Network dynamics handin 1

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1 Introduction

This report is divided into two sections. The first part analyzes an input-output network of Goods. The data set contains of several economies and which year the data was collected. The economies analyzed in this report are Sweden and Indonesia in the year 2000. The economies are divided into sectors e.g., 'Food Products' and 'Agriculture'. The sectors importance are analyzed depending on different centrality measures. The second part analyzes influence on twitter based on a subset of twitter accounts. The data was assembled by crawling twitter, starting at one account and collecting its followers and then the followers followers etc. PageRank centrality was calculated based on a graph were links $(i, j) \in E$ meant that i follows j

2 Theory

2.1 Centrality

The centrality of a section is a measure of how impactful that section is on the full graph. There are several different centrality measures and in this analysis we will use *in- and out-degree centrality*, *eigenvector centrality*, *katz centrality* and *pageRank centrality.*

2.1.1 In- and out-degree centrality

This centrality simple measure the amount of either in our out-degree of a node i.e. the number of outgoing or ingoing links. An issue with for example in-degree centraity is that every node contribute equally to the centrality of it's outneighbors independent of the nodes own centrality.

2.1.2 Eigenvector centrality

The eigenvector centrality takes into account that an nodes own centrality contributes to its outneighbors. The centrality in node i follows,

$$\lambda z_i = \sum_j W_{ij} z_j. \tag{1}$$

The problem with the eigenvector centrality measure is that it does not take into account the amount of outgoing links for each node. Normalizing the W matrix e.g the normalized weight matrix P and using that instead takes care of this problem and then the centrality measure is called the invariant distribution.

2.1.3 Katz centrality

In the *katz centrality* an intrinsic centrality μ is introduced which corresponds to a topological centrality independent of the links of the network. This is done to reduce manipulation of centrality. The equation follows,

$$z^{\beta} = \frac{1-\beta}{\lambda_W} W' z^{\beta} + \beta\mu. \tag{2}$$

2.1.4 PageRank centrality

The pageRank centrality is a commonly used centrality measures i.e. google used it to rank websites hence the name. The equation is almost the same as 2 but with the normalized weight matrix P used as follow,

$$z^{\beta} = \frac{1-\beta}{P}' z^{\beta} + \beta\mu. \tag{3}$$

and with $\beta = 0.15$. With unspecified beta this is refered to as the *Bonacich* centrality.

2.2 Linear averaging and flow dynamics

To determine what state the network converges when iteratively simulating the opinion vector one can use the *DeGroot opinion dynamics model*,

$$x = Px. \tag{4}$$

When stubborn nodes are introduced (nodes that won't change their opinion) the dynamics of the network change. The dynamics can the be discribed by,

$$x(t+1) = Qx(t) + Eu \tag{5}$$

where x is the opinion vector. $Q = P_{|RXR}$, $E = P_{|RXS}$ and $u \in \mathcal{R}^S$.

3 Results and discussion

3.1 Centrality

The most central sectors based on in-, out-degree, eigenvector and Katzcentrality can be shown in the five tables below.

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Table 1: Most Central Sectors in Sweden based on in- and out-degree

Country	**Top 5 Central Sectors**	
	Sweden	
	Out-degree	**In-degree**
Sector	43: Other Business Activities 39: Real estate activities 31: Wholesale & retail trade; repairs	19: Radio 21: Motor Vehicles 43 Other Business Activities

Table 2: Most Central Sectors in Indonesia based on in- and out-degree

Country	**Top 5 Central Sectors**	
	Indonesia	
	Out-degree	**In-degree**
Sector	31: Wholesale & retail trade; repairs 1: Agriculture 2: Mining and quarrying (energy	4: Food products 30: Construction 31: Wholesale & retail trade; repairs

Table 3: Most Central Sectors in Sweden and Indonesia based on eigenvector centrality

Country	**Top 5 Central Sectors**	
	Sweden	**Indonesia**
	Eigenvector	**Eigenvector**
Sector	37: Post & telecommunications 7: Pulp 33: Land transport; transport via pipelines	1: Agriculture 4: Food products 31: Wholesale & retail trade; repairs

Table 4: Most Central Sectors in Sweden based on katz centrality with intrinsic centrality μ as the unit vector in case 1 and Sector 31: Wholesale and retail trade; repairs as 1 and rest as 0 for case (2)

Country	**Top 5 Central Sectors**	
	Sweden	
	Katz case(1)	**Katz case(2)**
Sector	21: Motor vehicles 19: Radio 43: Other Business Activities	4: Food products 31: Wholesale & retail trade; repairs 32: Hotels & restaurants

Table 5: Most Central Sectors in Indonesia based on katz centrality with intrinsic centrality μ as the unit vector in case 1 and Sector 31: Wholesale and retail trade; repairs as 1 and rest as 0 for case (2)

Country	**Top 5 Central Sectors**	
	Indonesia	
	Katz case(1)	**Katz case(2)**
Sector	4: Food products 32: Hotels & restaurants 1: Agriculture	4: Food products 31: Wholesale & retail trade; repairs 32: Hotels & restaurants

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From the tables one can see that katz centrality case (2) gives all centrality to the sector 31 for the intrinsic centrality. For both Sweden and Indonesia three sectors closely related to food is weighed the highest. Compared to case (1) the difference in central sectors are larger in Sweden. As displayed the top three central sectors vary depending on which centrality measure to use.

3.2 Discrete time consensus based on different stubborn nodes

Based on figure 1 we can see that the higher the centrality of the stubborn node the more nodes reach an stationary opinion closer to the stubborn node's value. Depending if the most central nodes value is either one or zero obviously determines if the general opinion is more towards one or zero.

Note that the plotted nodes are the ten most central nodes based on pageRank centrality and the stubborn nodes. Comparing the top right plot with the bottom left plot, the conclusion can be drawn that difference in centrality between the 4th and 5th most central nodes is bigger than the difference in centrality between the 2nd and 3rd most central nodes, due to the opinion of the plotted nodes converging faster. Just looking at the top right corner the plot seems to suggest that more nodes converge to "weaker" stubborn node's opinion. In figure 3 the histogram of the stationary opinion is plotted and there we can see that there might be more nodes converging to a lower opinion but the nodes who converge to a higher opinion has a stronger opinion, by that I mean closer to 1 than the lower opinion nodes are to 0. The histograms a count of all the nodes where as the simulation plots only cover the ten most central nodes. One can also see in the bottom right plot where the difference in centrality between the stubborn nodes leads to a more distinct separation in opinion.
In this particular network all the nodes does not converge to a stationary opinion in 500 iterations.

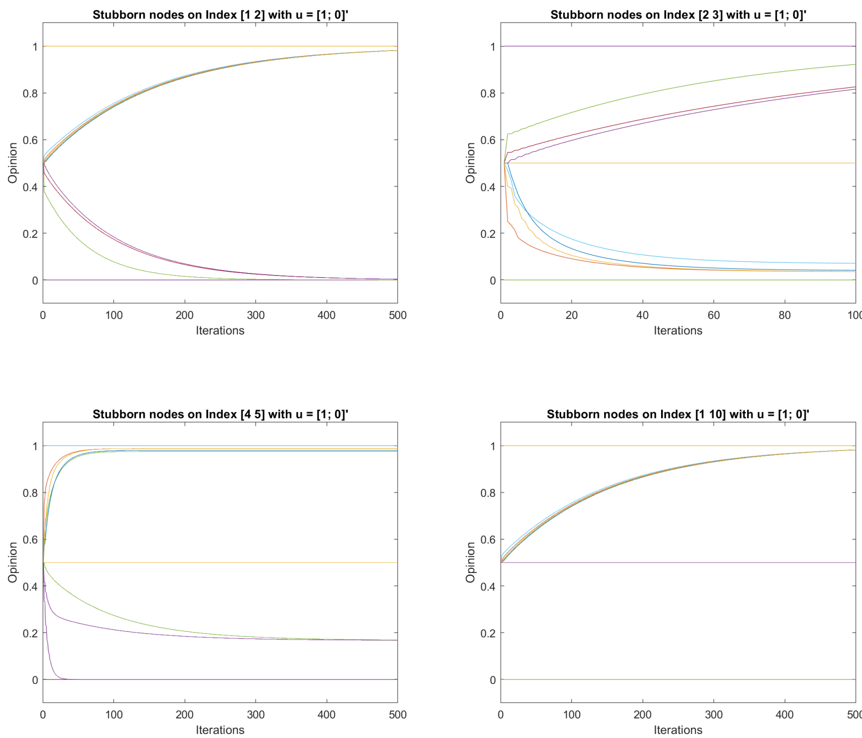

Figure 1: Discrete time consensus simulation with two stubborn nodes, with different centrality index

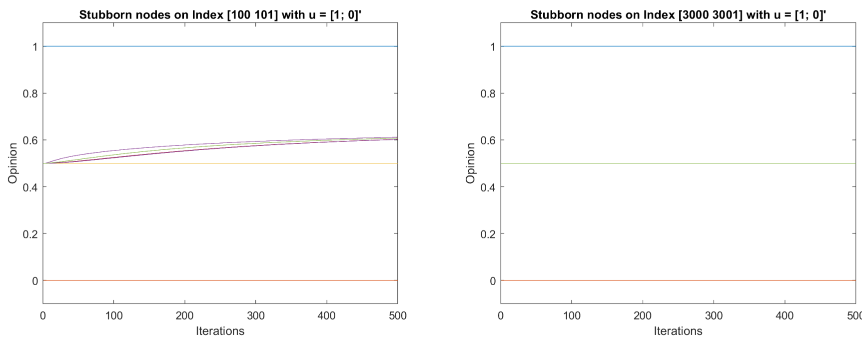

Figure 2: Plot of opinion over iteration where the stubborn nodes have lower centrality

In figure 2 the plots show that if the stubborn nodes have too low centrality (3000 and 3001 highest centrality) then it has very little effect the other nodes. This can also be seen in the histograms of figure 4.

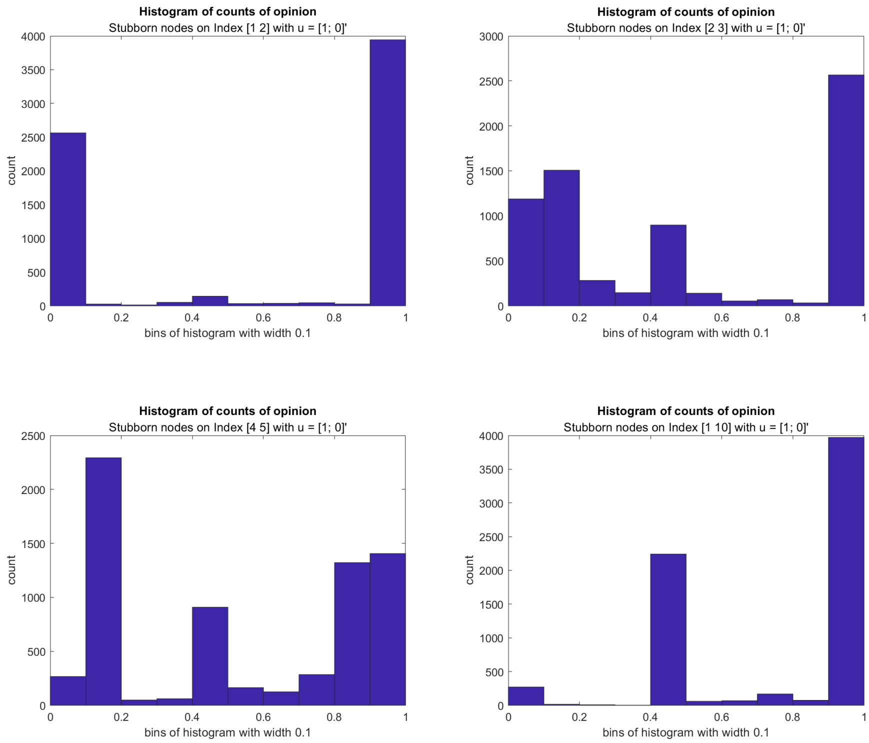

Figure 3: Histograms of the stationary opinion with different centrality on stubborn nodes.

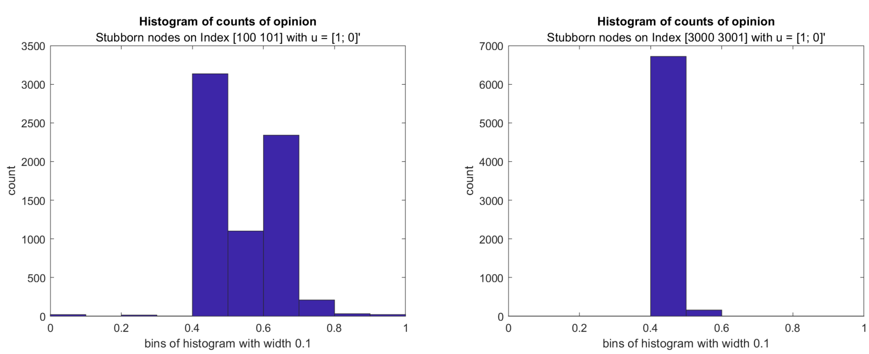

Figure 4: Histogram of stationary opinions with stubborn nodes lower centrality