EASE

$$\min_{B} \|X - XB\|_F^2 + \lambda \|B\|_F^2$$

s.t. $\quad diag(B) = 0$

$$\mathcal{L} = \|X - XB\|_F^2 + \lambda \|B\|_F^2 + 2\gamma^T diag(B)$$

$$\|X - XB\|_F^2 = Tr\Big[\underbrace{(X - XB)^T}_{X^T - B^T X^T}(X - XB)\Big] =$$

$$= \|X\|_F^2 - 2Tr(X^T X B) + Tr(B^T X^T X B)$$

не зав. от B

$\Rightarrow \nabla_B \ldots = 0$

$Tr(A) = a_{11} + a_{22} + \ldots = \sum a_{ii}$

$$\nabla_B \left(-2Tr(X^T X B)\right) = -2X^T X$$

$$\nabla_B \left(Tr(B^T X^T X B)\right) = 2X^T X B$$

$$\nabla_B \|B\|_F^2 = 2B$$

$$\nabla_B \left(2\gamma^T diag(B)\right) = 2\, diag(\gamma)$$

$$\nabla_B \mathcal{L} = 0 = (X^T X + \lambda I)B - (X^T X - diag\, \gamma)$$

$$\Rightarrow \boxed{\hat{B} = (X^T X + \lambda I)^{-1}(X^T X - diag(\gamma))}$$

$$\Rightarrow \hat{B} = \hat{P}\left(\hat{P}^{-1} - \lambda I - \mathrm{diag}(\gamma)\right) = I - \hat{P}\,\mathrm{diag}(\tilde{\gamma})$$

$$X^T X = \hat{P}^{-1} - \lambda I$$

$$\hat{P} = (X^T X + \lambda I)^{-1}$$

$$\tilde{\gamma} \overset{\text{def}}{=} \lambda \cdot \vec{1} + \gamma$$

elementwise product

$$\mathrm{diag}(\hat{B}) = 0 = \vec{1} - \mathrm{diag}(\hat{P}) \odot \tilde{\gamma}$$

$$\Rightarrow \tilde{\gamma} = \vec{1} \oslash \mathrm{diag}(\hat{P})$$

elementwise division

$$\Rightarrow \hat{B} = I - \hat{P}\,\mathrm{diag}\left(\vec{1} \oslash \mathrm{diag}(\hat{P})\right)$$

$$\begin{pmatrix} \frac{1}{p_{11}} & & \\ & \ddots & \\ & & \end{pmatrix} \neq I$$

$$\hat{B}_{ij} = \begin{cases} 0, & i = j \\ -\dfrac{\hat{P}_{ij}}{\hat{P}_{jj}}, & i \neq j \end{cases}$$

BPR

Bayesian Personalized Ranking

$$
\begin{array}{c|cccc}
 & i_1 & i_2 & i_3 & i_4 \\
\hline
u_1 & ? & + & + & ? \\
u_2 & + & ? & ? & + \\
u_3 & + & + & ? & ? \\
u_4 & ? & ? & + & ?
\end{array}
\rightarrow
\begin{bmatrix}
0 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
$$

$+ > ? \Rightarrow +$

$\underbrace{+ = +,\ ? = ?}$ $\Rightarrow ?$

$? < + \Rightarrow -$

$u_1 : \underline{i >_{u_1} j}$ . . .

	i_1	i_2	i_3	i_4
$\hat{i}_1 \to j_1$		+	+	?
$\to j_2$	−		?	−
$\to j_3$	−	?		−
$\to j_4$	?	+	+	

. . .

$u_4 : \underline{i >_{u_4} j}$

	i_1	i_2	i_3	i_4
$\hat{i}_1$ j_1		?	+	?
$\hat{i}_2$ j_2	?		+	?
$\hat{i}_3$ j_3	−	−		−
$\hat{i}_4$ j_4	?	?	+	

$i_3 >_{u_4} j_4 \Rightarrow +$

$i_4 <_{u_4} j_3 \Rightarrow -$

BPR-Opt:

$$\sum_{(u,i,j) \in D_s} \ln[\sigma(\hat{x}_{uij})] - \lambda_\Theta \|\Theta\|^2$$

$S \subseteq U \times I$ – implicit feedback matrix

$I_u^+ = \{ i \in I : (u,i) \in S \}$

$D_s = \{ (u,i,j) : \underbrace{i \in I_u^+}_{pos} \wedge \underbrace{j \in I \setminus I_u^+}_{neg} \}$

Matrix factorization:

$\hat{x}_{uij} = x_{ui} - x_{uj}$

$\hat{x}_{ui} = \sum_{f=1}^{k} w_{uf} \cdot h_{if}$

$\hat{X} = WH^T$, $\Theta = (W, H)$