Solving for Probability of Stablecoin Default with Kelly Vaults

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Abstract

The Kelly criterion formula for binary events is reversed to solve for a Kelly-neutral surface - a space where, no matter the combination of odds, it makes sense to remain in cash/cash-backed x+y=k stablecoin pools. Then a vault is constructed with the goal of maximizing compounded returns based on the Kelly criterion containing two assets - an unknown stablecoin and cash/cash-backed stablecoin/cash-backed stablecoin pool. By offering a vault consisting of cash and a yielding asset with unknown risk, the allocation of DeFi users winds up solving for the probability of the default on a yielding asset as one applies the Kelly criterion. In turn, this approach can help the DeFi ecosystem's users gauge the riskiness of a new protocol before deployng capital into it.

1 Introduction

The Kelly criterion f^*, the fraction of one's portfolio to allocate to a risky asset to maximize one's compounding, for a binary outcome with probability P, a potential gain of G, and a potential loss of L is given by:

$$\mathrm{f}^* = \mathrm{P} - \frac{(1-P)}{G/L}$$

For example, if we have a fair coin $P = 0.5$ with double the gains relative to losses $G = 2$ and $L = 1$, the criterion tells us to allocate a quarter of our portfolio to such a process:

$$\mathrm{f}^* = 0.5 - \frac{(1-0.5)}{2/1} = 0.25$$

If one allocates more, one gains unnecessary risk with worse compounding. If one underallocates, one does not fully take advantage of compounding[1]. The formula can be reversed to ask a different question:

Under what combination of probabilities, gains, and losses would Kelly advise to be in cash-backed stables if one wants to maximize compounding returns?

In other words, when is this true:

$$\mathrm{P} - \frac{(1-P)}{G/L} = 0$$

For example, a yield of 7% is offered on an unknown protocol that may depeg with $G = 0.07$ and $L = 1$. We can figure out what our probability would be to not make it worth allocating into such a protocol ($P \approx 0.935$) if we want to maximize our compounded returns:

$$0.935 - \frac{(1-0.935)}{.07/1} = 0$$

Some stablecoin yield protocols may even offer a limit to the downside with a reserve ratio as in the case of Gemini's yield which holds a 30% reserve ($L = 0.7$) which can push the probability of staying in cash down for some users ($P \approx 0.909$):

$$0.909 - \frac{(1-0.909)}{.07/.7} = 0$$

We can construct a Kelly-neutral surface for all combinations of yield G, losses L, and the probability level necessary to avoid such investments and to prefer cash-equivalents in **Figure 1**.

The problem in all of these cases is that while we're aware of what the yield is, we do not know the probability of earning a yield (P) and the probability of total loss/default could be $(1-P)$.

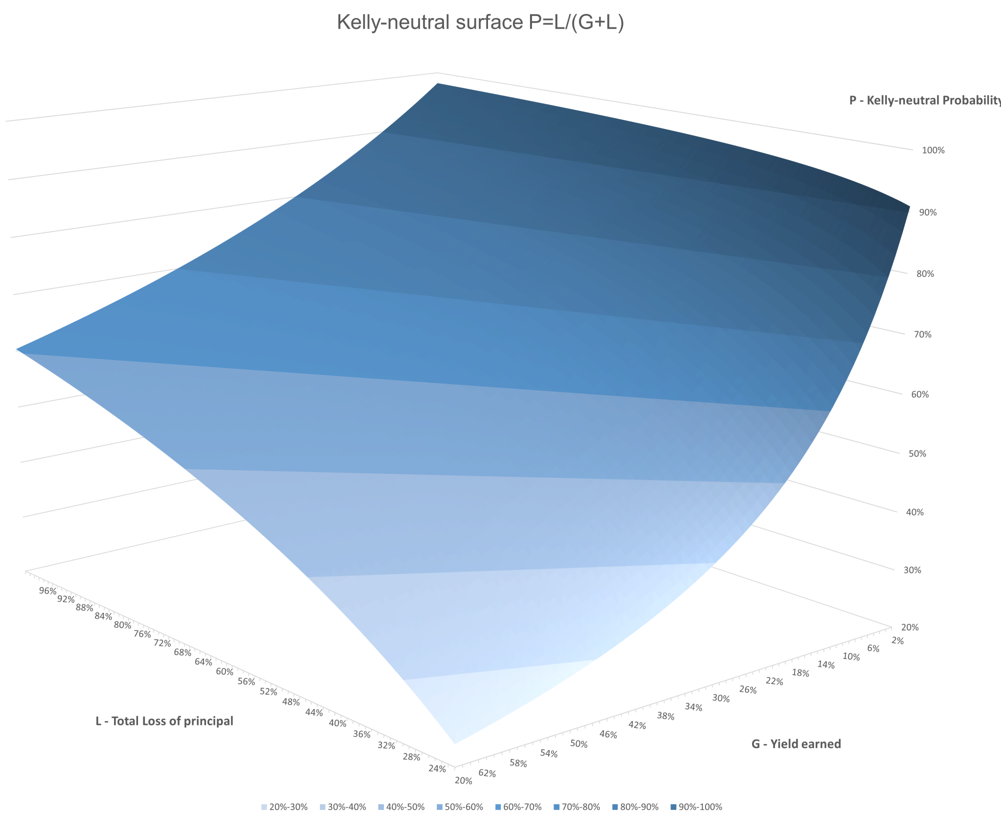

Figure 1: Kelly-neutral surface. Given a known gain in yield G and a known loss L, at what probability would one rather not invest in a protocol and sit in cash/stables: $P = \frac{L}{(G+L)}$. Every point on this surface corresponds to a decision to sit in cash/cash-backed stables instead of the unknown yield-generating asset.

2 Using Defi Vaults with Kelly to solve for P

While we are unaware of P, we can get the market to solve for it (similar to how the first person to allocate into x*y=k has to pick the right proportions). The trick is to create a vault containing both regular cash/cash-equivalents and the unknown yielding stablecoin protocol. The larger such a vault, the more certainty it brings to P.

Today, instead of allocating to a vault, users may rather allocate their money into the unknown yielding stablecoin protocol directly (e.g. Anchor), foregoing any knowledge of the probability of a default, resulting in depeg losses.

If one offers vaults with a cash-equivalent stablecoin component and an unknown yield protocol instead of just the unknown yield protocol, it can, counterintuitively, outperform as one applies LLN in the long run with the Kelly approach[2].

Consider a Kelly vault into which users start to allocate their funds between the unknown protocol X yielding 9% and cash/stables S. The first users of such a vault may allocate 50/50, 100/0, 20/80, but as time goes on and the TVL of the vault grows, one will start to see a pattern between TVL yielding from the unknown protocol and the TVL in cash/stables.

For example, if the TVL for X becomes 25% for the whole vault, we now can deduce the probability of default $(1 - P)$ by reversing the Kelly criterion since the collective allocation to the unknown protocol becomes the Kelly fraction!

$$\mathrm{f}^* = \mathrm{P} - \frac{(1-P)}{G/L}$$

Solving for P given Kelly fraction f:

$$\mathrm{P} = \frac{fG + L}{G + L}$$

For our 25% allocation with 9% yield example $P = 93.8\%$ and the **probability of loss** in DeFi for such a yield protocol stablecoin becomes $1 - P \approx 6.2\%$.

3 Conclusion

This probability can help new DeFi users be aware of the risk and avoid getting rekt by discouraging them from putting all their eggs in the unknown yield protocol and playing it safer with a Kelly vault instead if they're more adventurous on the risk frontier. Kelly vault concept can be used for StakeDAO, Curve pools, or various portfolio constructions containing stablecoins and stablecoin yield.

References

[1] Thorp, Ed. Kelly Capital Growth Investment Criterion. World Scientific, 2016.

[2] Carta, Andrea and Conversano Caludio. Practical Implementation of the Kelly Criterion: Optimal Growth Rate, Number of Trades, and Rebalancing Frequency for Equity Portfolios. Frontiers in Applied Mathematics and Statistics, 2020. url: https://www.frontiersin.org/article/10.3389/fams.2020.577050