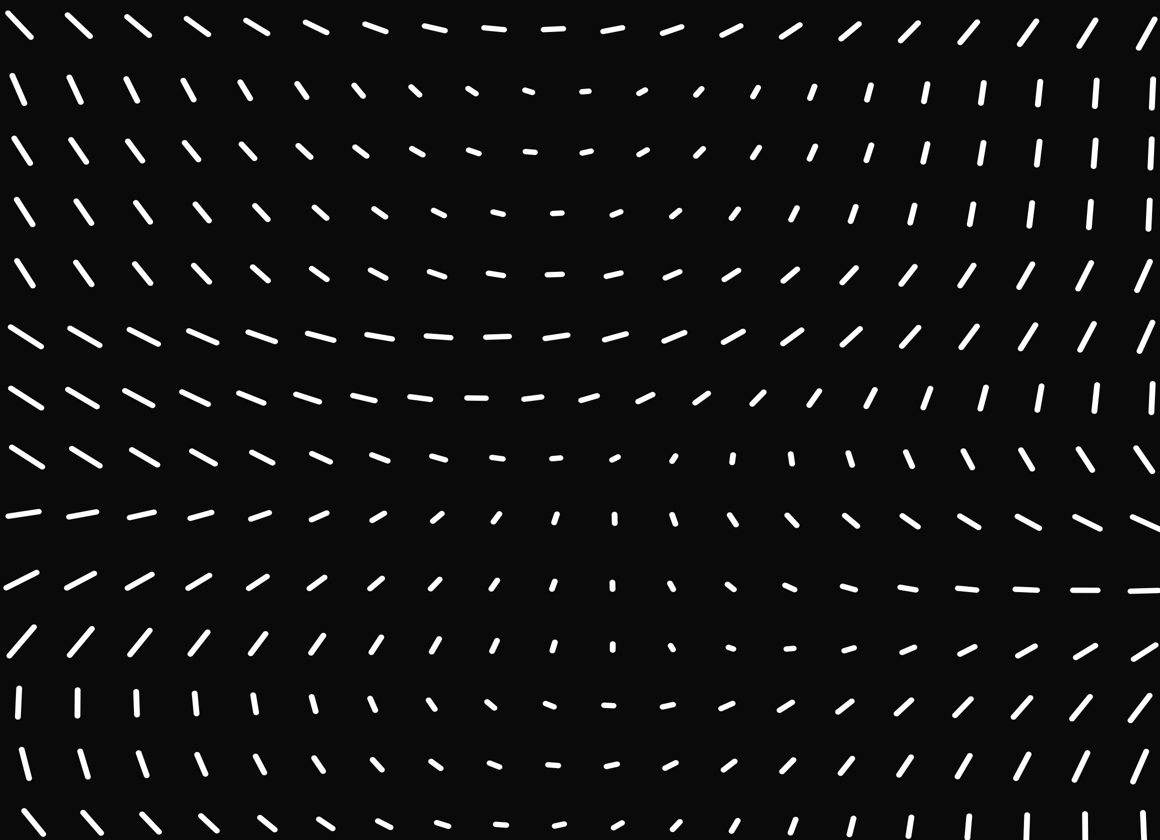


$$T(n) = C + T(n-1) + T(n-2)$$

$$T(n-2) \leq T(n-1)$$

$$T(n) \leq C + 2 \cdot T(n-1)$$

$$T(n) \leq C + 2(C + 2T(n-2))$$

$$T(n) \leq \underset{\times 2}{4}T(\underset{-1}{n-2}) + 3C$$

$$T(n) \leq 2^K \cdot T(n-K) + C \cdot \underbrace{(2^0 + \ldots + 2^{K-1})}_{= 2^K - 1}$$

$$T(n) \leq 2^n \cdot T(0) + (2^n - 1) \cdot C = \underbrace{2^n \cdot C_1 + C_2}$$

$$T(n) = O(2^n)$$

$$T(n) \geq C + 2T(n-2) \qquad 2^n = \Theta(2^{n/2})$$

$$T(n) \geq 2^{n/2} \cdot C_1 + C_2$$

$$T(n) = \Omega(\underline{2^{n/2}})$$

$$\boxed{T(n) = \Theta(2^{n/2})}$$

$$T(n) = \underbrace{T(\underbrace{n/2})}_{\text{subtask}} + \underbrace{n^2}_{f(n)}$$

$$\left.\begin{array}{ll} 1. & n^2 \\ 2. & n^2/4 \\ 3. & n^2/16 \\ 4. & n^2/64 \\ & \vdots \\ X & O(1) \end{array}\right\} \quad T(1) = C$$

$[\log_2 n]$

$$n^2 \underbrace{\left(1 + \frac{1}{4} + \frac{1}{16} \dots + \frac{1}{n^2}\right)}_{\frac{1}{1-x} = 4/3}$$

$$T(n) \leq 4/3\, n^2 = O(n^2)$$

$T(n) = \boxed{2}\, T(n/2) + n^2$

3

$k=2$

$b=2$

$a=2 \quad a=5$

$\log_b a = 1$

$\log_2 5 = 2.x$

$n^{\log_2 5}$

n^2

$n^2/4 \qquad n^2/4$

$n^2/16 \qquad n^2/16 \qquad n^2/16 \qquad n^2/16$

$O(1) \quad O(1) \quad \ldots \quad O(1) \quad O(1)$

n^2

$n^2/2 \cdot 3$

$n^2/4 \cdot 9$

$n^2/2^{2k} \cdot 3^k$

$n^2/n^2 \cdot 3^{\log_2 n}$

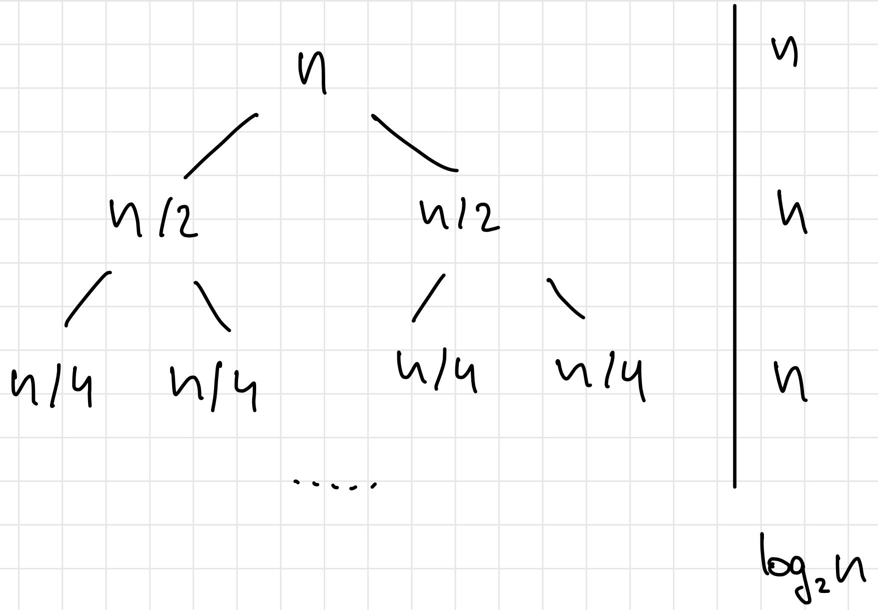
$n^2 \cdot \frac{3^k}{4^k}$

$// \frac{1}{1-3/4} = c$

$$n^2 \cdot \underbrace{\left(1 + \frac{1}{2} + \frac{1}{4} \ldots + \frac{1}{n}\right)}_{\leq 2 = \frac{1}{1-1/2}}$$

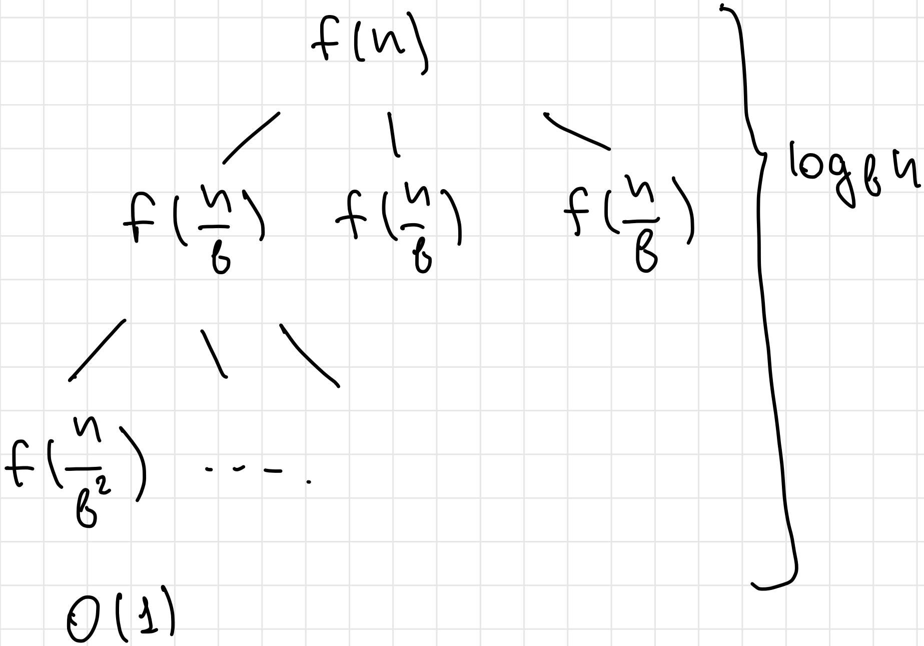
$T(n) = O(n^2)$

$$T(n) = 2T(n/2) + n$$



$$T(n) = \Theta(n \cdot \log n)$$

$$T(n) = a \cdot T\left(\frac{n}{b}\right) + f(n) \qquad f(n) = n^k \cdot \log^p n$$



cost

$f(n)$ // $\underline{n^k}$

$a f\left(\frac{n}{b}\right)$

$a^2 \cdot f\left(\frac{n}{b^2}\right)$

$a^k \cdot f\left(\frac{n}{b^k}\right)$

$$\boxed{a^{\log_b n}}$$

$$\Downarrow$$

$$n^{\log_b a}$$

n^k vs $n^{\log_b a}$

1. $k > \log_b a \Rightarrow n^k$

2. $k < \log_b a \Rightarrow n^{\log_b a}$

$$T(n) = \underbrace{T\left(\frac{n}{b}\right)}_{=\, T\left(\frac{n}{b^2}\right) + f\left(\frac{n}{b}\right)} + f(n)$$

$$T(n) = a \cdot T\left(\frac{n}{b}\right) + f(n), \quad f(n) = n^k \cdot \log^p n$$

1. $\log_b a > k \Rightarrow \Theta(n^{\log_b a})$

2. $\log_b a < k \Rightarrow \begin{cases} p \geq 0 & n^k \cdot \log^p n \\ p < 0 & n^k \end{cases}$

3. $\log_b a = k \quad \begin{cases} p > -1, & f(n) \cdot \log n = n^k \cdot \log^{k+1} n \\ p = -1, & n^k \cdot \log\log n \\ p < -1, & n^k \end{cases}$

$$T(n) = 4T(n/2) + \frac{n^2}{\log n}$$

$a = 4$

$b = 2 \quad \Rightarrow \quad n^2 \cdot \log\log n$

$k = 2$

$p = -1$

$$T(n) = 7 \cdot T(n/2) + n^2$$

$a = 7$ $\qquad$ $\log_2 7 \sim 2.81$

$b = 2$

$k = 2$

$$T(n) = \Theta\left(n^{\log_2 7}\right)$$