Lecture 3
Neural Networks basics

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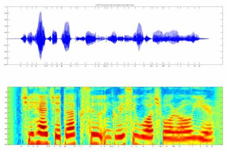
Moscow, 2021

Outline

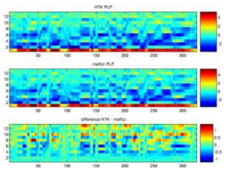
1. Neural Networks in different areas, historical overview
2. Backpropagation
3. More on backpropagation
4. Activation functions
5. Playground

Real world applications

Audio Features

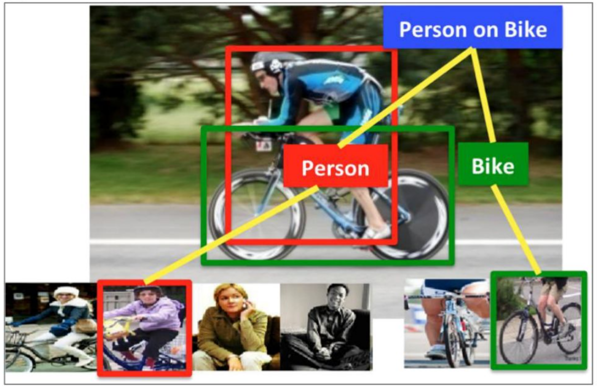


Spectrogram



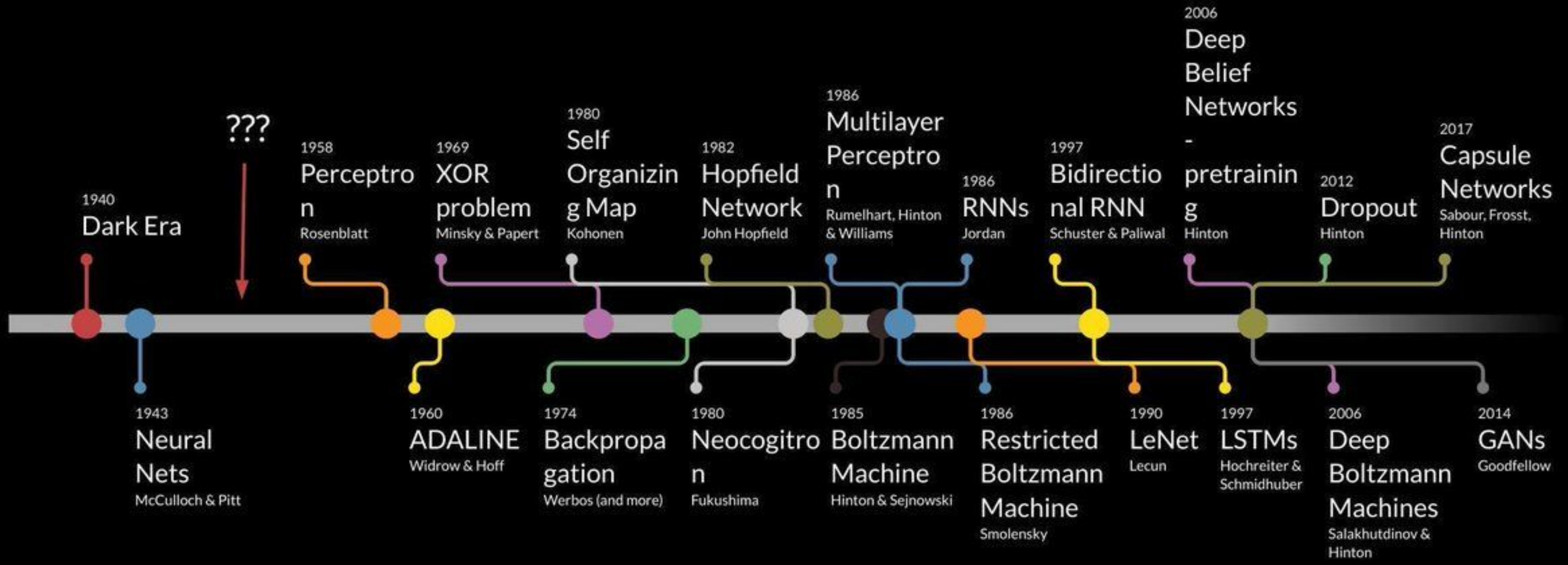
MFCC

- Object detection
- Action classification
- Image captioning
- …



"man in black shirt is playing guitar."

Deep Learning Timeline



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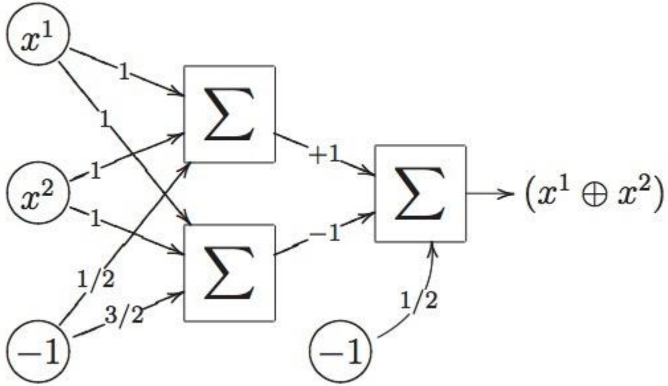
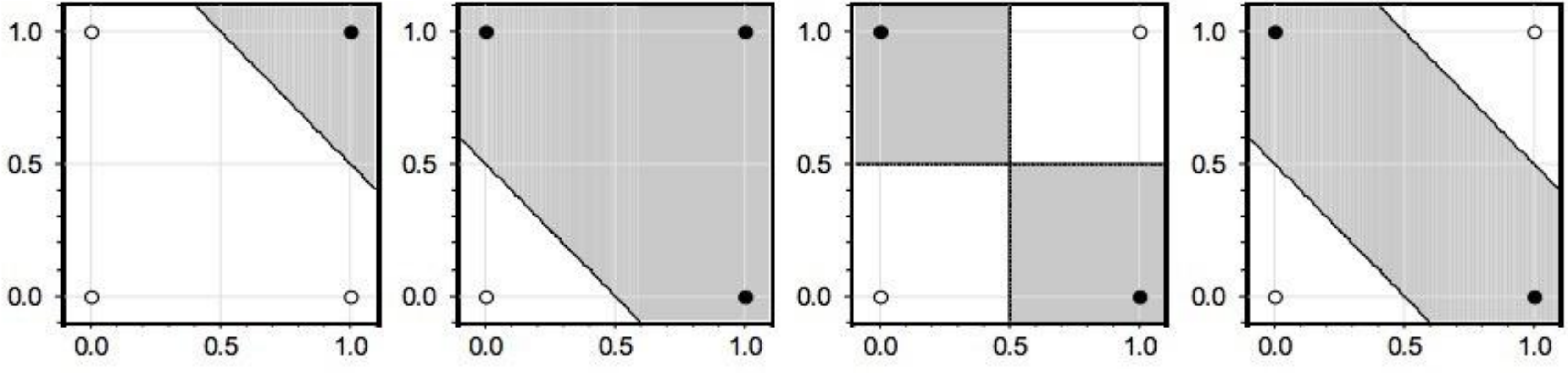
XOR problem



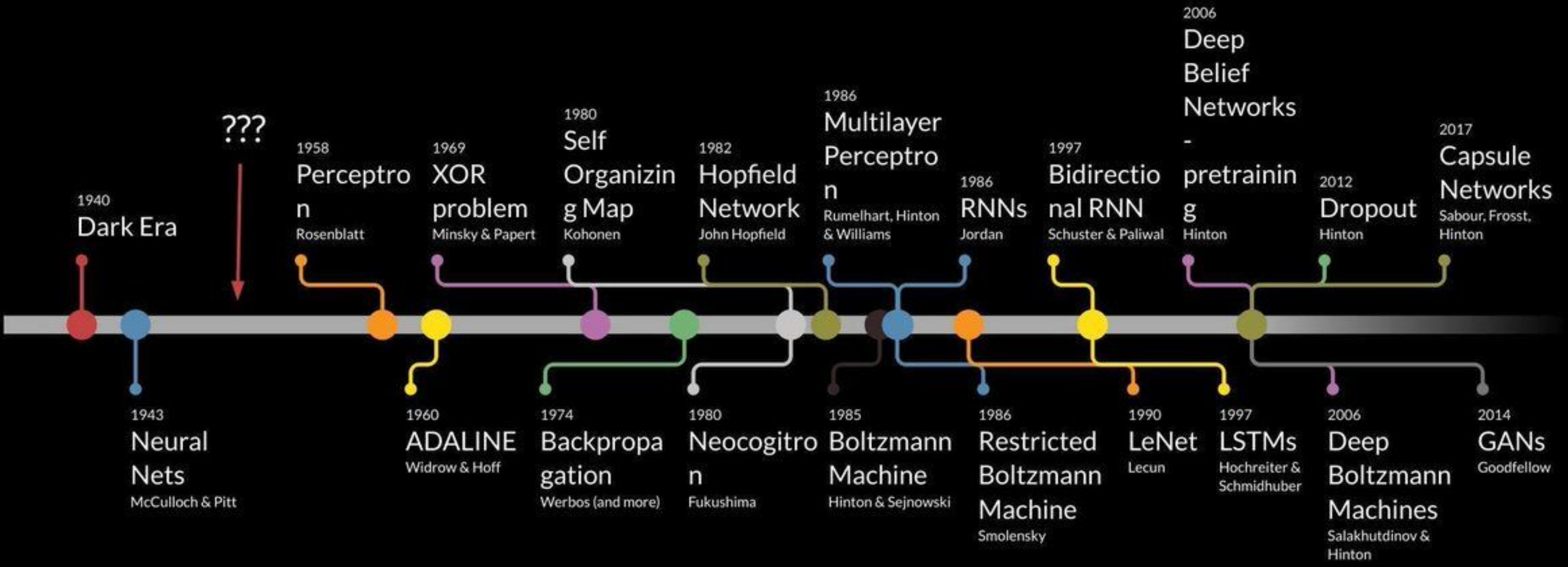
This 2-layer NN (on the left) implements XOR with only x1 and x2 features.

1-layer NN also can succeed, but only with extra feature x1*x2.



AND OR XOR(with x1*x2) XOR

Deep Learning Timeline



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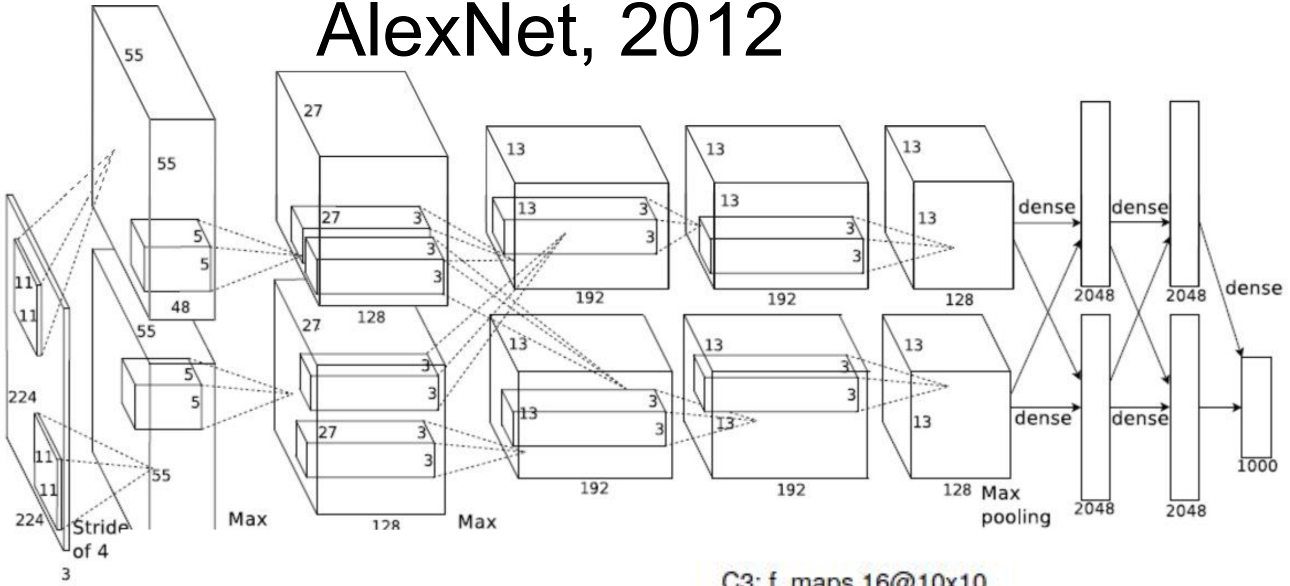
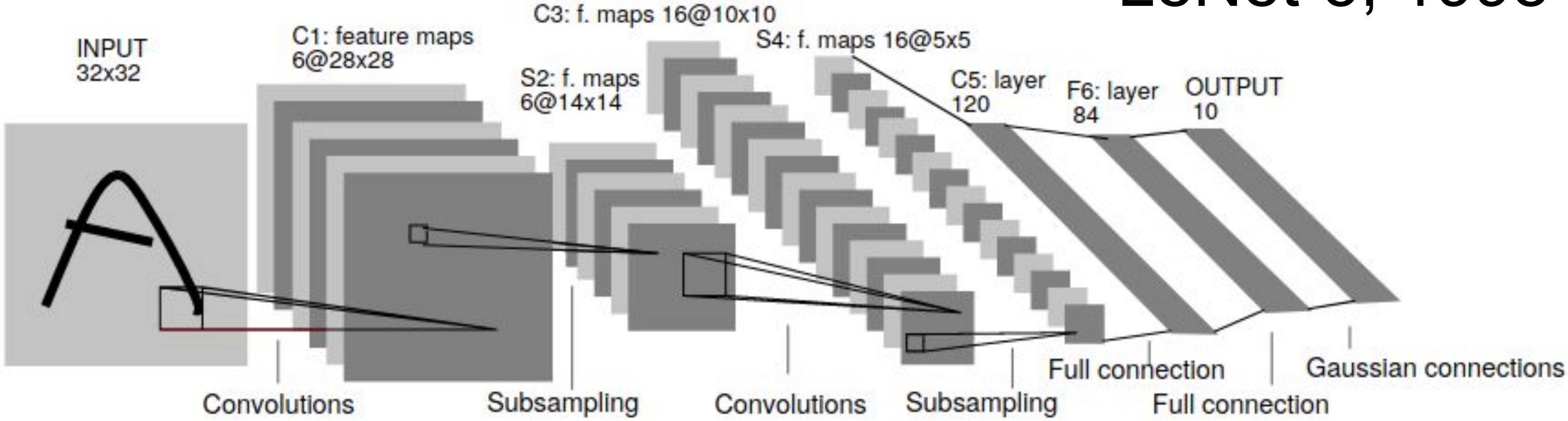
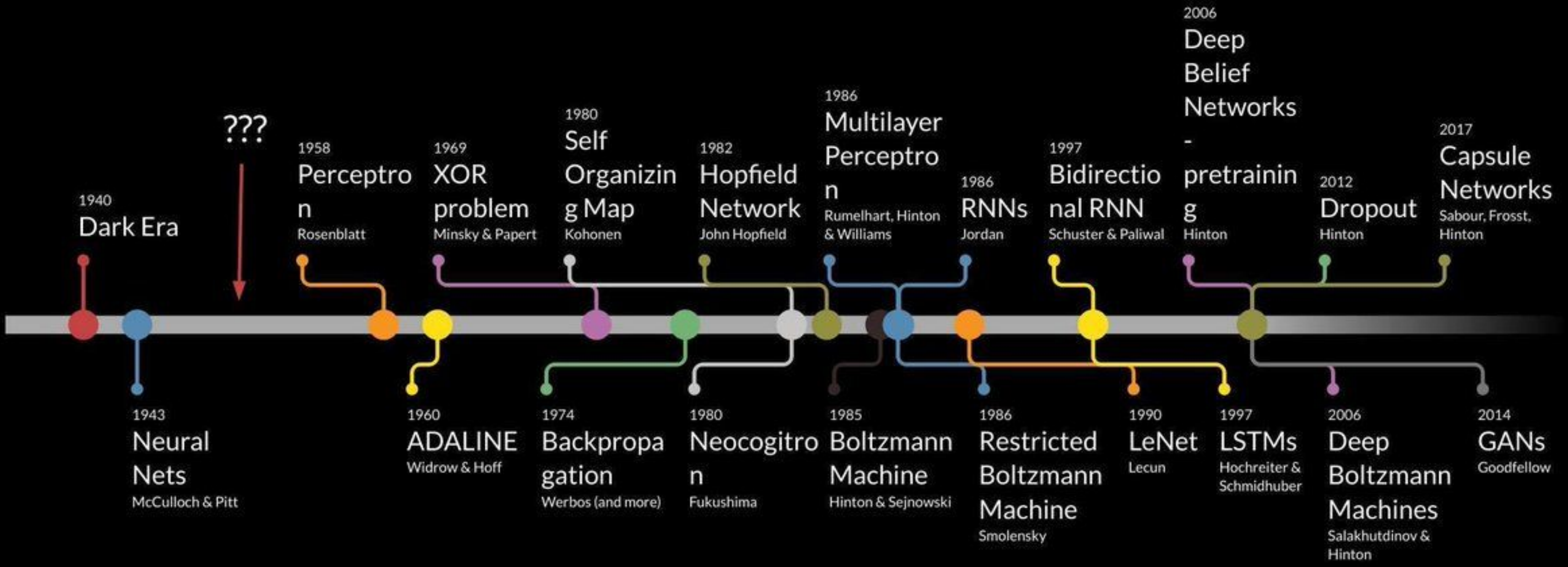
AlexNet, 2012



LeNet-5, 1998



Deep Learning Timeline



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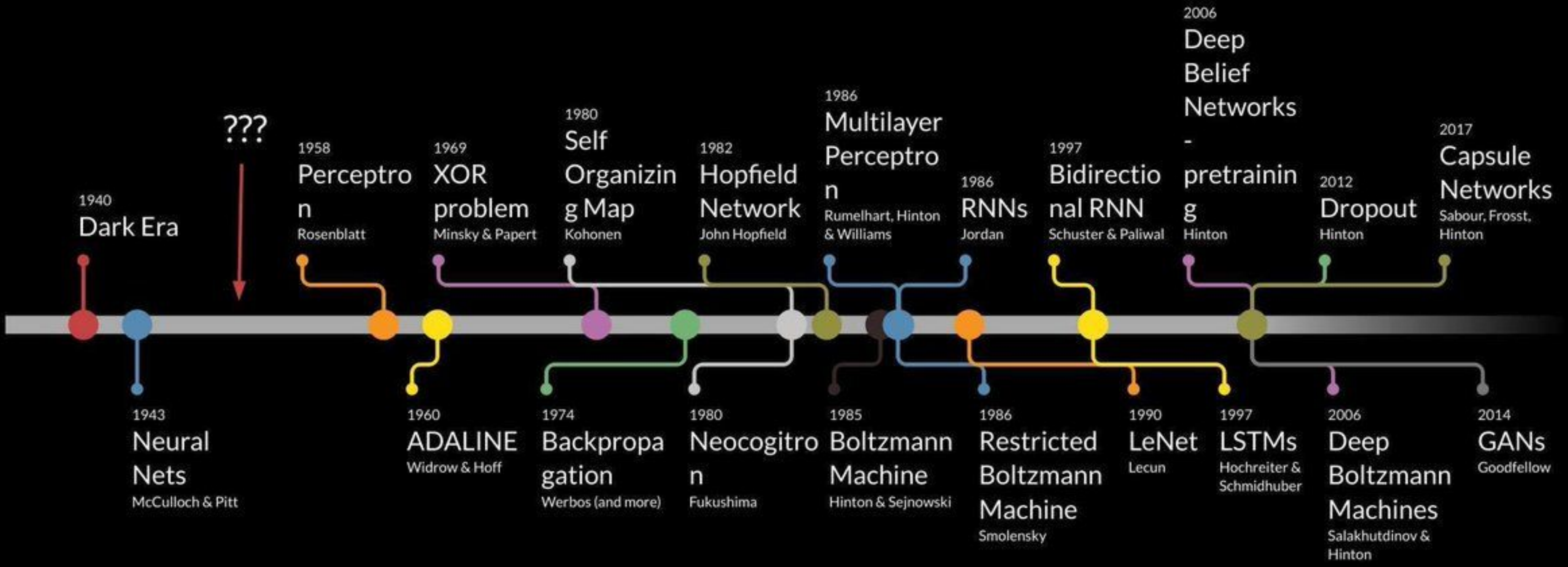
GANs, 2014+

https://thispersondoesnotexist.com/

Deep Learning Timeline



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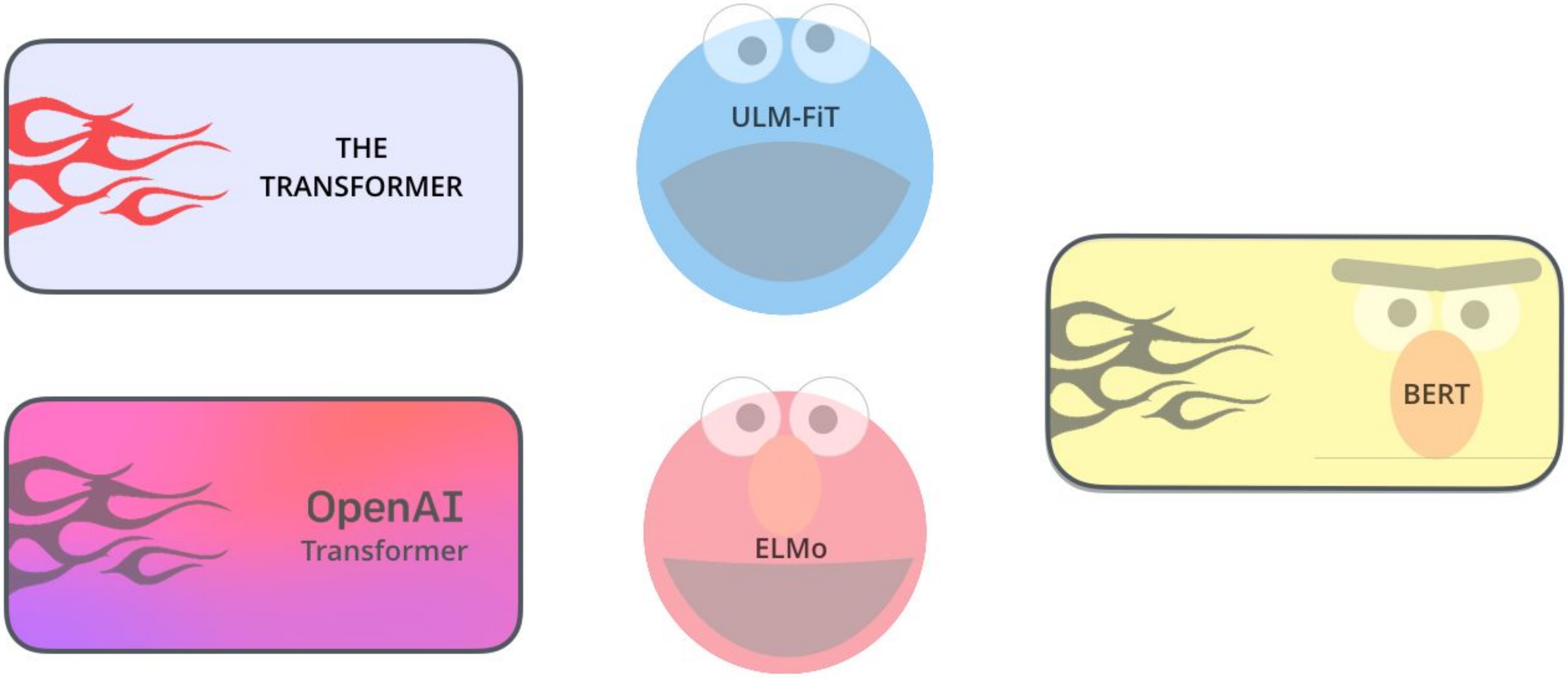
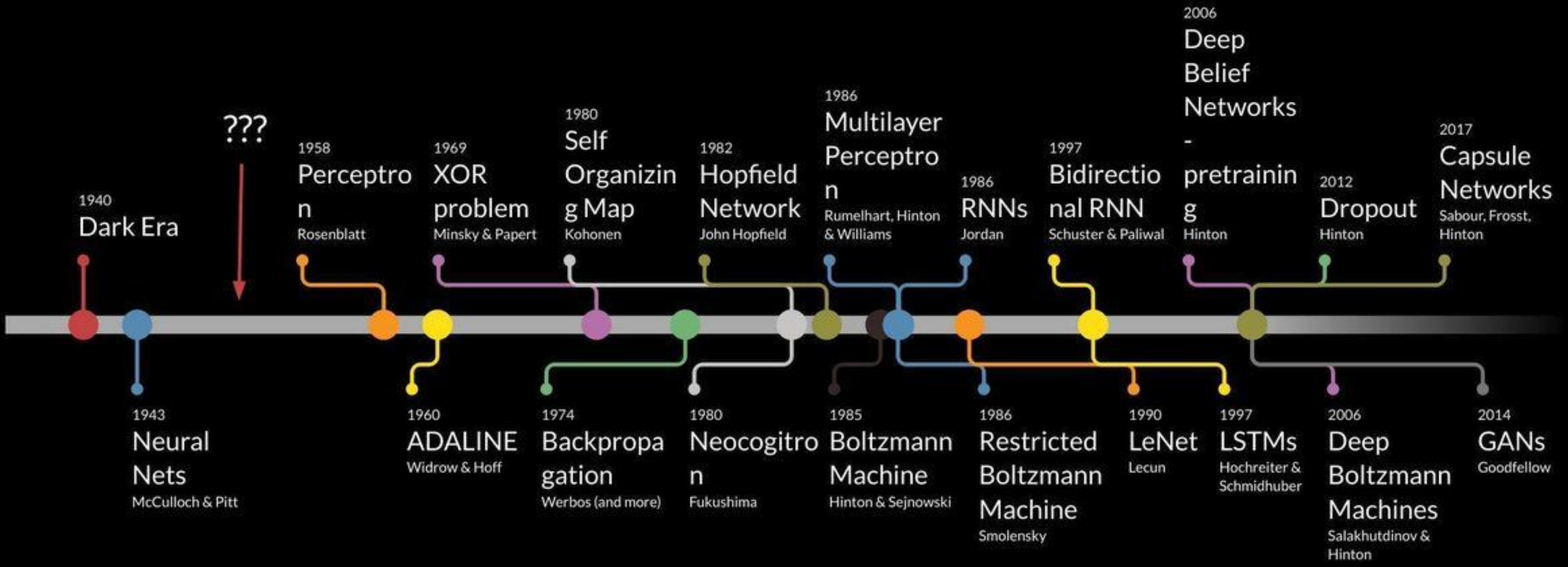
Transformer, BERT, GPT-2 and more, 2017+



Deep Learning Timeline



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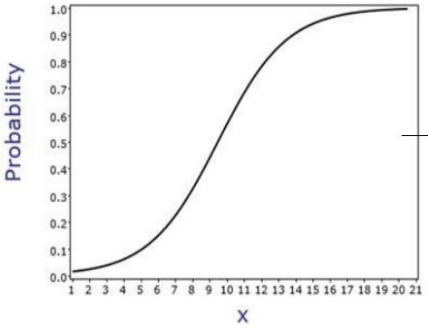
Backpropagation

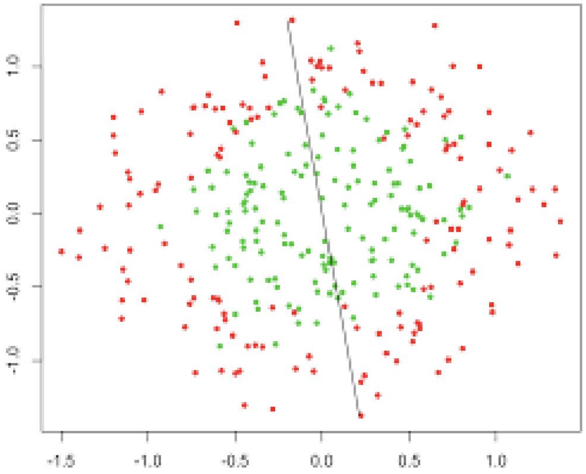
Logistic regression

X → Wx +b →  → P(y)

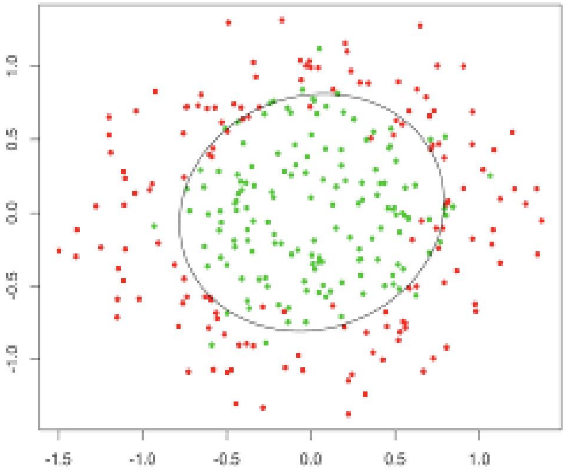
$$P(y|x)=\sigma(w\cdot x+b)$$

$$L=-\sum_i y_i \log P(y|x_i)+(1-y_i)\log(1-P(y|x_i))$$

Problem: nonlinear dependencies



What we have

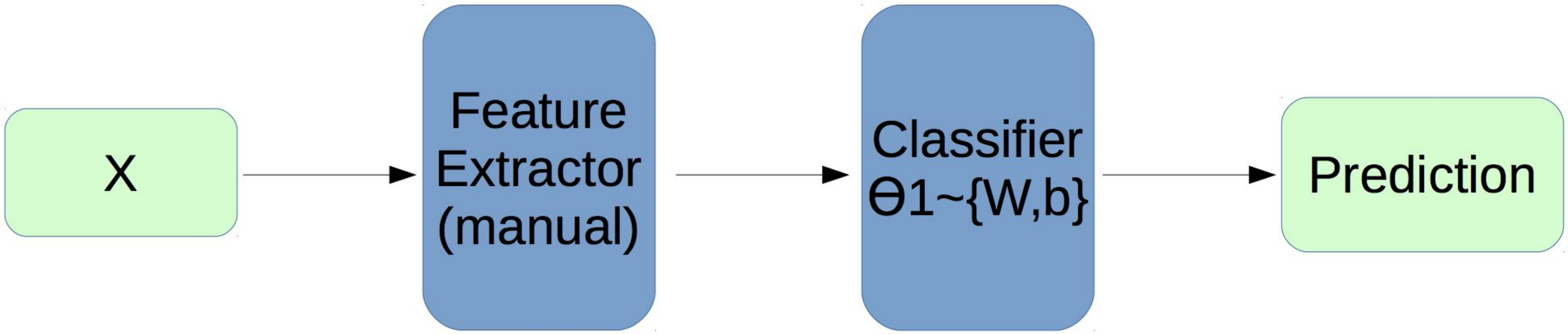


What we want

Logistic regression (generally, linear model) need feature engineering to show good results.

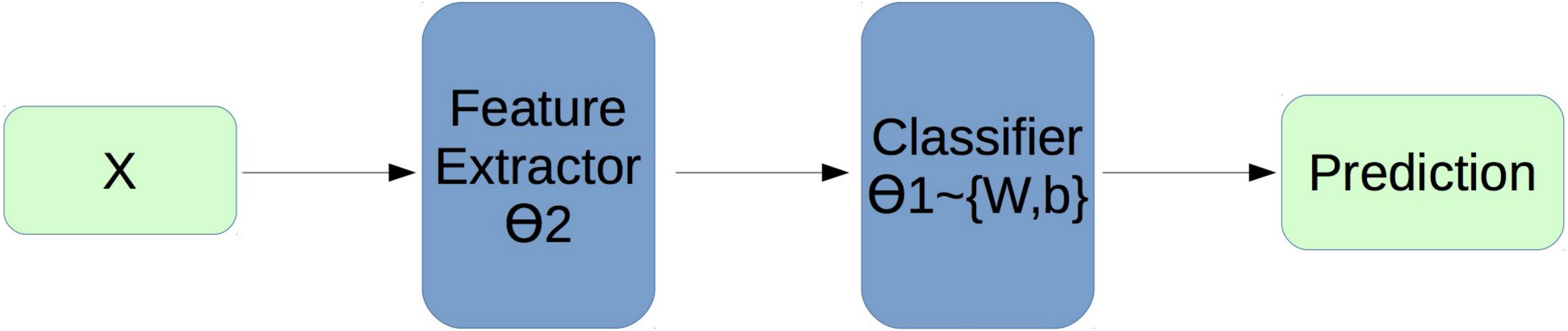
And feature engineering is an *art*.

Classic pipeline



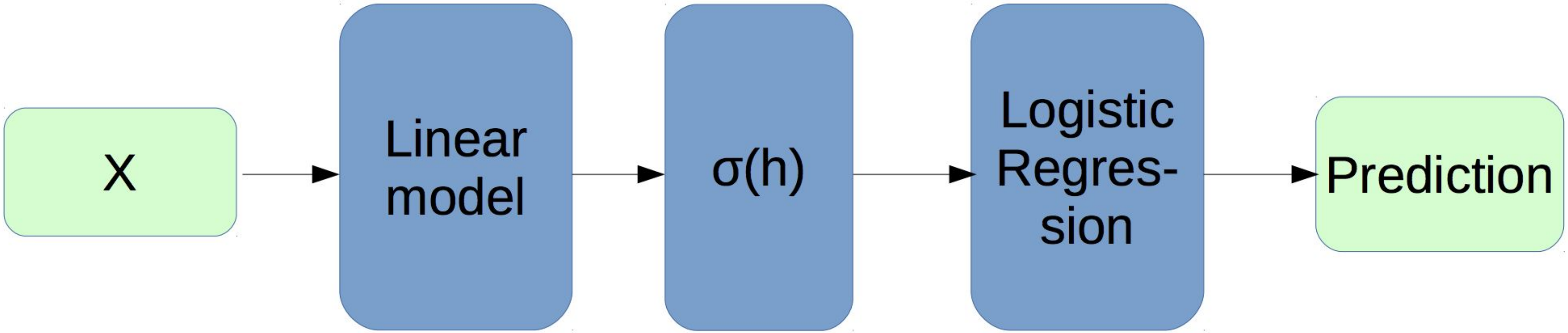
Handcrafted features, generated by experts.

NN pipeline



Automatically extracted features.

NN pipeline: example



E.g. two logistic regressions one after another.

Actually, it's a neural network.

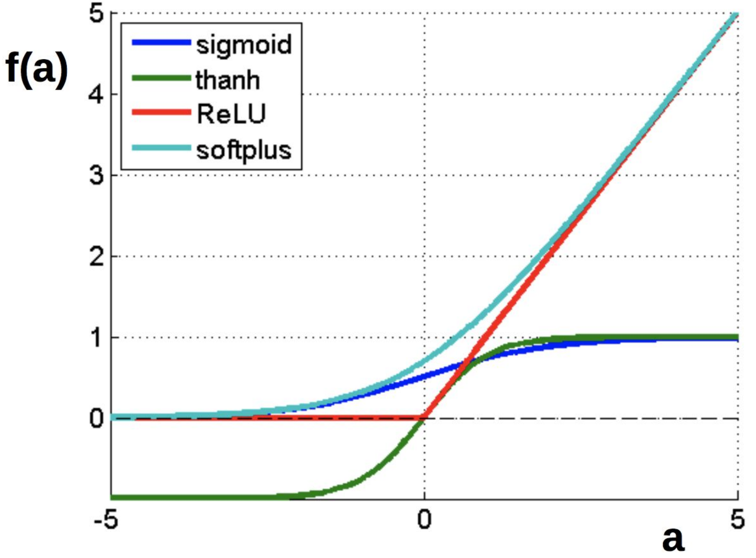
Activation functions: nonlinearities

$$f(a) = \frac{1}{1 + e^{a}}$$

$$f(a) = \tanh(a)$$

$$f(a) = \max(0, a)$$

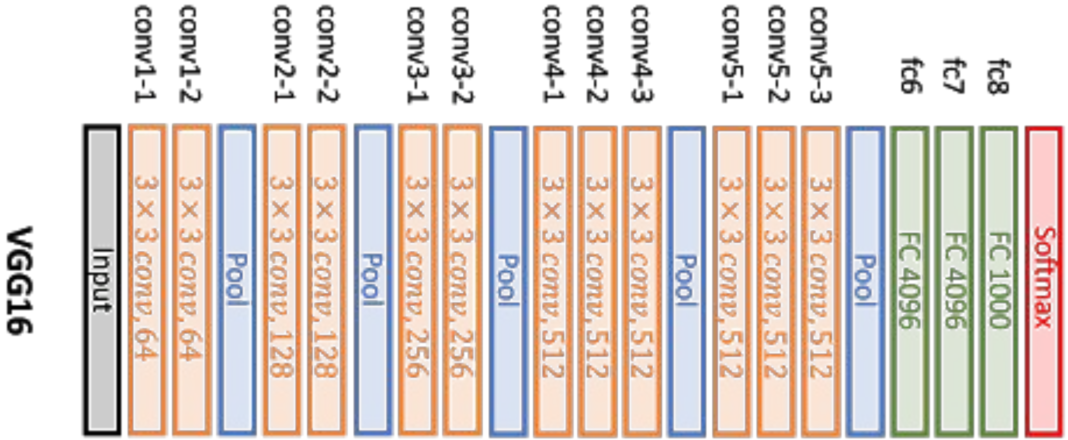
$$f(a) = \log(1 + e^{a})$$



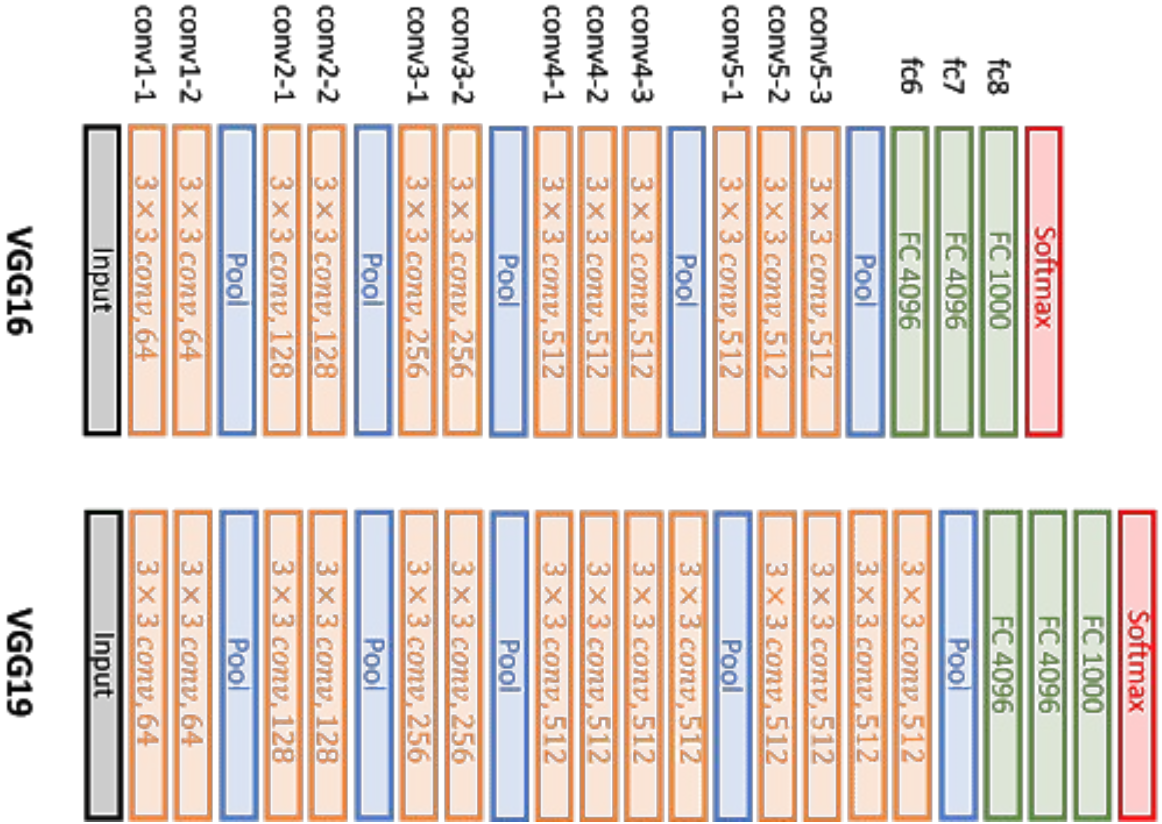
Some generally accepted terms

- Layer – a building block for NNs :
  - Dense/Linear/FC layer: $f(x) = Wx+b$
  - Nonlinearity layer: $f(x) = \sigma(x)$
  - Input layer, output layer
  - A few more we will cover later
- Activation function – function applied to layer output
  - Sigmoid
  - tanh
  - ReLU
  - Any other function to get nonlinear intermediate signal in NN
- Backpropagation – a fancy word for "chain rule"

Actually, networks can be deep



And deeper...



VGG16
conv1-1
conv1-2
conv2-1
conv2-2
conv3-1
conv3-2
conv4-1
conv4-2
conv4-3
conv5-1
conv5-2
conv5-3
fc6
fc7
fc8
Input
3 × 3 conv, 64
3 × 3 conv, 64
Pool
3 × 3 conv, 128
3 × 3 conv, 128
Pool
3 × 3 conv, 256
3 × 3 conv, 256
Pool
3 × 3 conv, 512
3 × 3 conv, 512
3 × 3 conv, 512
Pool
3 × 3 conv, 512
3 × 3 conv, 512
3 × 3 conv, 512
Pool
FC 4096
FC 4096
FC 1000
Softmax
VGG19
Input
3 × 3 conv, 64
3 × 3 conv, 64
Pool
3 × 3 conv, 128
3 × 3 conv, 128
Pool
3 × 3 conv, 256
3 × 3 conv, 256
Pool
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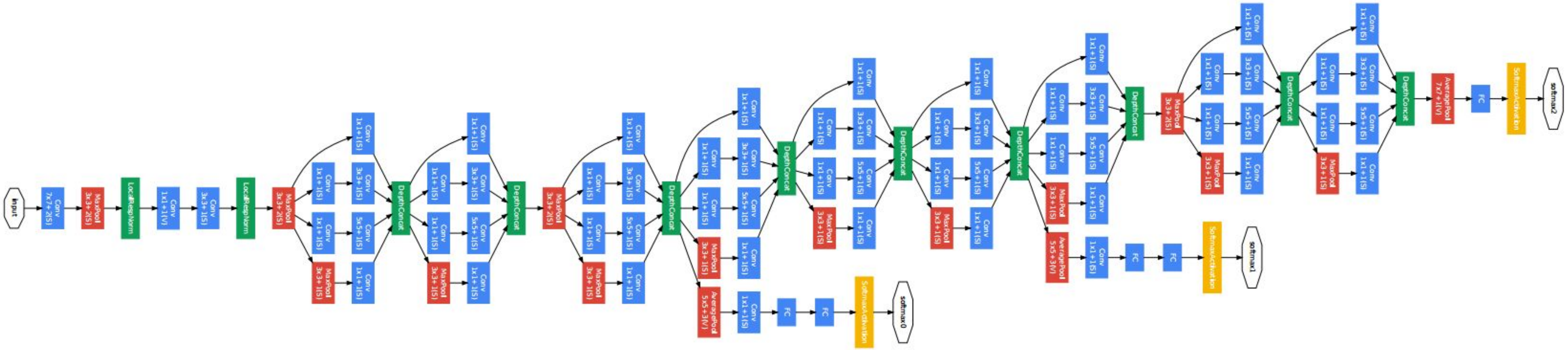
3 × 3 conv, 512
Pool
FC 4096
FC 4096
FC 1000
Softmax

Much deeper...

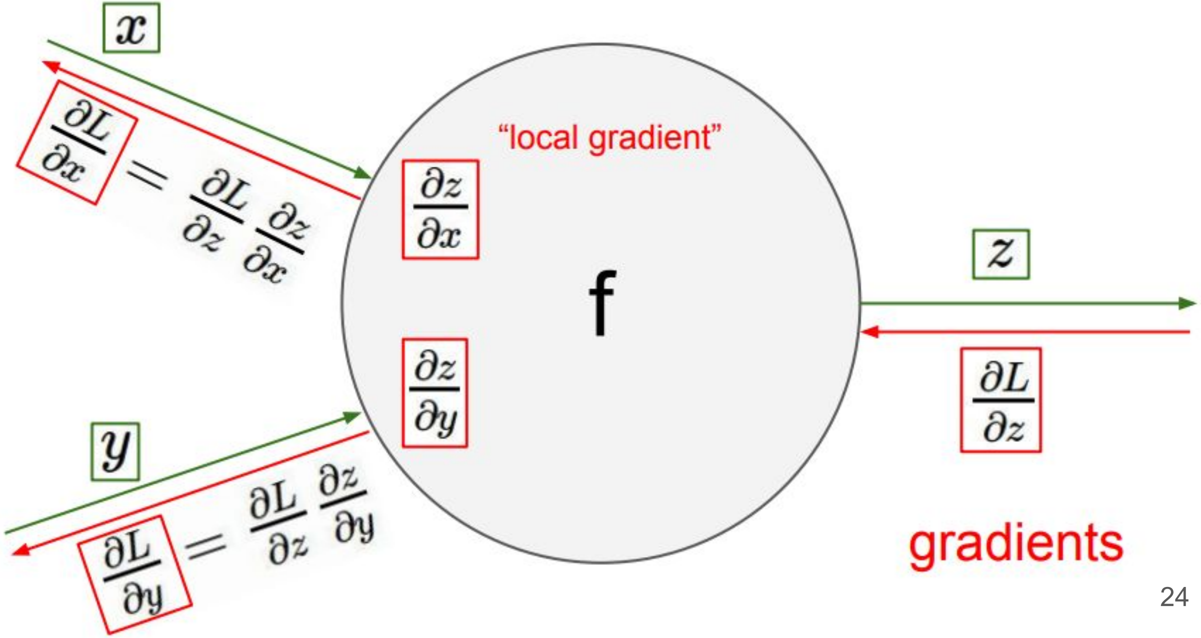


How to train it?

Backpropagation and chain rule

Chain rule is just simple math: $$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial x}$$

Backprop is just way to use it in NN training.

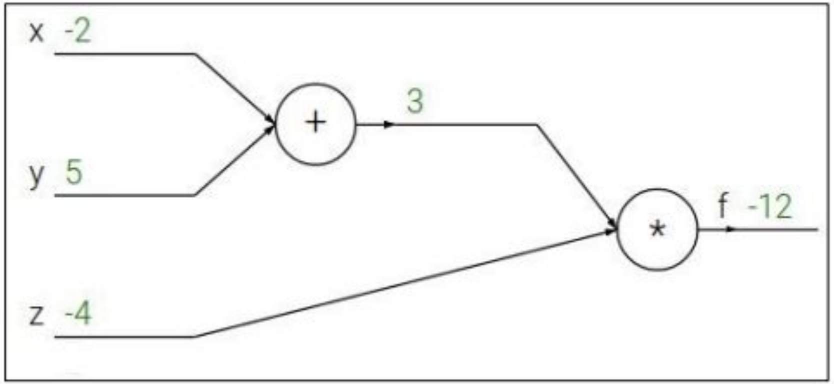


source: http://cs231n.github.io

Backpropagation example

$$f(x, y, z) = (x + y)z$$

e.g. x = -2, y = 5, z = -4



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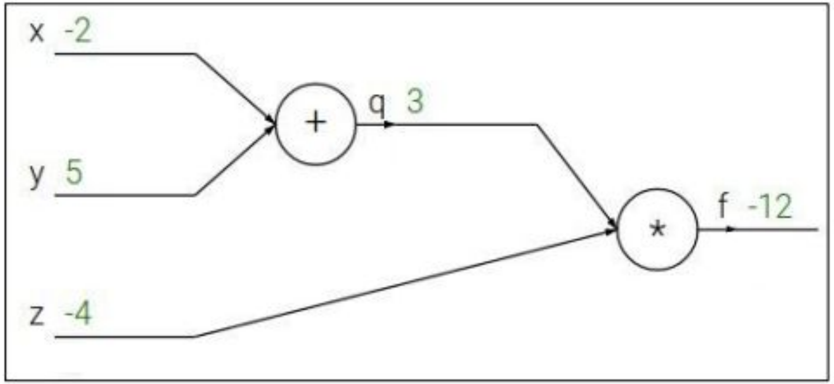

$$q = x + y \qquad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$f = qz \qquad \frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

Want: $\dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}, \dfrac{\partial f}{\partial z}$

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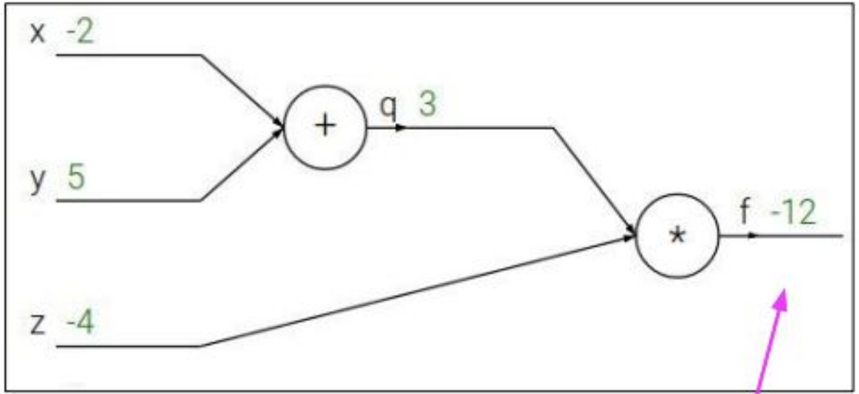

$$\frac{\partial f}{\partial f}$$

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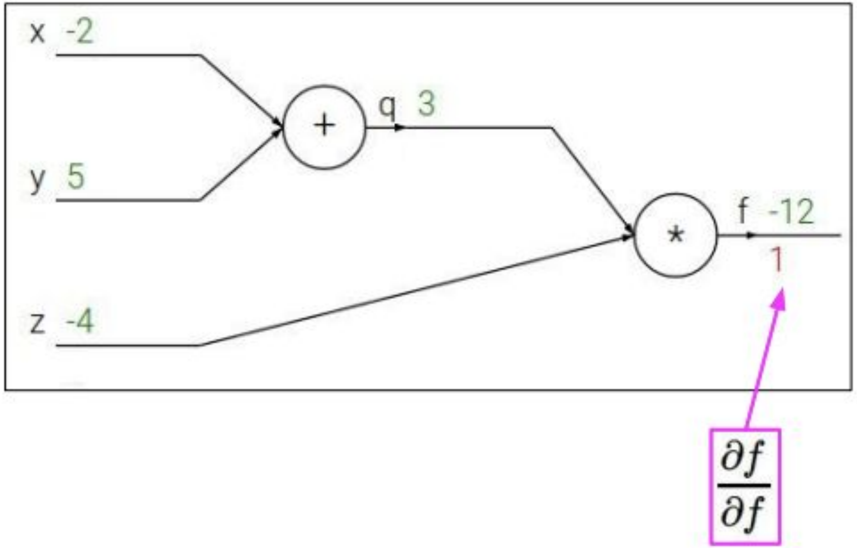

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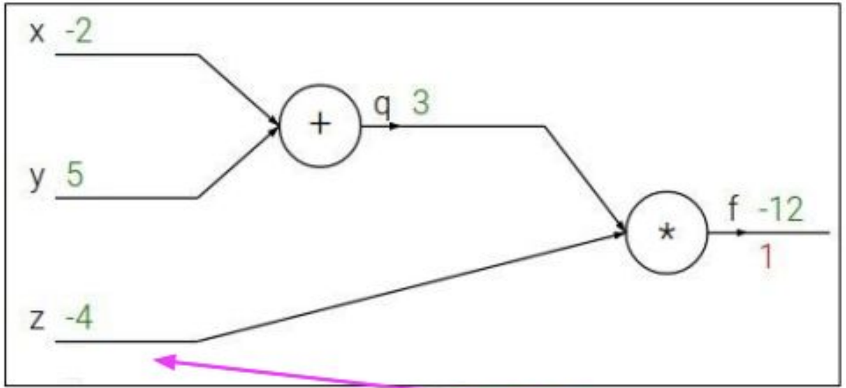

$$\frac{\partial f}{\partial z}$$

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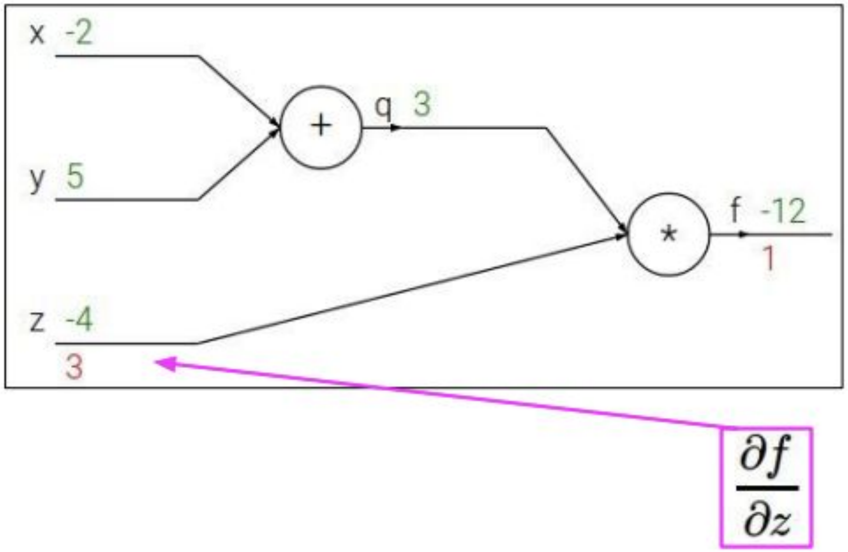

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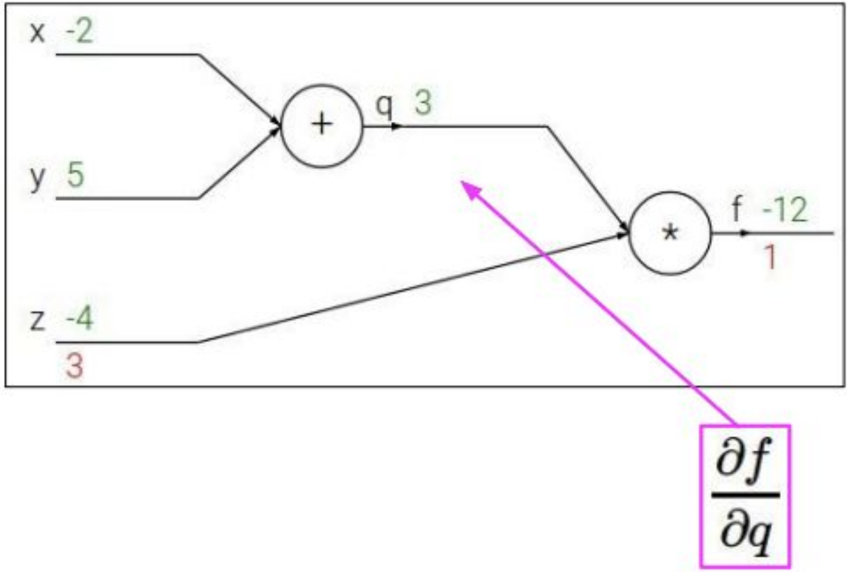

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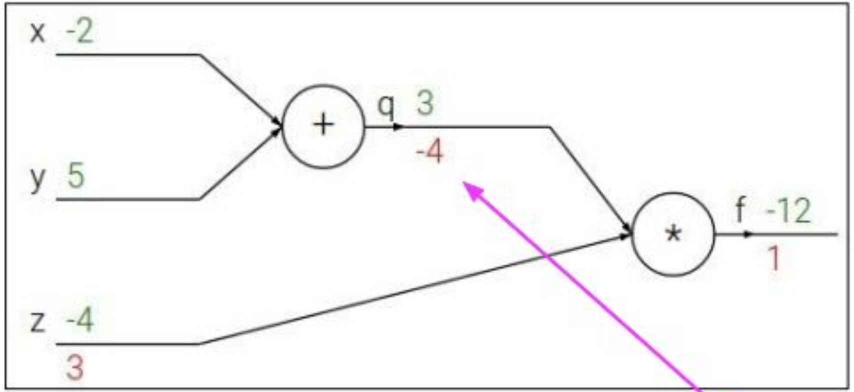

$$\frac{\partial f}{\partial q}$$

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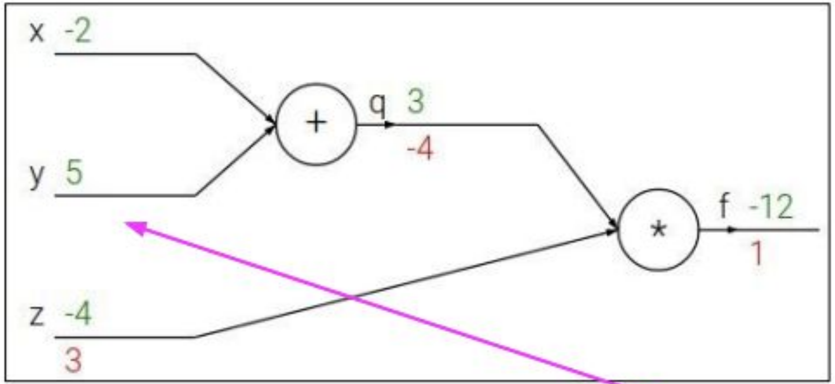

$$\boxed{\frac{\partial f}{\partial y}}$$

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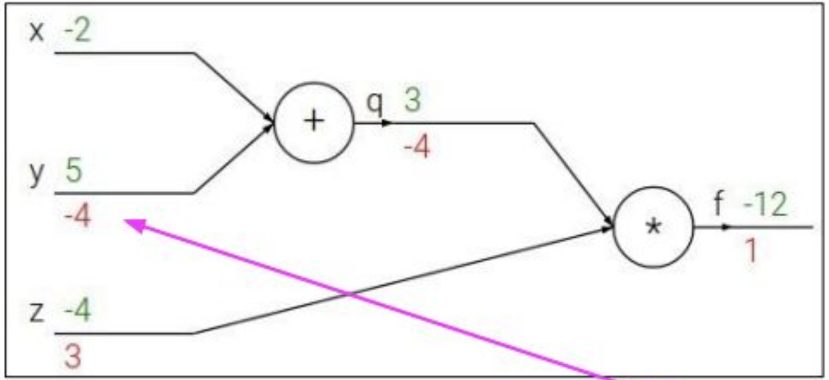

$\frac{\partial f}{\partial y}$

Chain rule:

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial q}\frac{\partial q}{\partial y}$$

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Backpropagation example

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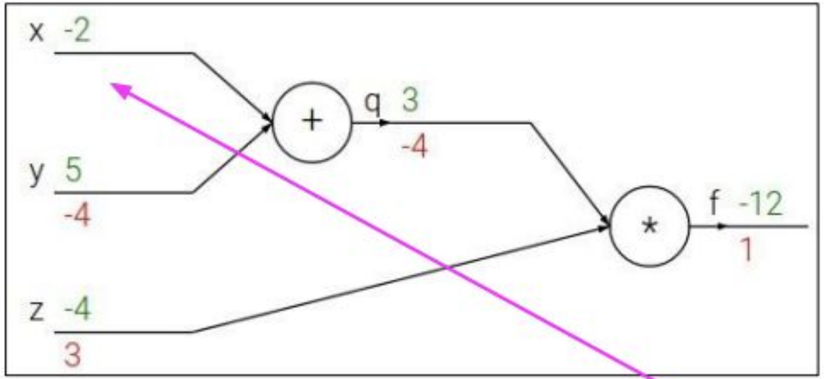

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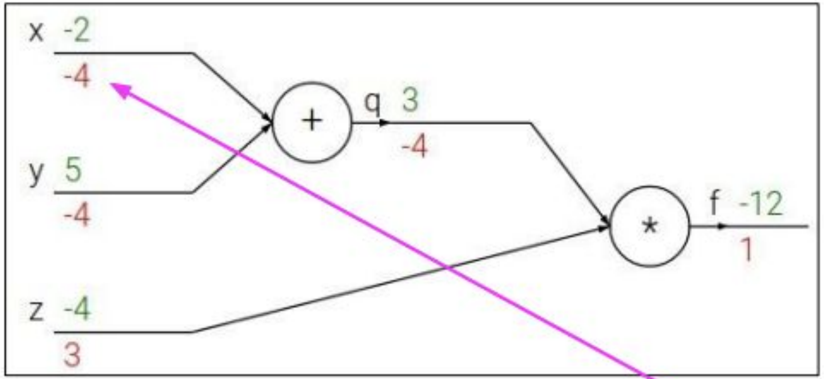

$$\frac{\partial f}{\partial x}$$

Chain rule:

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial x}$$

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Backpropagation and chain rule

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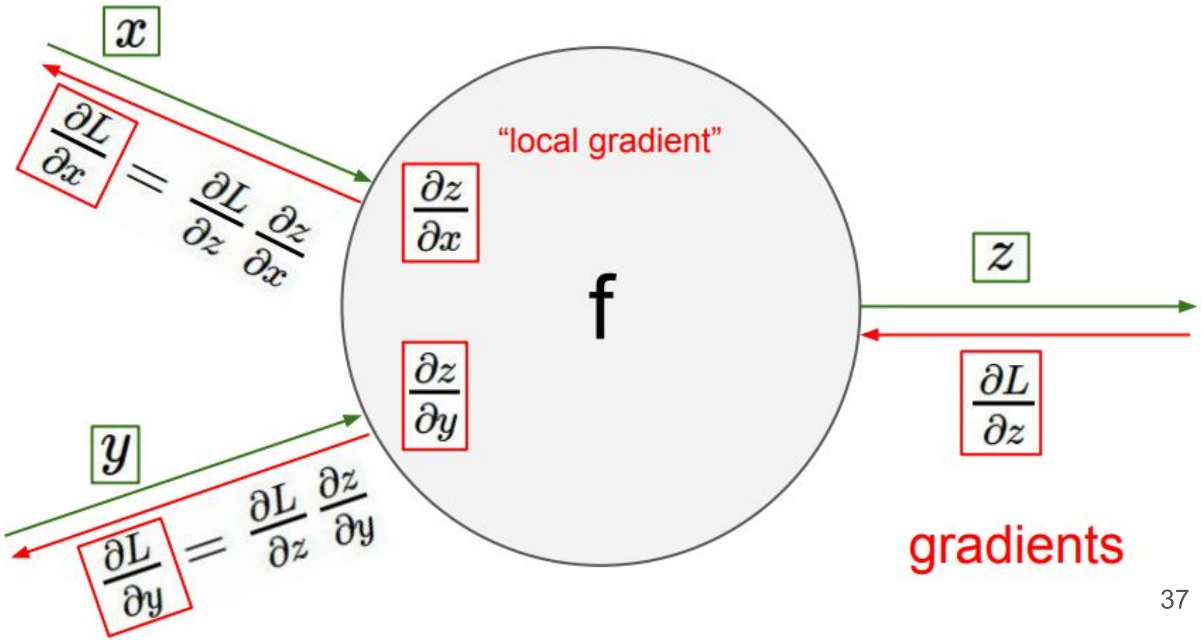
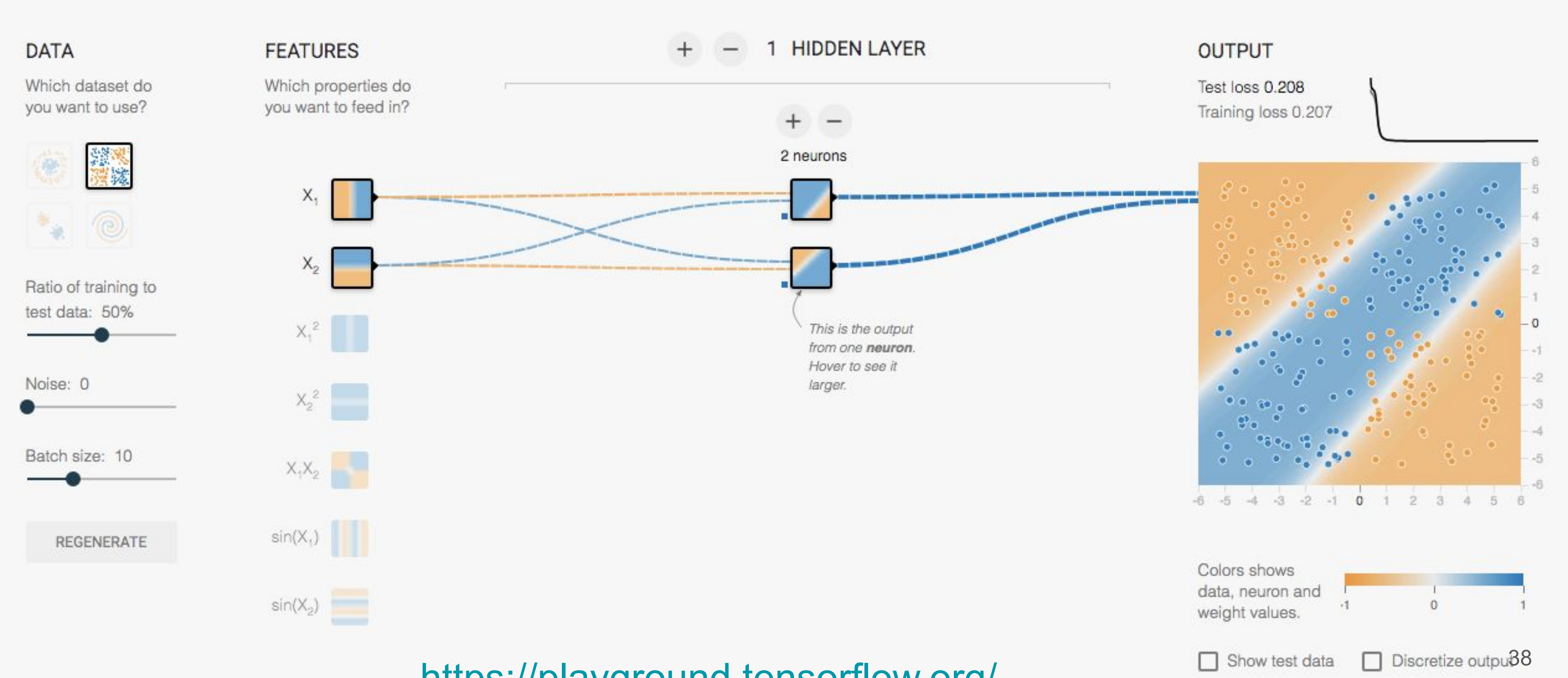
Backprop is just way to use it in NN training.



source: http://cs231n.github.io

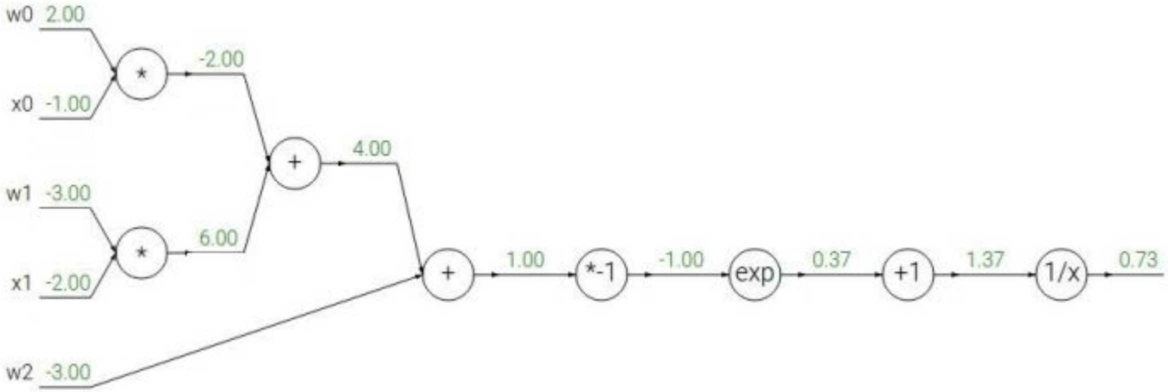
Interactive playground



https://playground.tensorflow.org/

Backpropagation example

Another example: $$f(w,x)=\frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



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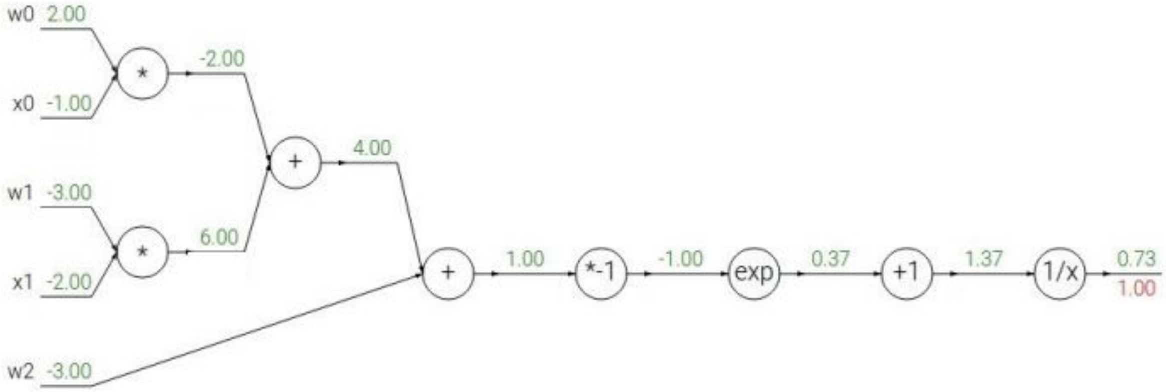

$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x \qquad\qquad f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a \qquad\qquad f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$$

source: http://cs231n.github.io

Backpropagation example

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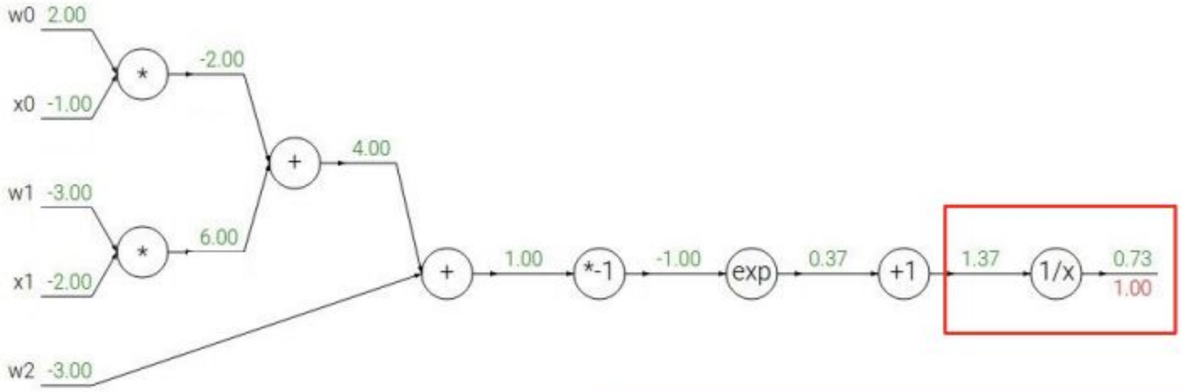

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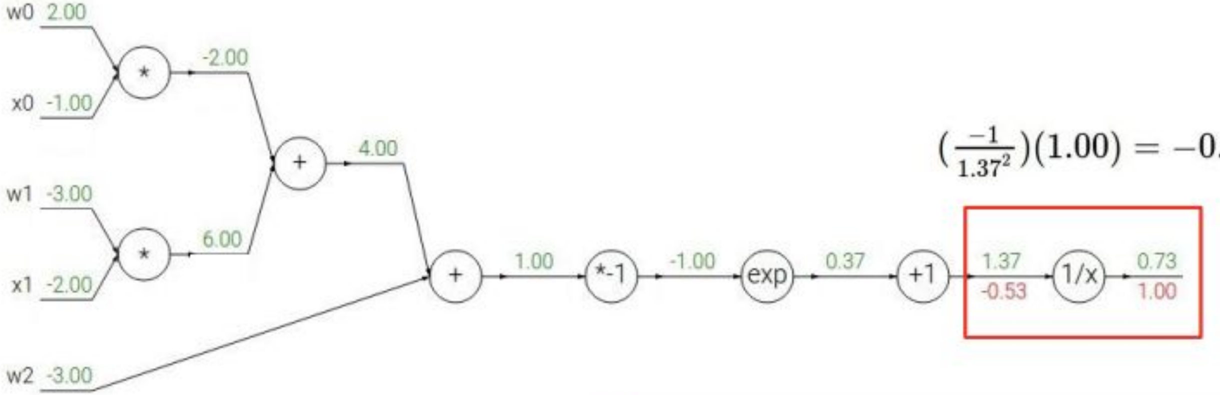
Backpropagation example

Another example: $$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

$$\left(\frac{-1}{1.37^2}\right)(1.00) = -0.53$$



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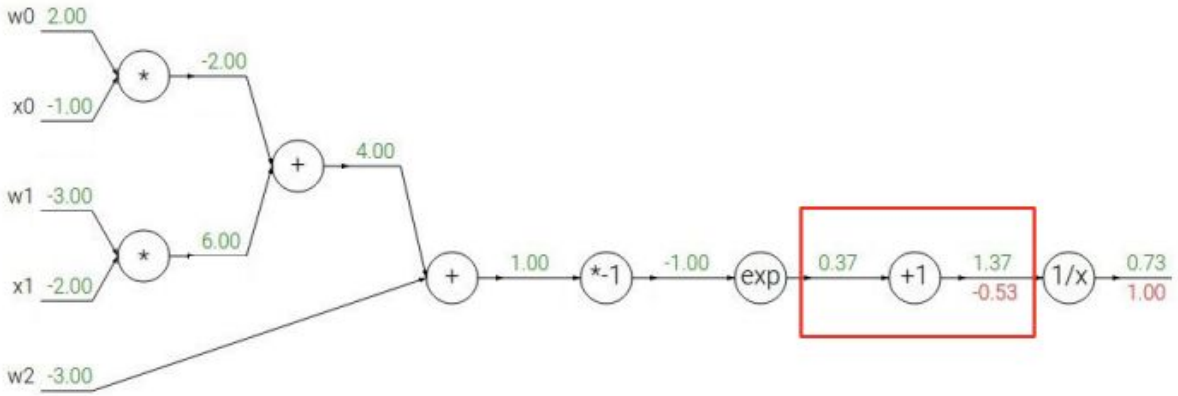
$\boxed{f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2}$

$f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$

source: http://cs231n.github.io

Backpropagation example

Another example: $$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



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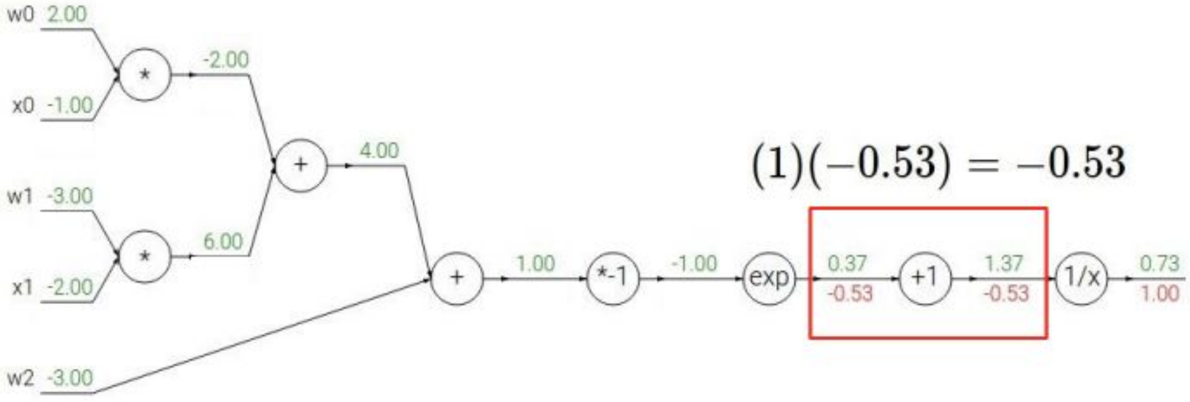

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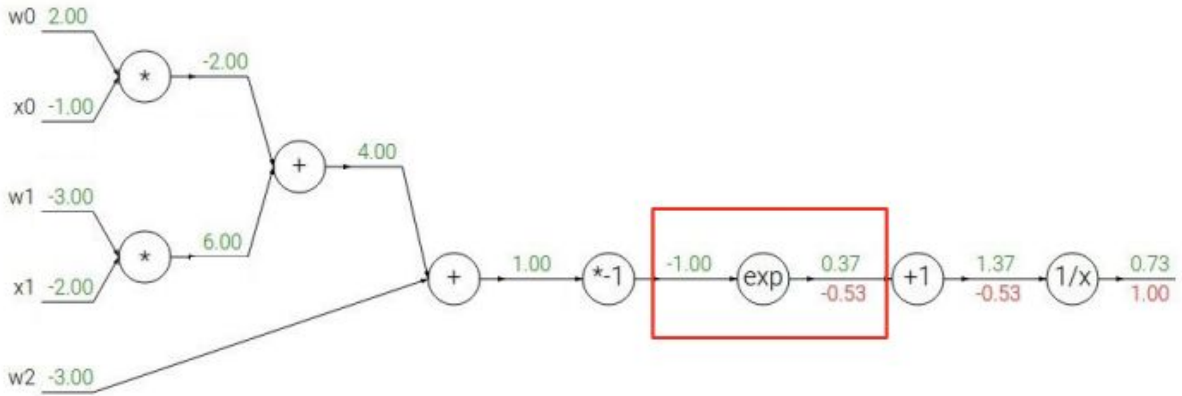

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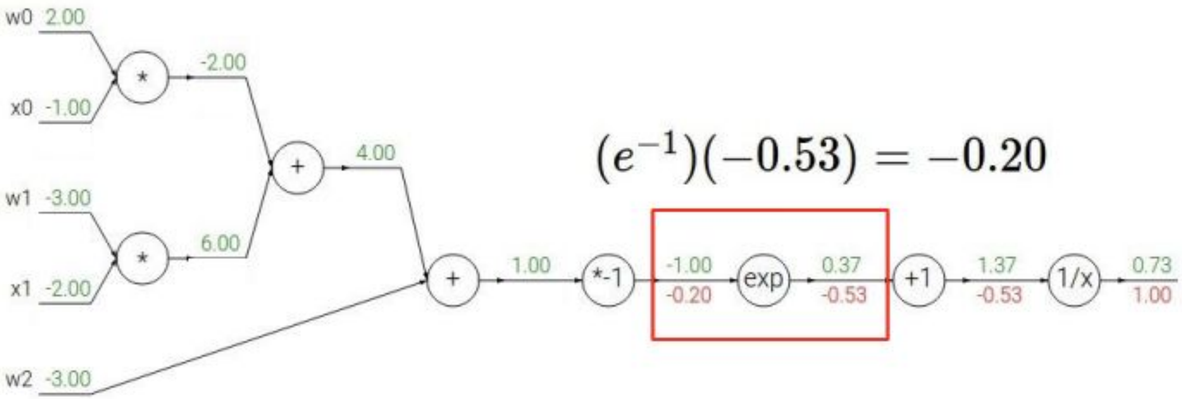

$$(e^{-1})(-0.53) = -0.20$$

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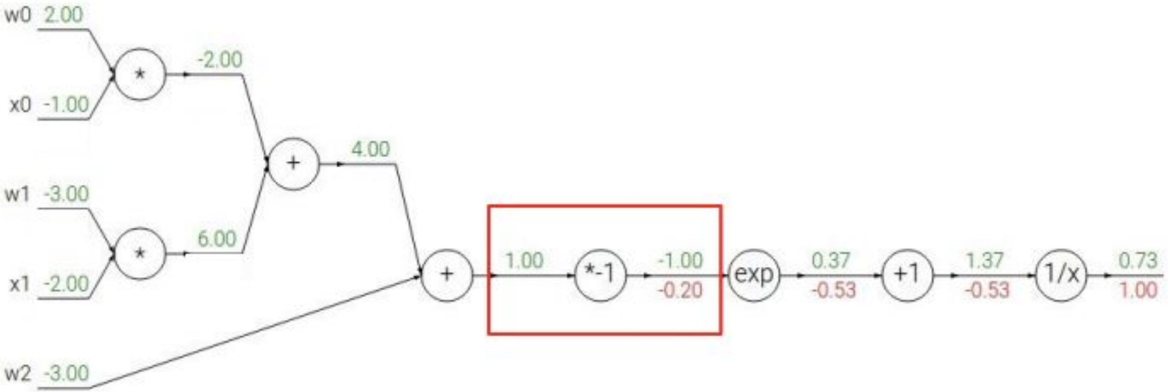

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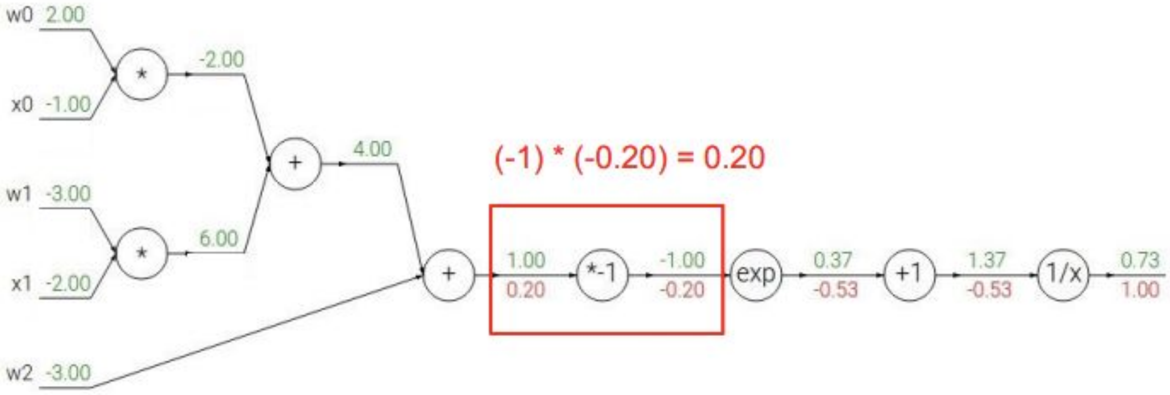

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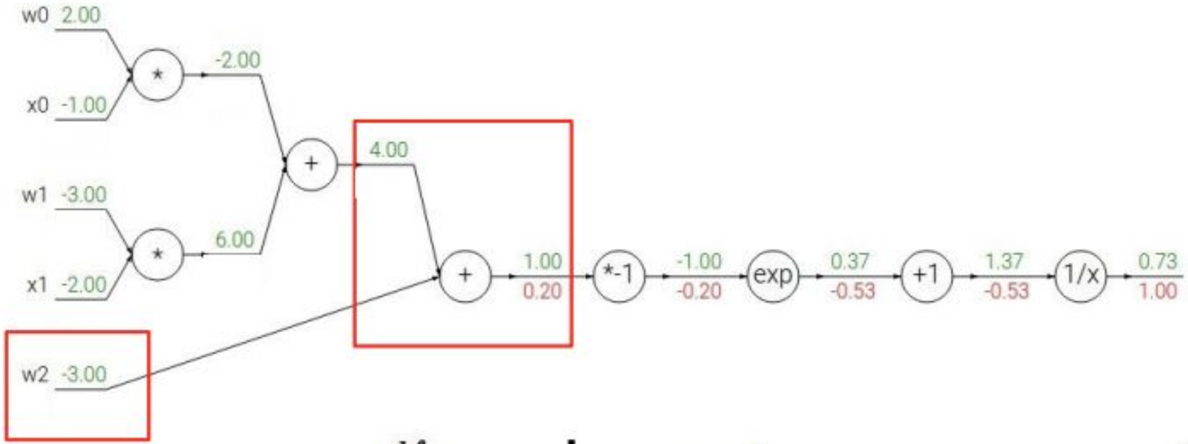

$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x \qquad\qquad f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

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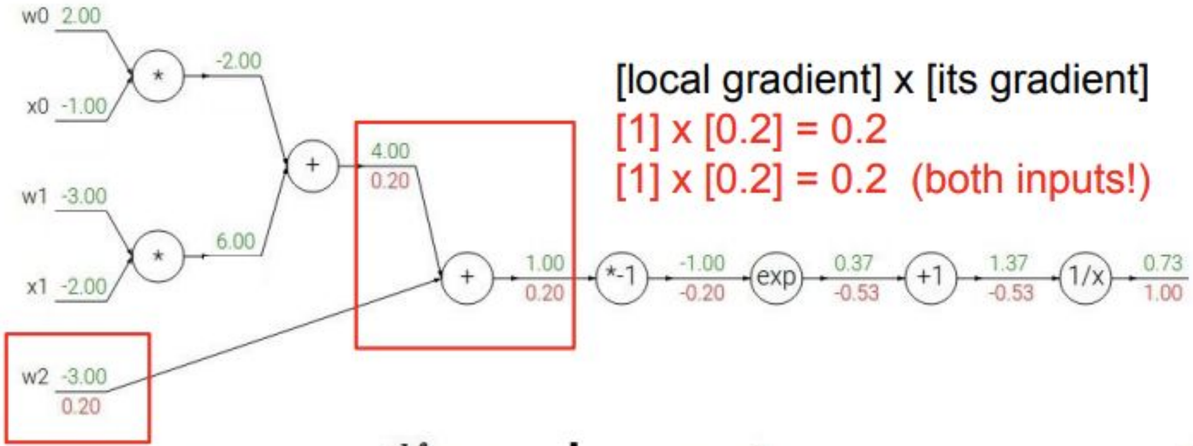

[local gradient] x [its gradient]

[1] x [0.2] = 0.2

[1] x [0.2] = 0.2 (both inputs!)

$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$$

$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$$

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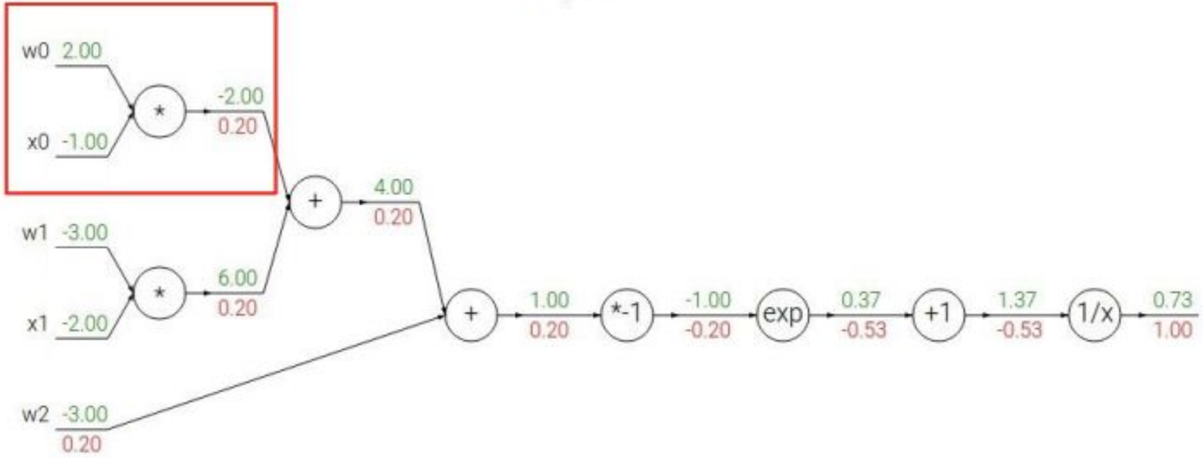

$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$		$f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$	
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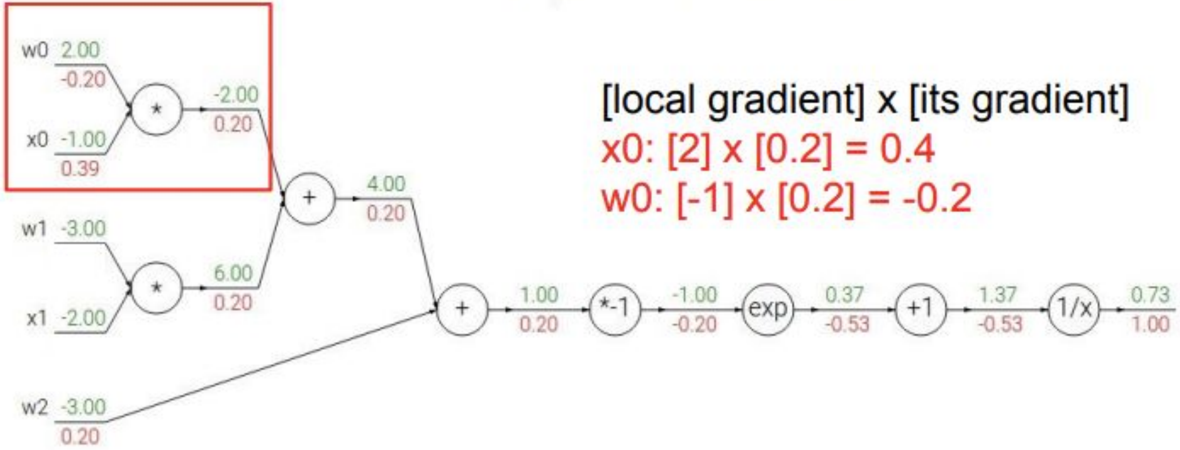
source: http://cs231n.github.io

Backpropagation example

Another example: $f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$



[local gradient] x [its gradient]
x0: [2] x [0.2] = 0.4
w0: [-1] x [0.2] = -0.2

$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$ | $f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$

$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$ | $f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$

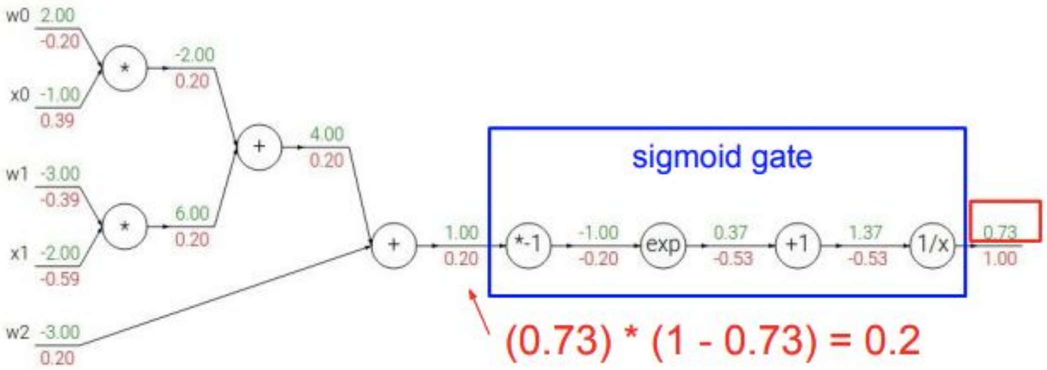
source: http://cs231n.github.io

Backpropagation example

$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

$$\boxed{\sigma(x) = \frac{1}{1+e^{-x}}}$$ sigmoid function

$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1+e^{-x})^2} = \left(\frac{1+e^{-x}-1}{1+e^{-x}}\right)\left(\frac{1}{1+e^{-x}}\right) = (1-\sigma(x))\,\sigma(x)$$

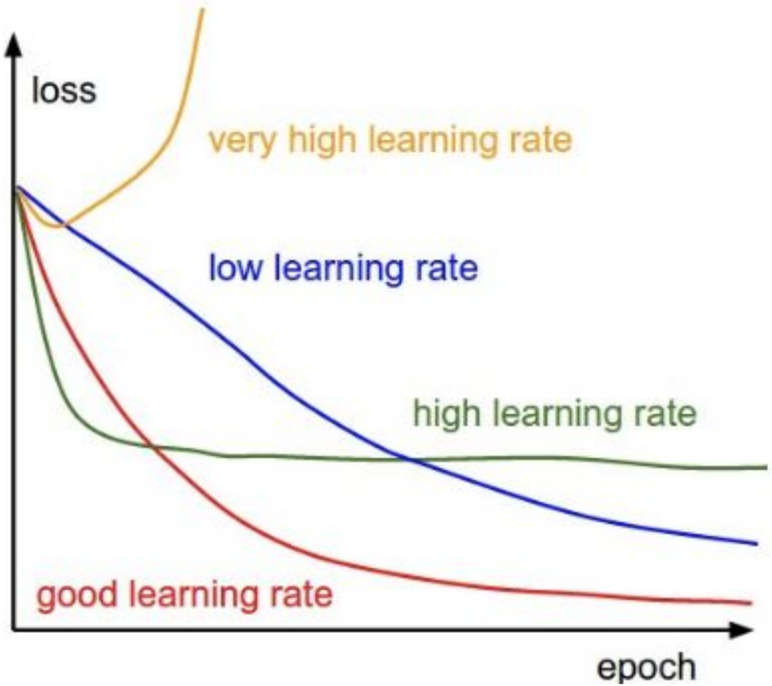


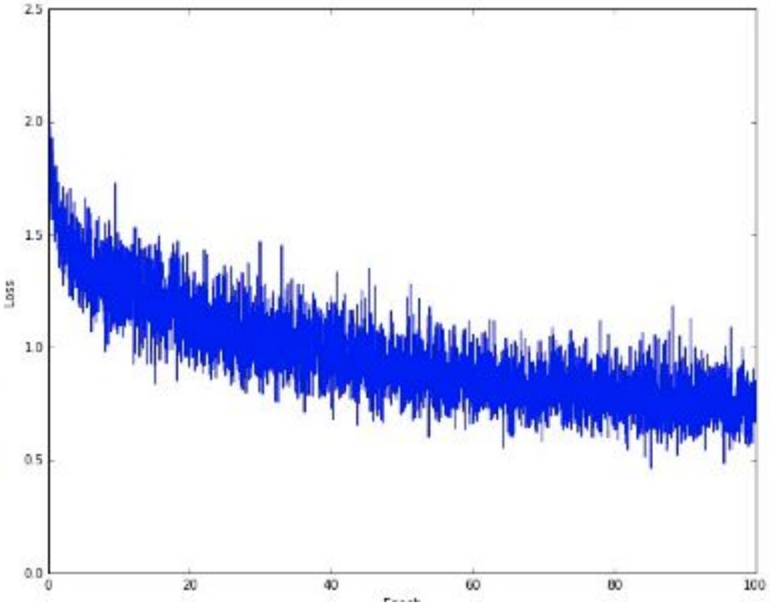
source: http://cs231n.github.io

Gradient optimization

Stochastic gradient descent (and variations) is used to optimize NN parameters.

$$x_{t+1} = x_t - \text{learning rate} \cdot dx$$





source: http://cs231n.github.io/neural-networks-3/

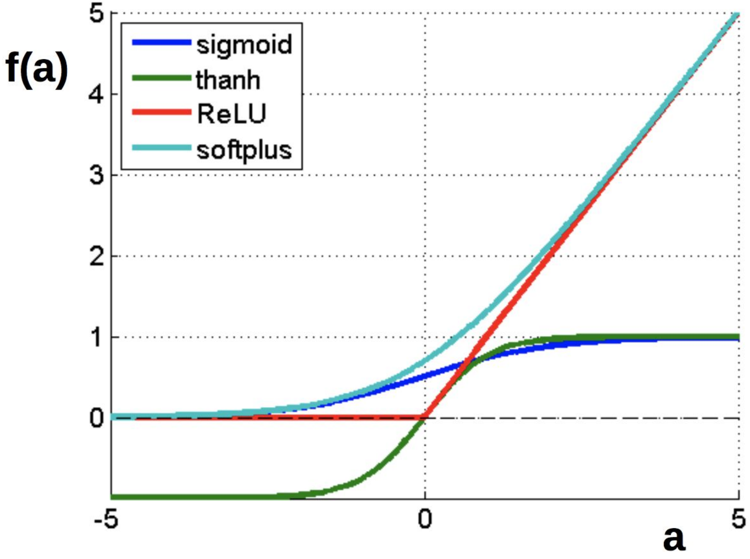
Once more: nonlinearities

$$f(a) = \frac{1}{1+e^{a}}$$

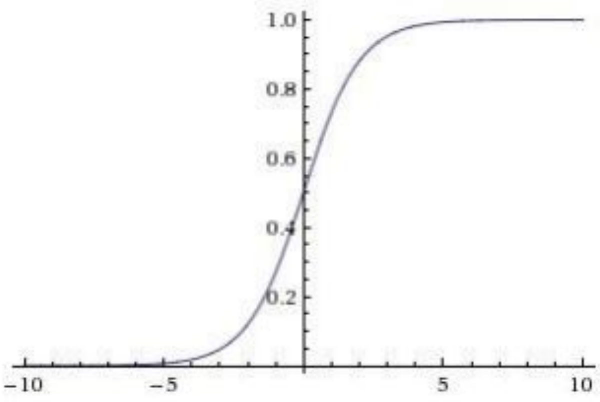
$$f(a) = \tanh(a)$$

$$f(a) = \max(0, a)$$

$$f(a) = \log(1 + e^{a})$$



Activation functions



Sigmoid

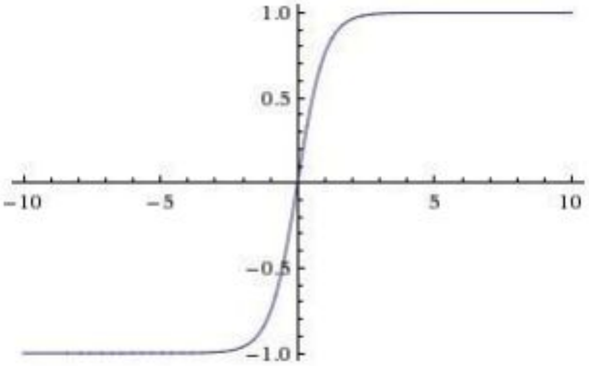
$$f(a) = \frac{1}{1 + e^{a}}$$

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating "firing rate" of a neuron

3 problems:

1. Saturated neurons "kill" the gradients
2. Sigmoid outputs are not zero-centered
3. exp() is a bit compute expensive

source: http://cs231n.github.io



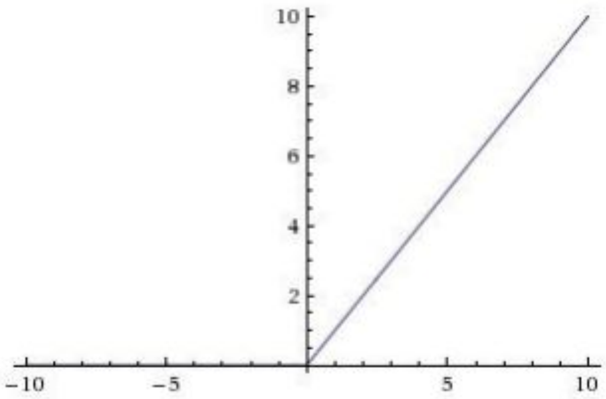
tanh(x)

- Squashes numbers to range [-1,1]
- zero centered (nice)
- still kills gradients when saturated :(

$$f(a) = \tanh(a)$$

source: http://cs231n.github.io

Activation functions



ReLU
(Rectified Linear Unit)

$$f(a) = \max(0, a)$$

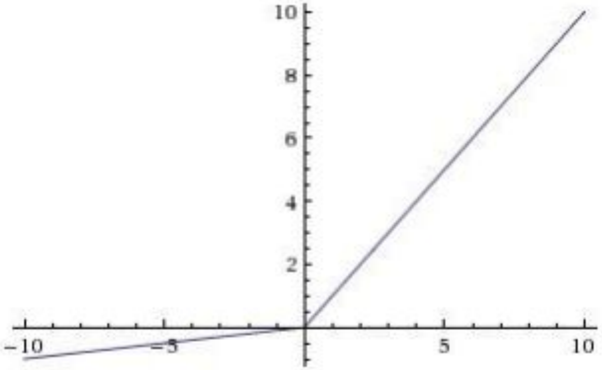
- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)

- Not zero-centered output
- An annoyance:

hint: what is the gradient when x < 0?

source: http://cs231n.github.io

Activation functions



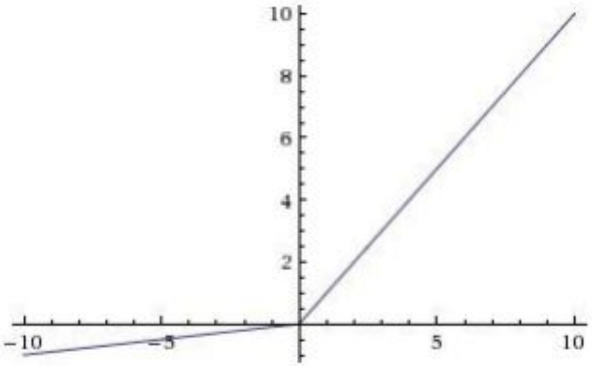
- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- **will not "die".**

Leaky ReLU

$$f(x) = \max(0.01x, x)$$

source: http://cs231n.github.io

Activation functions



- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- **will not "die".**

Leaky ReLU

$$f(x) = \max(0.01x, x)$$

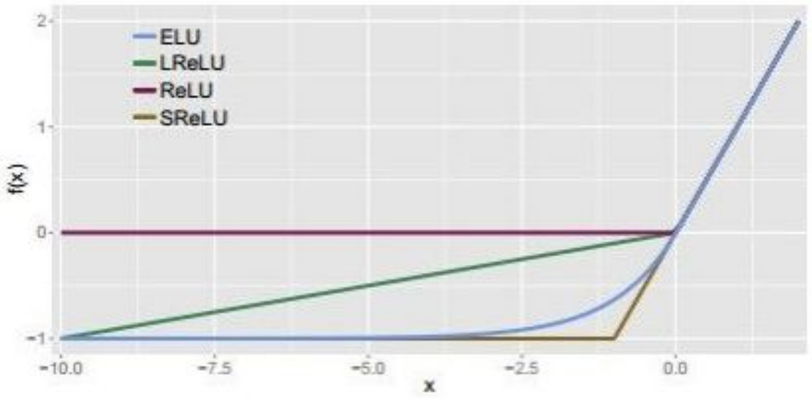
Parametric Rectifier (PReLU)

$$f(x) = \max(\alpha x, x)$$

backprop into \alpha
(parameter)

source: http://cs231n.github.io

Exponential Linear Units (ELU)



$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha\,(\exp(x) - 1) & \text{if } x \leq 0 \end{cases}$$

- All benefits of ReLU
- Does not die
- Closer to zero mean outputs

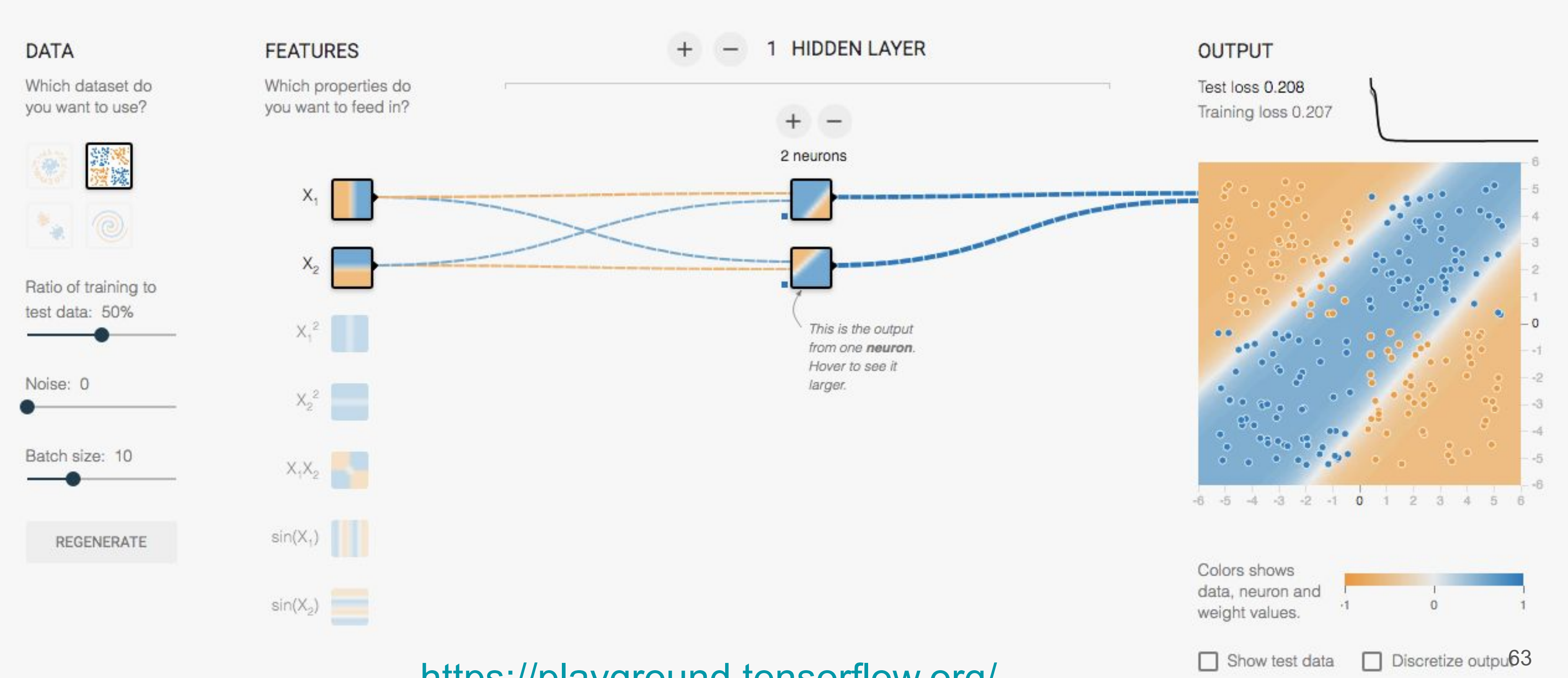
- Computation requires exp()

source: http://cs231n.github.io

Activation functions: sum up

- Use ReLU as baseline approach
- Be careful with the learning rates
- Try out Leaky ReLU or ELU
- Try out tanh but do not expect much from it
- Do not use Sigmoid

Don’t miss the interactive playground



https://playground.tensorflow.org/


WHO'S AWESOME?
You're Awesome