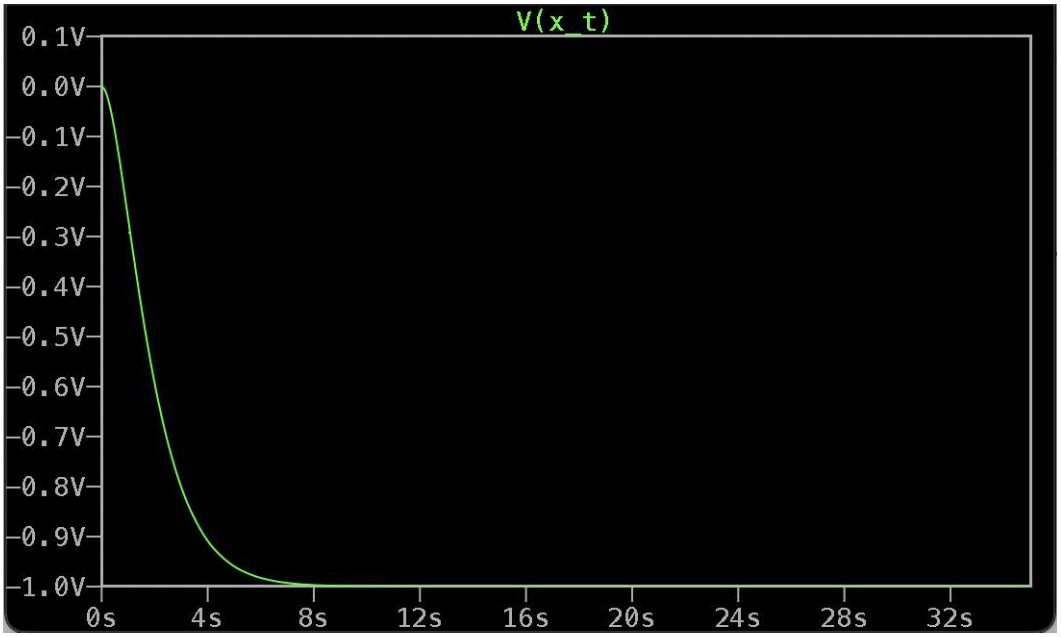


- In shorting R3, the output becomes a standard sinusoidal waveform. By shorting the bottom resistor, we eliminated part of the damping feedback path and effectively reduced the damping factor to zero. With no damping present, the system shifts into sustained oscillation.

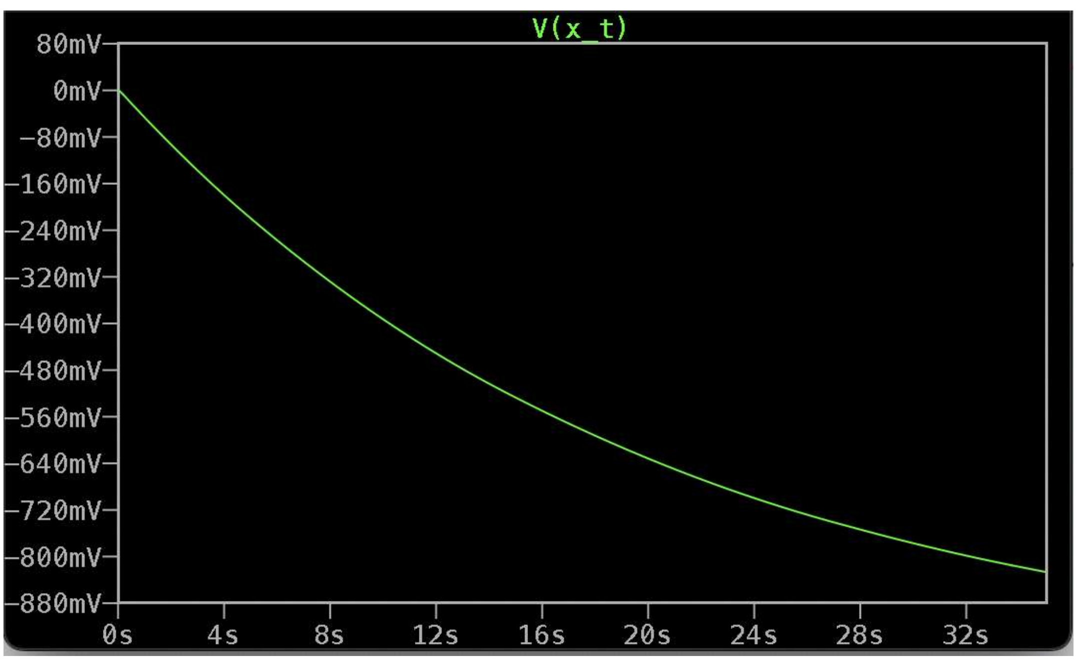
c)

- To achieve critical damping: 2(K1/t1t2)^{½}t1/K3 = 1. This can be done by doubling R3 to 2Meg.

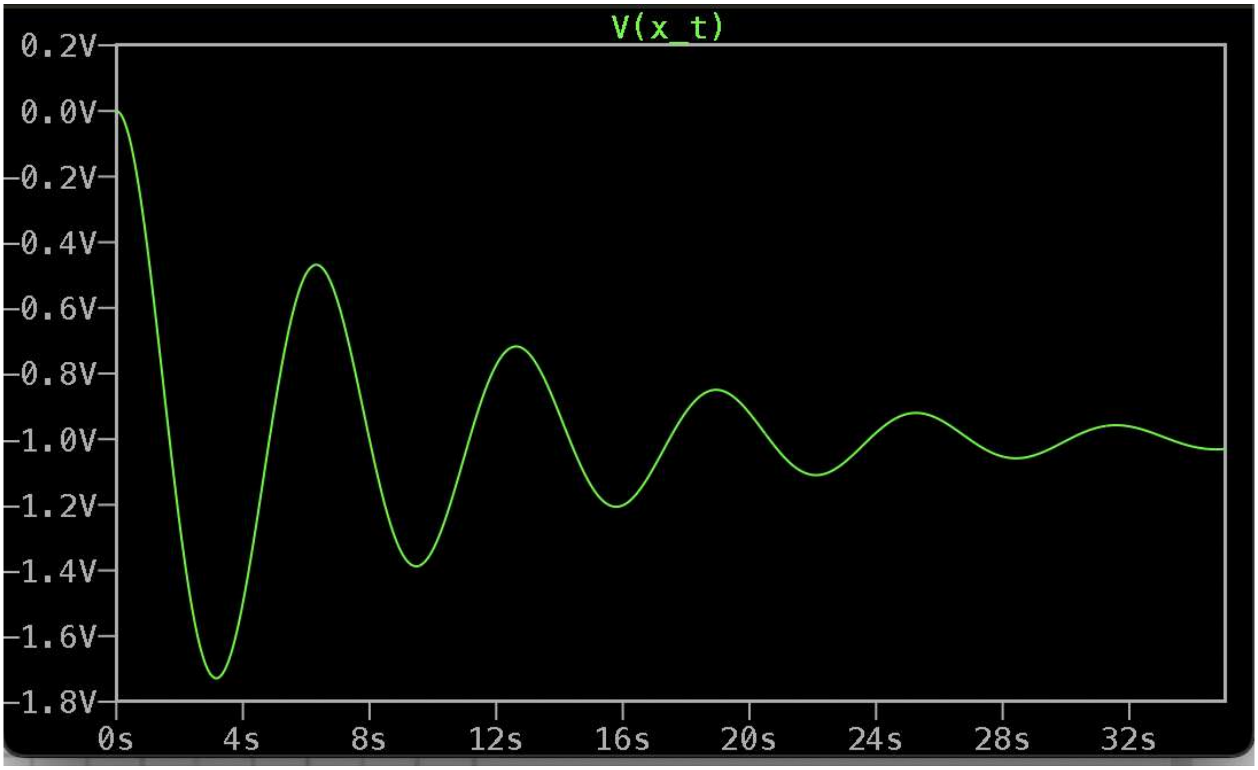
- Critically damped (R3 at 2Meg):

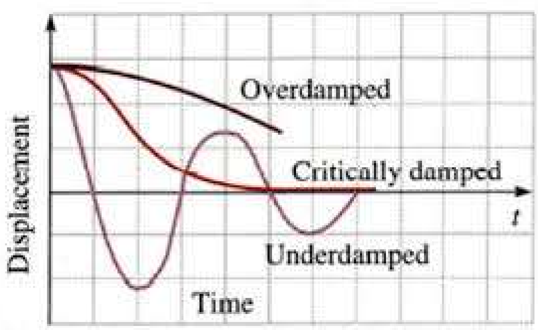


- Overdamped (R3 at 20Meg):



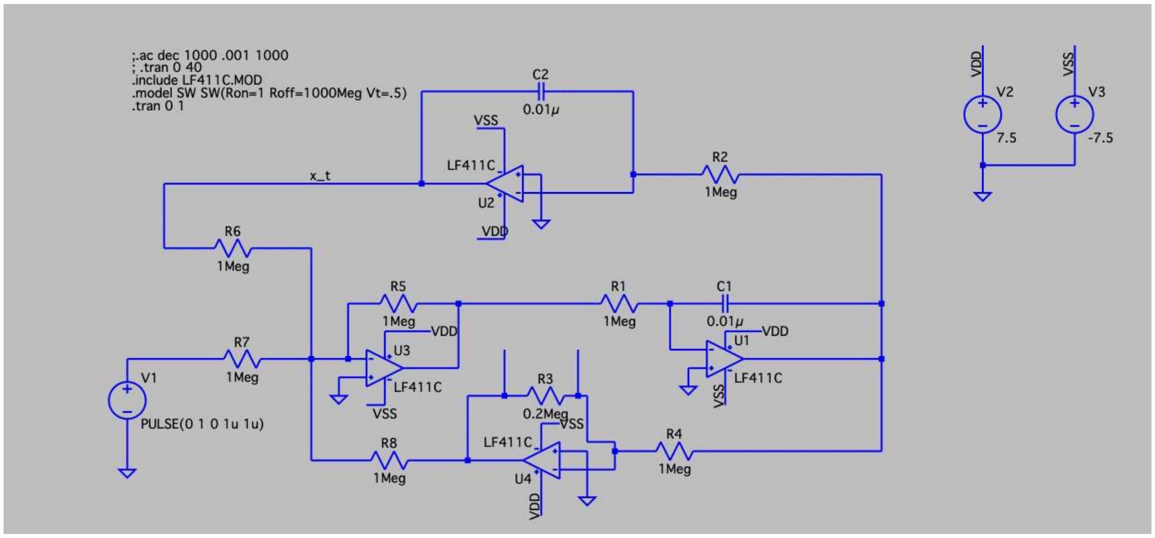
- Underdamped (R3 at 0.2 Meg):



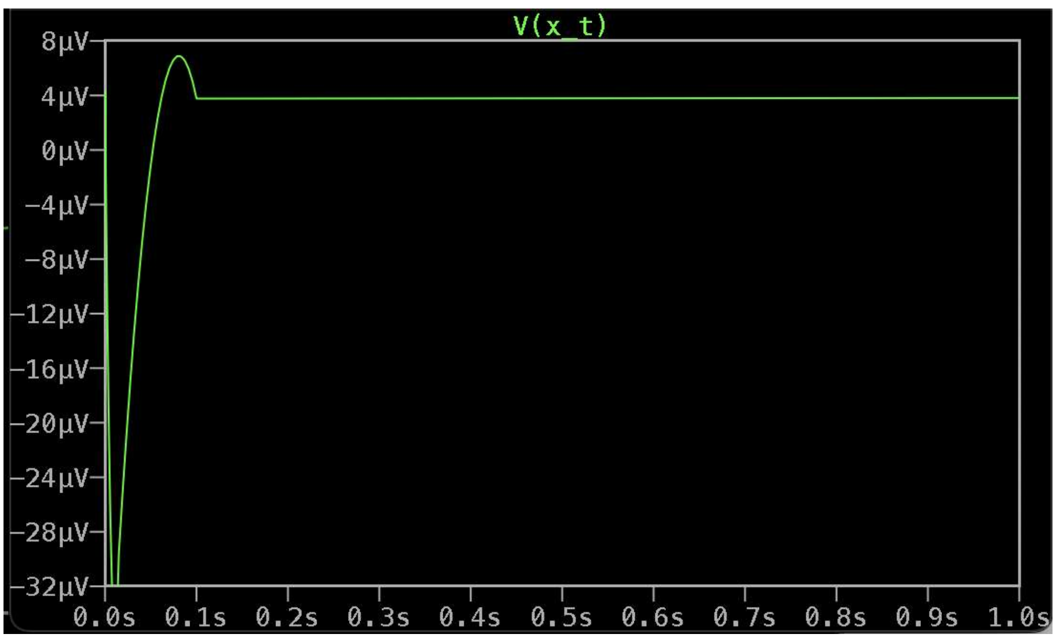


d)

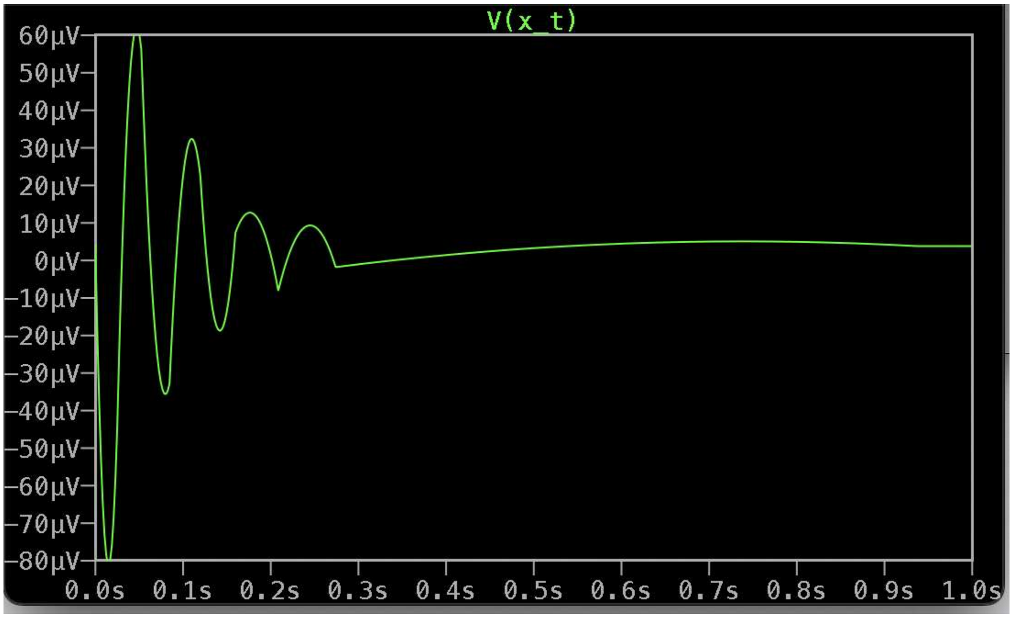
- To raise the critical frequency to 100, the time constants were changed to 0.01 and R3 was set to 2 MΩ to preserve critical damping.



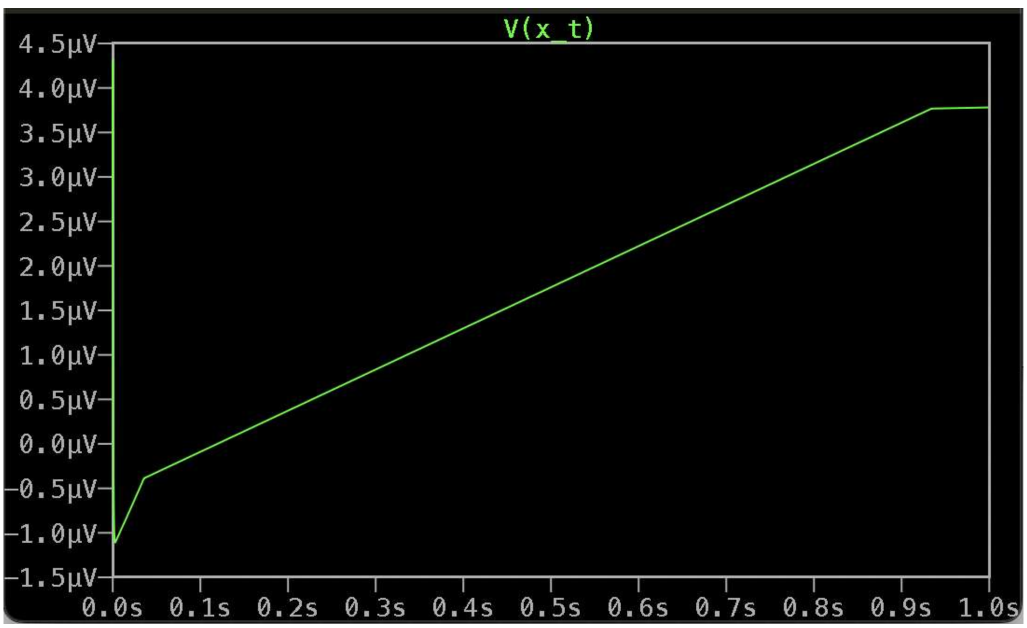
- Critically damped case:



- Underdamped case:

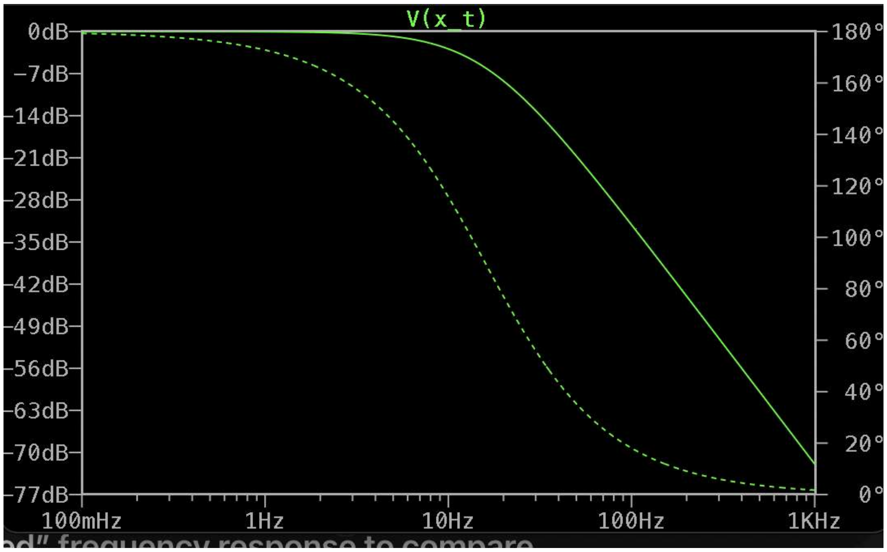


- Overdamped case:

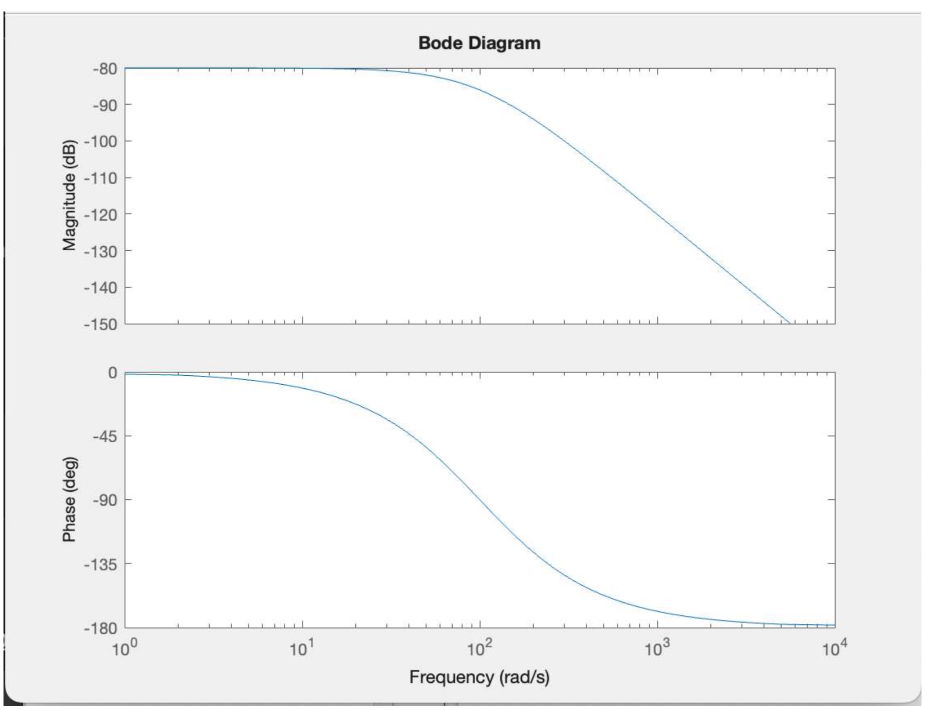


- After increasing ω_0 by 100×, all responses speed up proportionally, but their shapes stay the same: the underdamped case oscillates quickly, the critically damped case rises fastest without overshoot, and the overdamped case is slowest.

e)



$$H(s) = \frac{K}{s^2 + 200s + 10{,}000}$$



f)

Calculations:

$R1 = R2 = R4 = R5 = R6 = R7 = R8 = 1\ M\Omega$

$C1 = C2 = 0.01\ \mu F$

Therefore $\tau 1 = R1 * C1 = 1\ M\Omega * 0.01\ \mu F = 0.01\ s$

$K1 = R5 / R6 = 1$

$\omega 0 = \mathrm{sqrt}(\ K1 / (\tau 1 * \tau 2)\ ) = \mathrm{sqrt}(\ 1 / (0.01 * 0.01)\ ) = 100\ rad/s$

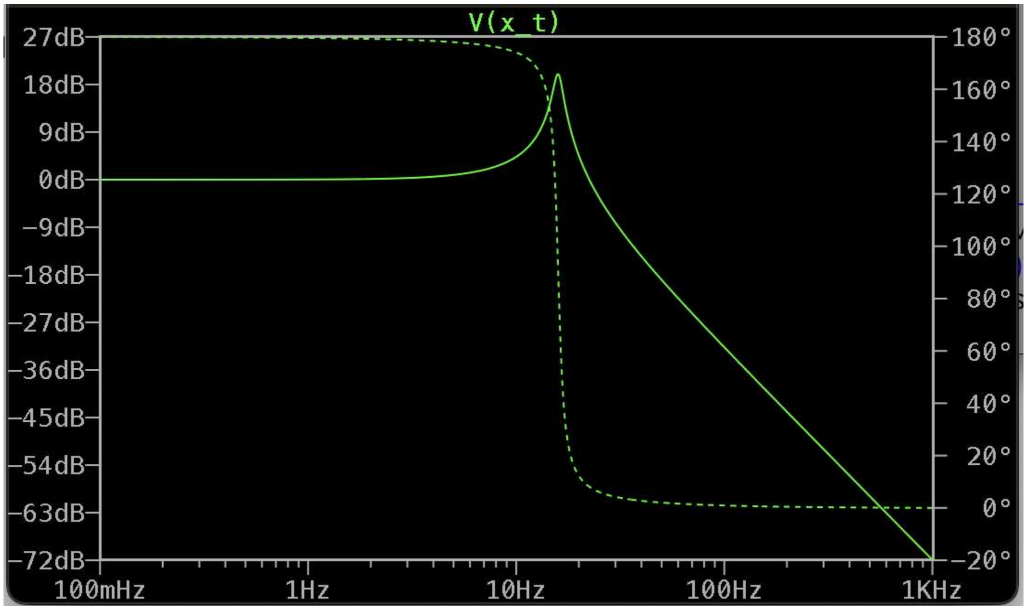
$Q = (\omega 0 * \tau 1) / K3$
$10 = (100 * 0.01) / K3$
$10 = 1 / K3$
$K3 = 0.1$

$K3 = (R3 * R5) / (R4 * R8)$

$0.1 = (R3 * 1\ M\Omega) / (1\ M\Omega * 1\ M\Omega)$
$0.1 = R3 / 1\ M\Omega$
$R3 = 0.1 * 1\ M\Omega$
$R3 = 100\ k\Omega$

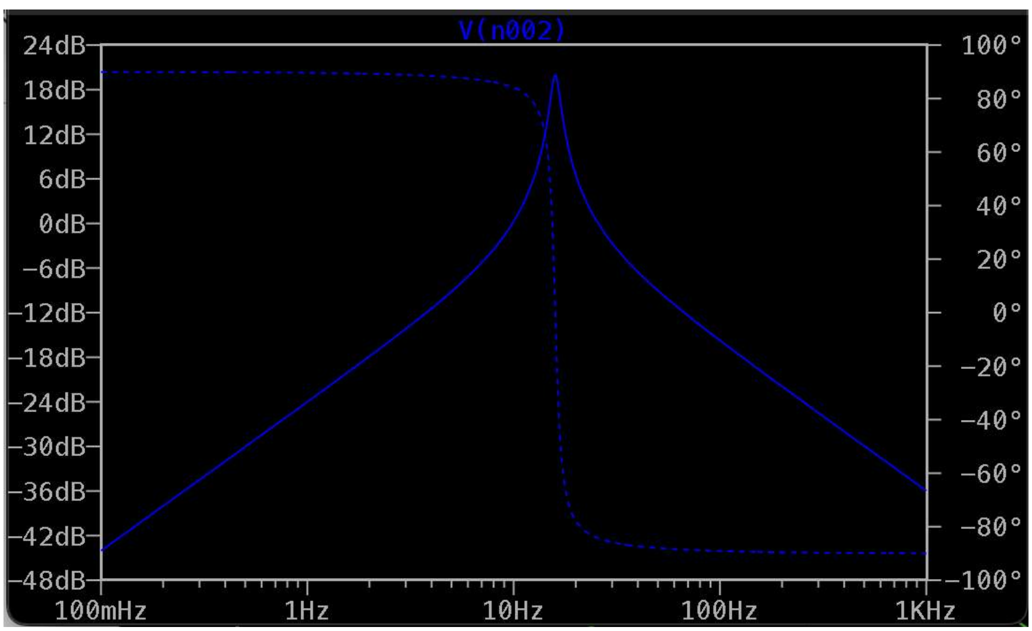
$R3 = 100\ k\Omega$

$K3 = 0.1$



- Filter started acting slightly after 10Hz. When R_3 is adjusted to give $Q \approx 10$ (still with ω_0 = 100 rad/s), the frequency response develops a tall, narrow resonant peak near ω_0 and a much narrower bandwidth. This agrees with the calculated response and corresponds to the strongly underdamped, ringing behavior seen in the time-domain simulations.

g)



- The output of U1 exhibits a band-pass response that peaks at the natural frequency ω_0. When the quality factor is high ($Q = 10$), the passband becomes very narrow and the peak gain increases significantly.

h)

Based on the band-pass response, the resonant peak appears at approximately f_0 = 15.9 Hz. The peak magnitude is about 20 dB, so the half-power level occurs at 17 dB. The frequencies at which the response crosses 17 dB are $f_1 \approx 15.0$ Hz and $f_2 \approx 16.8$ Hz, giving a measured bandwidth of:

Bandwidth = $f_2 - f_1$ = 16.8 Hz – 15.0 Hz = 1.8 Hz

The corresponding quality factor is:

Q_measured = f_0 / Bandwidth = 15.9 Hz / 1.8 Hz ≈ 8.8

This is reasonably close to the theoretical value Q = 10, with deviations explained by op-amp non-idealities and the resolution limits of the frequency sweep.

This confirms that a higher Q produces a narrower bandwidth.

i)

In addition to the low-pass transfer function from V_1 to x(t) and the band-pass response at U1, the circuit also produces a second-order high-pass response at U3, completing the standard trio of low-, band-, and high-pass behaviors.

2. **Build it:**