SLIM and factorization machines

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Lecture plan



- SLIM
- Factorization machines
- Content-based recommendations
- Neural network recommendations.

Sparse Linear Methods

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SLIM – Sparse Linear Methods



- **A** is a binary matrix of **M x N** user-item interactions
- We will define the **aui** interaction as weighing events from the past:

$$\hat{a}_{ui} = \sum_{j=1}^{N} w_{ij} a_{uj}$$

- Weights wij ≥ 0, that is, the model takes into account similar items. For example, for a photo
 with a cat, it is much easier to say who looks the most like it than who looks the least like it
- ullet Moreover, **w**ii = 0 allows you to explicitly avoid an elementary solution $\,W=I_N\,$
- Thus, **w**ij is a similarity score **j**-th item to **i**-th

SLIM – Sparse Linear Methods



• Optimized MSE loss with *L1* and *L2* regularizations:

$$\frac{1}{2} \sum_{u,i} \left(a_{ui} - \sum_{j} w_{ij} a_{uj} \right)^{2} + \lambda \sum_{i,j} |w_{ij}| + \frac{\beta}{2} \sum_{i,j} (w_{ij})^{2} \to \min_{W}$$

• Note that according to the lines **W**i, the task is divided into m independent:

$$\frac{1}{2} \sum_{u} \left(a_{ui} - \sum_{j} w_{ij} a_{uj} \right)^{2} + \lambda \sum_{j} |w_{ij}| + \frac{\beta}{2} \sum_{j} (w_{ij})^{2} \to \min_{w_{i1}, \dots, w_{iN}}$$
 (\forall i)

- We can solve each problem by coordinate descent:
 - 1. fix all **W**i except one coordinate **w**ij
 - 2. go to the optimum by **w**ij go to the next coordinate
 - 4. repeat until convergence

3.



SLIM – Sparse Linear Methods



• The process of building recommendations:

$$\hat{a}_{ui} = \sum_{j=1}^{N} w_{ij} a_{uj}$$

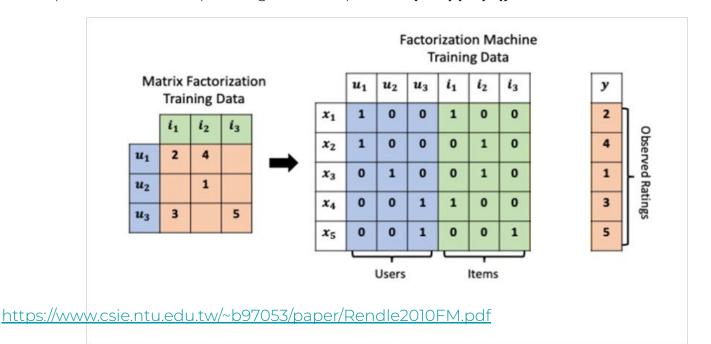
- 1. We take the vector of user interactions \mathbf{A}_{u} .
- 2. We count \hat{a}_{ui} for all unseen aitems
- 3. We sort the unseen items by \hat{a}_{ui} and take the top products with the highest value
- ullet Due to the presence of **L1**-regularization, the matrix **W** will be sparse;
- The event matrix **A** is also sparse;
- This makes it possible to significantly speed up the asymptotics of the model application

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• Let $x \in \mathbb{R}^n$ — one-hot be the interaction vector of the user-item pair, where 1 stands in place of the corresponding user and product (n = |I| + |U|):





• Let's consider the regression model in this statement:

$$a(x) = w_0 + \sum_{i=1}^n w_i x_i$$

• We will add second-order interactions to our regression model, which will allow us to take into account more complex relationships between features:

$$a(x) = w_0 + \sum_{i=1}^{n} w_i x_i + \sum_{i=1}^{n} \sum_{j=i+1}^{n} w_{ij} x_i x_j$$

- In the resulting model, **n(n+1)/2 + n + 1** parameters;
- Since **n = ||| +|U|**, the model size becomes too large.



$$a(x) = w_0 + \sum_{i=1}^{n} w_i x_i + \sum_{i=1}^{n} \sum_{j=i+1}^{n} w_{ij} x_i x_j$$

• Let's compare each feature with the x_i vector $v_i \in \mathbb{R}^k$ and present the model in the form:

$$a(x) = w_0 + \sum_{i=1}^{n} w_i x_i + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \langle v_i, v_j \rangle x_i x_j$$

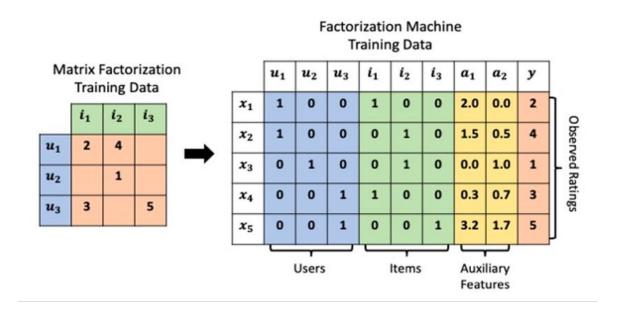
- The number of model parameters decreased to nk + n + 1;
- The last term can be calculated as **O(nk)**:

$$a(x) = w_0 + \sum_{i=1}^n w_i x_i + \frac{1}{2} \left\| \sum_{i=1}^n v_i x_i \right\|_2^2 - \frac{1}{2} \sum_{i=1}^n \left\| v_i \right\|_2^2 x_i^2$$

• Such a model is called a factorization machine.



• In addition to one-hot coded interaction, you can add content attributes of a user or product to vector **x**:





$$a(x) = w_0 + \sum_{i=1}^{n} w_i x_i + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \langle v_i, v_j \rangle x_i x_j$$

- The model is trained by gradient descent;
- The main point of factorization machines is that the weights for paired interactions of features are factorized;
- In addition, the predictions of the model can clearly be considered linearly, which gives a good performance of the algorithm.

Field-aware Factorization Machines

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- Example: there are 3 signs that are different in nature: year of manufacture, color and model of the car;
- In the FM model, the same vector for the year is used to account for the year-color and year-brand interaction;
- But since these signs are different in meaning, the nature of their interaction may differ;
- Idea: use 2 different vectors for the "year of manufacture" attribute when taking into account the year-color and year-brand interactions;

| | Feature vector x | | | | | | | | | | | | | | | | Target y | | | | | |
|--------------|-------------------------|---|---|--|---|---|---|---|--|---|---|---|---|--|----|---|----------|---|---|----|---|-------------------------|
| ((1) | 1 | 0 | 0 | | 1 | 0 | 0 | 0 | | 0 | 0 | 0 | 1 | | 13 | 0 | 0 | 0 | 0 | [] | 5 | y ⁽¹⁾ |
| (2) | 1 | 0 | 0 | | 0 | 1 | 0 | 0 | | 1 | 0 | 0 | 0 | | 14 | 1 | 0 | 0 | 0 | | 3 | y ⁽²⁾ |
| (3) | 1 | 0 | 0 | | 0 | 0 | 1 | 0 | | 0 | 1 | 0 | 0 | | 16 | 0 | 1 | 0 | 0 | | 1 | y ⁽²⁾ |
| (4) | 0 | 1 | 0 | | 0 | 0 | 1 | 0 | | 0 | 1 | 0 | 0 | | 5 | 0 | 0 | 0 | 0 | | 4 | y ⁽³⁾ |
| (5) | 0 | 1 | 0 | | 0 | 0 | 0 | 1 | | 0 | 0 | 1 | 0 | | 8 | 0 | 0 | 1 | 0 | | 5 | y ⁽⁴⁾ |
| (6) | 0 | 0 | 1 | | 1 | 0 | 0 | 0 | | 0 | 0 | 0 | 1 | | 9 | 0 | 0 | 0 | 0 | | 1 | y ⁽⁵⁾ |
| (7) | 0 | 0 | 1 | | 0 | 0 | 1 | 0 | | 0 | 1 | 0 | 0 | | 12 | 1 | 0 | 0 | 0 | | 5 | y ⁽⁶⁾ |
| | | | | | | | | | | | | | | | | | | | | | | |

Field-aware Factorization Machines



- Let's divide the signs into groups, let **f** be the index of the group **i** of that sign;
- Then the FFM model looks like:

$$a(x) = w_0 + \sum_{i=1}^{n} w_i x_i + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \langle v_{i,f_j}, v_{j,f_i} \rangle x_i x_j$$

- It is trained by gradient descent, similar to FM;
- Similarly, the quadratic sum can be calculated linearly by n;
- They work best with groups of the type "categorical sign of large cardinality";

Neural networks in recommendations

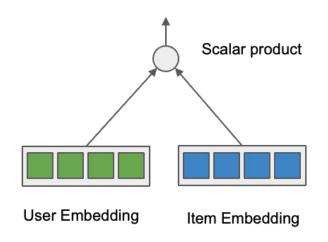
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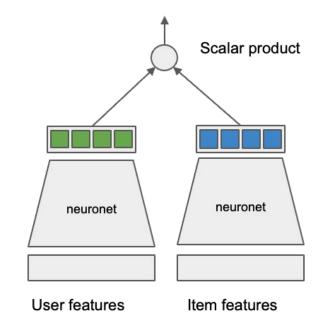
SVD is a kind of neural network



Let's go back to the matrix decomposition and think about how we can take into account additional features in it.



We teach using SGD



Deep Structured Semantic Models



$$R(Q,D) = \operatorname{cosine}(y_Q,y_D) = \frac{y_Q^T y_D}{\|y_Q\| \|y_D\|}$$

$$\operatorname{neuronet}$$

$$M_Q$$

$$\operatorname{dictionary size vector}$$

$$\operatorname{dictionary size vector}$$

DSSM is a classic search and ranking model

- **Q** text query, **D** document
- X_Q and X_D are their representations, for example, in the form of bag of words (~100k)
- The request and the document are translated by neural networks into embeddings of a small size (~300)
- Between them we consider the proximity function, cosine, scalar product, etc
- We rank documents by the proximity value





We will consider the conditional probability of a click on the document \boldsymbol{D} under the condition of the query \boldsymbol{Q} .

$$P(D|Q) = \frac{\exp(\gamma R(Q,D))}{\sum_{D} \exp(\gamma R(Q,D))}$$

$$R(Q, D) = \text{cosine}(y_Q, y_D) = \frac{y_Q^T y_D}{\|y_Q\| \|y_D\|}$$

Here

y – k-t smoothing, established empirically

D – the set of all documents

Calculating the gradient of such a functional for each example is expensive. What can be done?

Deep Structured Semantic Models: how to train?



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Here

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Negative sampling options:

- 1. It is equally likely to select a subset of documents from non-selected ones
- 2. It is more likely to choose those unlisted documents whose popularity is higher
- 3. At each epoch of training, choose non-called documents with the maximum score (the score is taken from the previous epoch)





Taking into account negative sampling, the probability of a click in the document is described by the formula

$$P(D|Q) = \frac{\exp(\gamma R(Q,D))}{\exp(\gamma R(Q,D)) + \sum_{d \in \mathbf{D}^{-}} \exp(\gamma R(Q,d))}$$

During the training process, we will maximize the likelihood of the sample, or, what is the same thing, minimize the loss:

$$L(\Lambda) = -\log \prod_{(Q,d \in \mathbf{D}^+)} P(d|Q)$$

where Λ are the parameters of the layers

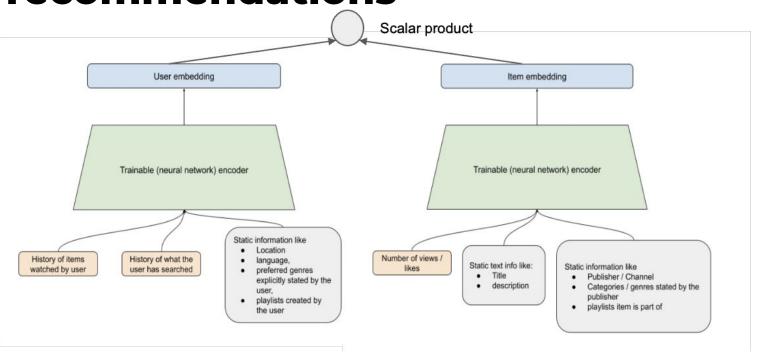
D* – a lot of clicked documents

D-lots of negative sampling documents









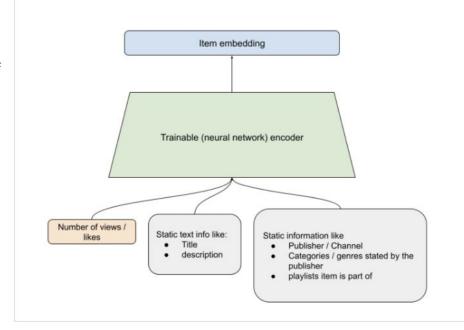
The search for relevant items can be presented as a ranking task, where the user, his history and features are used as a query.

Document features



As signs, you can use:

- standard document statistics: number of likes, clicks, subscriptions
- author's signs: number of subscribers, genre
- unstructured data: document text (you can use BOW format, or you can use it before trained embeddings), videos and pictures (also pre-submit them as embedding)



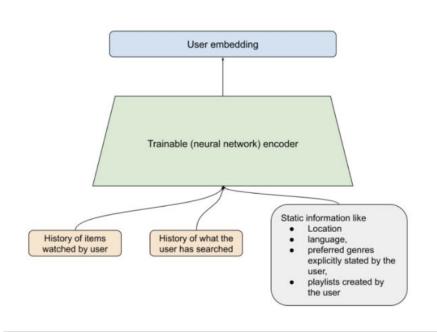
User features



As features, you can use:

- information about the user: age, gender, language, how long the service
- has been using information about the context of the request: from which device was made, at what time
- information about the user's friends/subscribers and their interactions

It makes sense to use the user's history as an average embedding of those articles that he read. Or train RNN or Transformer on history and concatenate the result to the rest of the features.



Learning strategies

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- MSE
- Cross Entropy Loss (CE)
- Pairwise loss
- Full Product Softmax loss (aka: Infancy, InfoMAX, SINCLAR)

Cross entropy loss



The probability that user u will click on item i can be represented as:

$$\widehat{p}_{ui} = \sigma(R(u, i)) = \sigma(\det(y_u, y_i))$$

Were:

 $\left|y_i
ight|$ – embedding of aitema

 y_u – embedding the user

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

Then the loss has the form ($r_{u,i} \in \{0,1\}$ -rating):

$$L = -\sum_{u,i} (r_{ui} \log \widehat{p}_{u,i} + (1 - r_{u,i}) \log(1 - \widehat{p}_{u,i}))$$

Pairwise losses



Consider a pair of items in which i_1 – positive, i_2 – negative, there are several options for pairwise loss:

$$L(R(u, i_1), R(u, i_2)) = \text{CrossEntropy}(1.0, \sigma(R(u, i_1) - R(u, i_2)))$$

the network learns to rank positive examples above negative ones

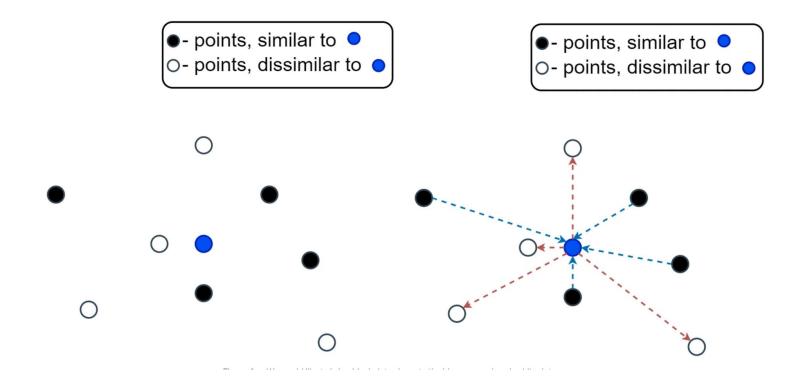
$$L(R(u, i_1), R(u, i_2)) = \max(0, \alpha - R(u, i_1) + R(u, i_2))$$

the network makes sure that the positive and negative examples differ as much as possible (known as triplet loss, which is used to train Siamese networks)

https://www.v7labs.com/blog/contrastive-learning-guide https://medium.com/@maksym.bekuzarov/losses-explained-contrastive-loss-f8f57fe32246

Pairwise losses



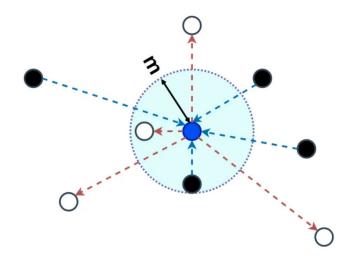


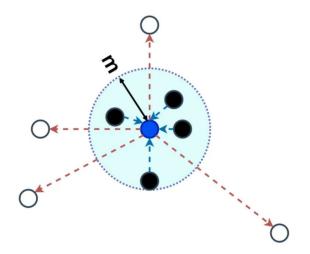
Pairwise Iosses



- points, similar to
- O- points, dissimilar to

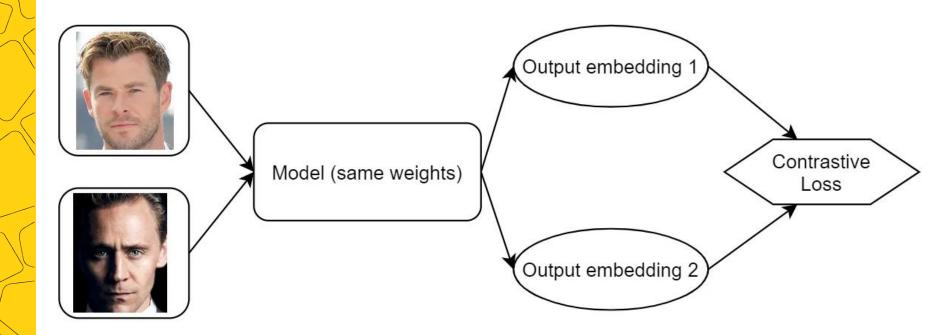
- points, similar to
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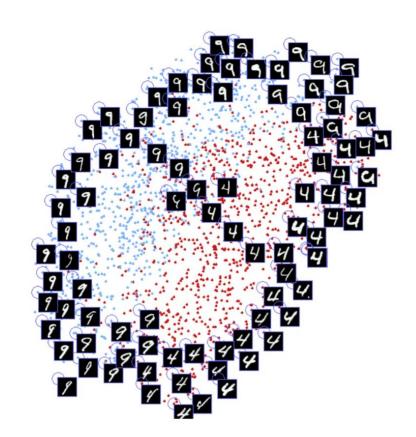
Pairwise Iosses





Pairwise losses





Full Product Softmax loss

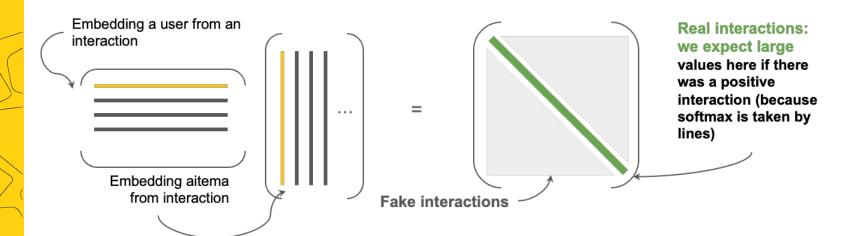


Consider a batch of interactions of size m, consisting of

User embedding matrices $U \in \mathbb{R}^{m \times d}$ Embedding matrices of aitems $I \in \mathbb{R}^{m \times d}$

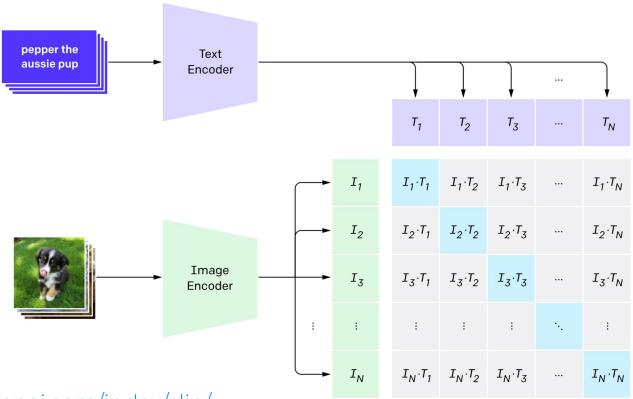
Target vectors $r \in \mathbb{R}^m$

Consider a matrix $Softmax(\alpha \cdot UI^T + \beta)$, $UI^T \in \mathbb{R}^{m \times m}$ where softmax is taken by rows.



CLIP pre-training

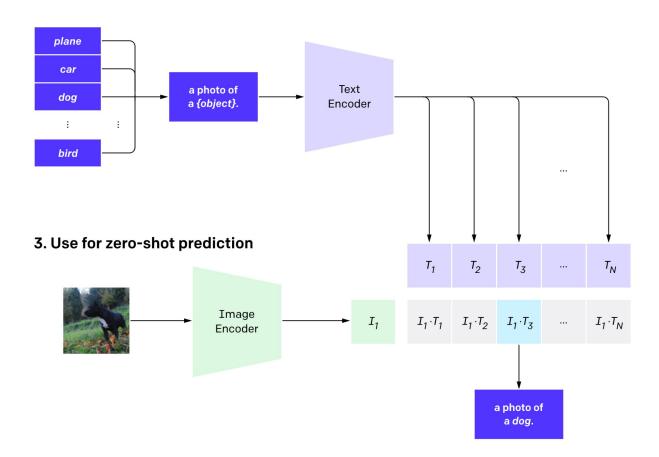




https://openai.com/index/clip/

CLIP usage







Full Product Softmax loss



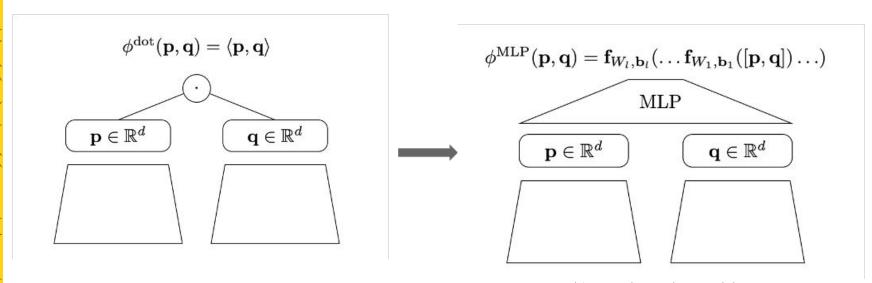
Consider a loss of the form

$$L = -(r > 0)^T \cdot \log(\operatorname{diag}(\operatorname{softmax}(\alpha \cdot UI^T + \beta)))$$

The loss makes the diagonal elements of the matrix larger than the rest of the elements: so in a dataset with unique users and documents on the diagonal of the optimal matrix will be r>0

What else have we tried?

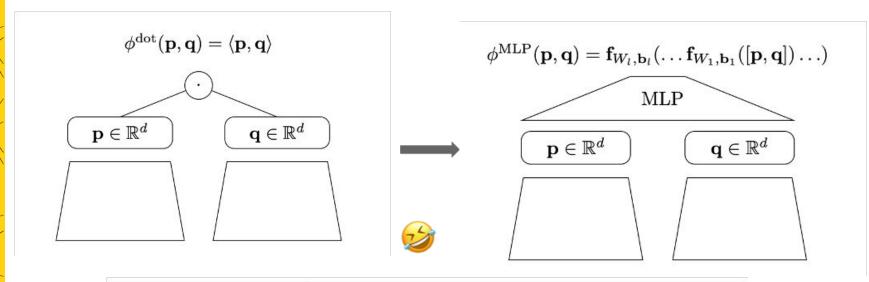




- 1. Users and items have learnable embedding
- Embedding is concatenated and passed through MLP
- The neural network provides a prediction of the rating

What else have we tried?





| Method | Movielens | | Pinterest | | Result |
|----------------------|-----------|---------|-----------|---------|--------|
| | HR@10 | NDCG@10 | HR@10 | NDCG@10 | from |
| Popularity | 0.4535 | 0.2543 | 0.2740 | 0.1409 | [8] |
| SLIM [25, 30] | 0.7162 | 0.4468 | 0.8679 | 0.5601 | [8] |
| iALS [20] | 0.7111 | 0.4383 | 0.8762 | 0.5590 | [8] |
| NeuMF (MLP+GMF) [17] | 0.7093 | 0.4349 | 0.8777 | 0.5576 | [8] |
| Matrix Factorization | 0.7294 | 0.4523 | 0.8895 | 0.5794 | Fig. 2 |

Conclusions



- 1. The fundamental difference between neural network models and factorization machines is that we do not limit ourselves only to linear transformations and add non-linearity
- 2. Two-tower architecture (in particular DSSM) has a number of advantages:
 - A. A large space for creativity in the design of features 🤞
 - B. Fast inference, since embedding and items can be pre-calculated offline
 - C. The ability to build an offline (and even online) selection of candidates for trained embedding
- 3. Teaching an alternative dot-y measure of proximity using MLP is a flexible idea

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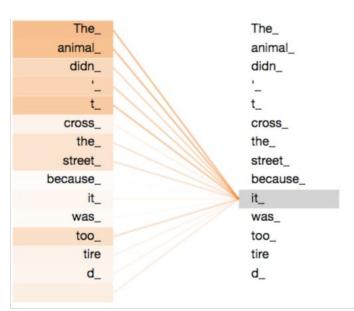




The layer receives the embedding x_1, \ldots, x_t of sequence elements as input and converts them into embeddings z_1, \ldots, z_t that take into account information about the elements in all other positions.

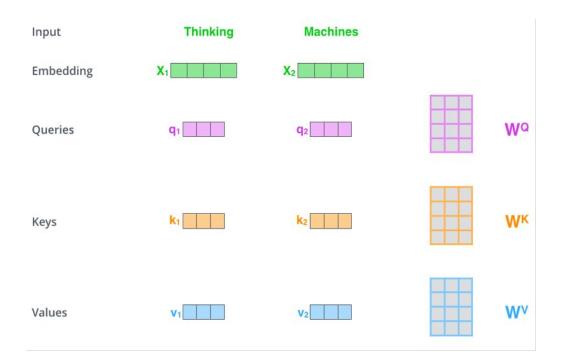
Let's look at the illustration for the texts:

- In each position we look at all the words in the text
- We determine which words need to be looked at more strongly, and which ones are weaker
- Based on this information, we are building a new embedding position





The first step is to build three vectors Key, Query and Value for each word in the sentence

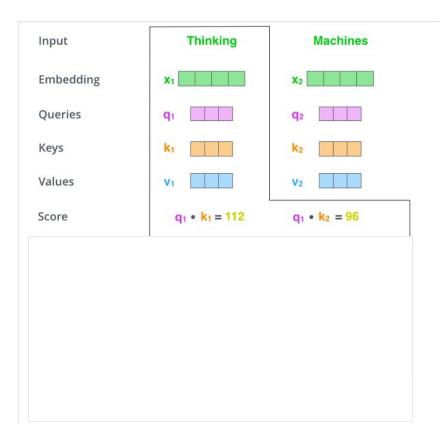




*

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Further, the scalar product of the vectors **q**i and **k**i lo shows how much attention should be paid to the word **j** in position **i**.



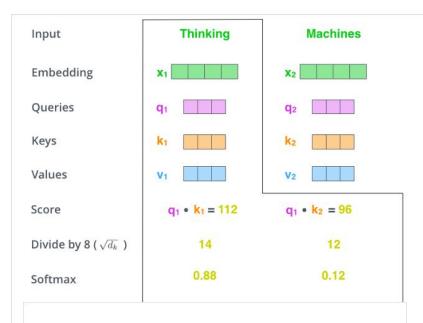
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Further, the scalar product of the vectors **q**i and **k**i lo shows how much attention should be paid to the word **j** in position **i**.

Then we normalize the scores by the root of the dimensions of the key and query vectors and calculate the softmax of the scores for each position.

We get the weights with which we look at each word in the sentence.





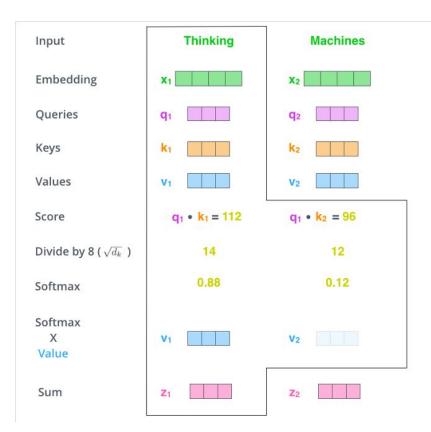
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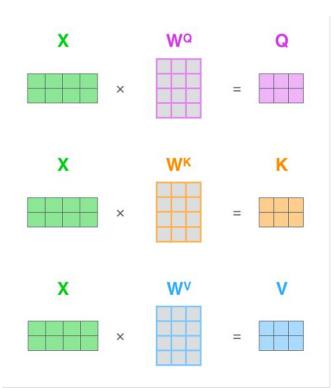
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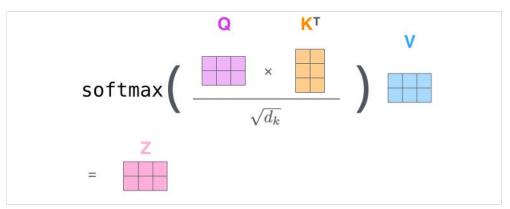
With these weights, we average the value vectors and get a new embedding of a word in a sentence that takes into account all other words



Self-attention in matrix form



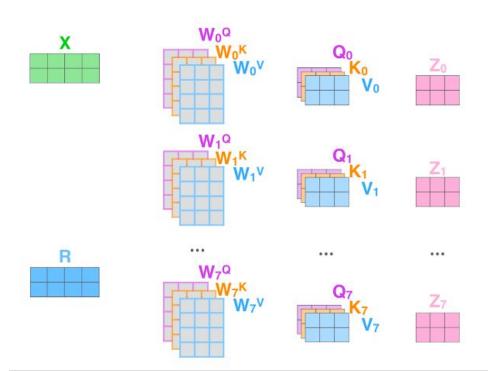






We will use several parallel heads of self-attention.

For each head we have our own projection matrices

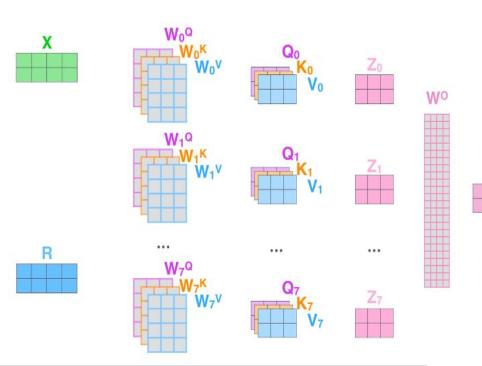




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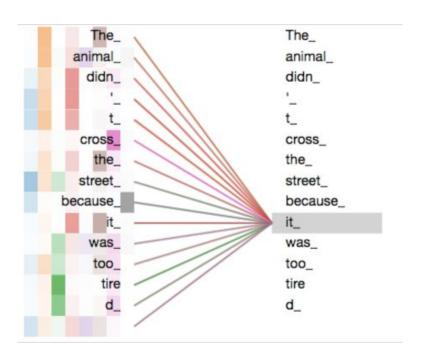
For each head we have our own projection matrices

At the end we concatenate the embedding and from all the heads and linearly project it into the embedding of the correct size





The main idea: different heads pay attention to the different dependencies between the elements of the sequence



Positional Encoding

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Problem: in this formula there is no dependence on the position in the sentence, we work with its elements as with a set





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Solution: to embedding positional embedding and, instead of xi, consider xi + pi, where positional embedding pi depends only on position i.

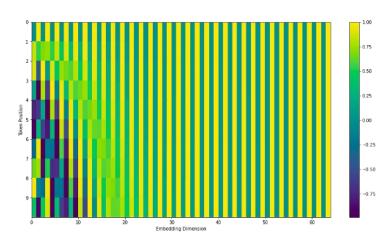
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Option 1: positional embedding is deterministically calculated by cosine formulas



Positional Encoding

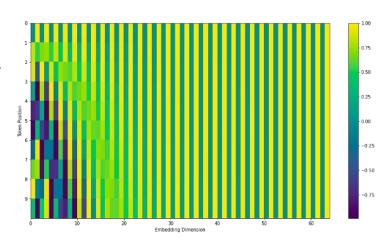


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Option 1: positional embedding is deterministically calculated by cosine formulas

Option 2: Positional embedding is trained together with the entire model



Bert4Rec

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Bert4Rec



- The self-attention approach shows itself well in tasks related to texts, audio and images
- The Bert4Rec model is an attempt to apply this approach to the task of recommendations
- The model is applied to a sequence of items from the user's history and predicts the next item



The layer receives the embedding h_1, \ldots, h_t of sequence elements as input and converts them into embeddings h'_1, \ldots, h'_t hat take into account information about all other positions.

Let's define the basic Attention operation, which takes the input of the matrix Q,K,V and returns

Attention
$$(Q, K, V) = \operatorname{softmax}\left(\frac{QK^T}{\sqrt{m}}\right)V, \ Q, K, V \in \mathbb{R}^{t \times m}$$

• Attention $(Q, K, V)_i = \operatorname{softmax}\left(\frac{Q_i K^T}{\sqrt{m}}\right) V$

This is a linear combination of strings from V with weights $\operatorname{softmax}\left(\frac{Q_iK_1}{\sqrt{m}}\right),\ldots,\operatorname{softmax}\left(\frac{Q_iK_t}{\sqrt{m}}\right)$

- Accordingly, Q_i and K express how much contribution information about all other elements should have to the element of position i.
- ullet The matrix **V** carries the information that each position should transmit.



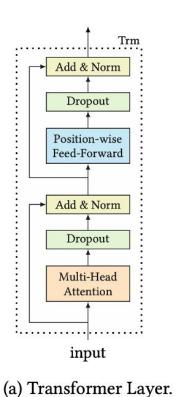
```
Let the input embedding form a H\in \mathbb{R}^{t	imes d} matrix Define MH(H)=[\mathrm{head}_1,\ldots,\mathrm{head}_h]W^O,\ W^O\in \mathbb{R}^{d	imes d}
```

$$head_i = Attention(HW_i^Q, HW_i^K, HW_i^V), \ W_i^Q, W_i^K, W_i^V \in \mathbb{R}^{d \times d/h}$$

$$W^O,W_i^Q,W_i^K,W_i^V$$
 – trainable parameters

Transformers layer





Consists of:

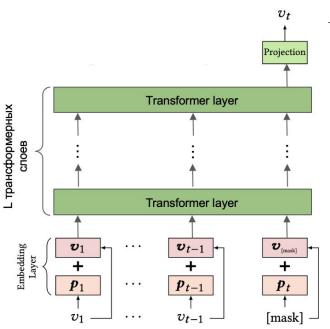
- Multi-Head Self-Attention Layer
- Position-wise Feed-Forward, applied to outputs piecemeal

$$FFN(x) = GELU(xW^{(1)} + b^{(1)})W^{(2)} + b^{(2)}$$

Dropout, skip connection and LayerNorm after both previous ones

Bert4Rec Architecture





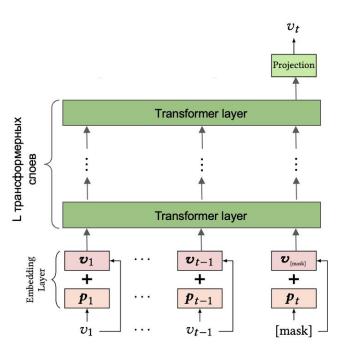
The model consists of

- Layers of embeddings of aitems
- **L** Transformer layers
- A projecting head that performs predictions

(b) BERT4Rec model architecture.

Bert4Rec Architecture





(b) BERT4Rec model architecture.

The model consists of

- Layers of embeddings of aitems
- **L** Transformer layers
- A projecting head that performs predictions

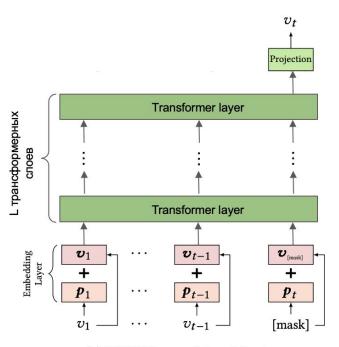
Itemes of positive user interactions are submitted to the input in order of time.

A special [mask] item is added to the end of the story.

To the output embedding corresponding to this special item, we apply a projection head that predicts the relevant next item.

Bert4Rec Architecture





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The projection layer is arranged as

$$P(v) = \operatorname{softmax} \left(GELU(h_i^L W^P + b^P) E^T + b^O \right),$$

$$E \in \mathbb{R}^{|V|}$$

where $E \in \mathbb{R}^{|V| \times d}$ the trainable matrix of aitems



Bert4Rec Training



• We will randomly mask the share of the items ρ from the user's history, that is, we will replace it with a special item [mask]

Input:
$$[v_1, v_2, v_3, v_4, v_5] \xrightarrow{\text{randomly mask}} [v_1, [\text{mask}]_1, v_3, [\text{mask}]_2, v_5]$$
Labels: $[\text{mask}]_1 = v_2$, $[\text{mask}]_2 = v_4$

• Let S_u^m be the set of masked positions of the user $oldsymbol{u}$,

 S_u' - a sequence of aitems with replaced by [mask] aitems,

 v_m^st - predictions of aitems in disguised positions

Then we optimize the likelihood, that is:

$$\mathcal{L} = -\frac{1}{|S_u^m|} \sum_{m \in S_u^m} \log P(v_m^* = v_m | S_u')$$

• In other words, learning is similar to Masked Language Model learning in text tasks

Thanks for attention!

Questions?



