


EASE

$$\min_B \|X - XB\|_F^2 + \lambda \|B\|_F^2$$

$$\text{s.t. } \text{diag}(B) = 0$$

$$\mathcal{L} = \|X - XB\|_F^2 + \lambda \|B\|_F^2 + 2f^T \text{diag}(B)$$

$$\|X - XB\|_F^2 = \text{Tr}[(X - XB)^T (X - XB)] =$$

$$= \underbrace{\|X\|_F^2}_{\text{neg. ab. of } B \Rightarrow \nabla_B \dots = 0} - 2 \text{Tr}(X^T X B) + \text{Tr}(B^T X^T X B)$$

$$\text{Tr}(A) = a_{11} + a_{22} + \dots = \sum a_{ii}$$

$$\nabla_B (-2 \text{Tr}(X^T X B)) = -2 X^T X$$

$$\nabla_B (\text{Tr}(B^T X^T X B)) = 2 X^T X B$$

$$\nabla_B \|B\|_F^2 = 2B$$

$$\nabla_B (2f^T \text{diag}(B)) = 2 \text{diag}(f)$$

$$\nabla_B \mathcal{L} = 0 = (X^T X + \lambda I)B - (X^T X - \text{diag}(f))$$

$$\Rightarrow \hat{B} = (X^T X + \lambda I)^{-1} (X^T X - \text{diag}(f))$$

$$\Rightarrow \hat{B} = \hat{P} (\hat{P}^{-1} - \lambda I - \text{diag}(f)) \ominus$$

$$X^T X = \hat{P}^{-1} - \lambda I$$

$$\ominus I - \hat{P} \text{diag}(\tilde{f})$$

$$\hat{P} = (X^T X + \lambda I)^{-1}$$

$$\tilde{f} \stackrel{\text{def}}{=} \lambda \cdot \vec{1} + f$$

$$\text{diag}(\hat{B}) = 0 = \vec{1} - \text{diag}(\hat{P}) \odot \tilde{f}$$

$$\Rightarrow \tilde{f} = \vec{1} \odot \text{diag}(\hat{P})$$

$$\Rightarrow \hat{B} = I - \hat{P} \text{diag}(\vec{1} \odot \text{diag}(\hat{P}))$$

elementwise
product

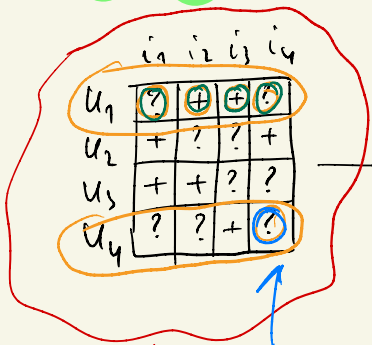
elementwise
division

$$\left(\frac{p_{11}}{p_{11}}, \frac{1}{p_{11}}, \dots \right) = I$$

$$\hat{B}_{ij} = \begin{cases} 0, & i=j \\ -\frac{\hat{P}_{ij}}{\hat{P}_{ji}}, & i \neq j \end{cases}$$

BPR

Bayesian Personalized Ranking



0	1	1	0
1	0	0	1
1	1	0	0
0	0	1	0

$+$ > ? $\Rightarrow +$
 $+$ = +, ? = ? $\Rightarrow ?$
 $? < + \Rightarrow -$

$u_1: i >_{u_1} j$

i_1, i_2, i_3, i_4

i_1	i_2	i_3	i_4
j_1	/	+	+
j_2	-	/	?
j_3	-	?	/
j_4	?	+	+

$u_4: i >_{u_4} j$

i_1, i_2, i_3, i_4
 j_1, j_2, j_3, j_4

i_1	i_2	i_3	i_4
/	?	+	?
?	/	+	?
-	-	/	-
?	?	+	/

$i_3 >_{u_4} j_4 \Rightarrow +$

$i_4 <_{u_4} j_3 \Rightarrow -$

BPR-Opt:

$$\sum_{(u,i,j) \in \mathcal{D}_S} \ln[\sigma(\hat{x}_{uij})] - \frac{1}{2} \|\Theta\|^2$$

$S \subseteq U \times I$ - implicit feedback matrix

$$I_u^+ = \{i \in I : (u,i) \in S\}$$

$$\mathcal{D}_S = \{(u,i,j) : \underbrace{i \in I_u^+}_{pos} \wedge \underbrace{j \in I \setminus I_u^+}_{neg}\}$$

Matrix factorization:

$$\hat{x}_{uij} = x_{ui} - x_{uj}$$

$$\hat{x}_{ui} = \sum_{f=1}^F w_{uf} \cdot h_{if}$$

$$\hat{X} = WH^T, \Theta = (W, H)$$