

**Review questions and study problems, week 3**

1. Consider the two-point boundary value problem

$$y'' = x^2 + y^2 \quad y(0) = 0, y(1) = 0.$$

Approximate  $y''$  by  $\frac{y_{n-1} - 2y_n + y_{n+1}}{\Delta x^2}$  and write the corresponding discretization for this BVP. Take  $N = 4$ ; write the nonlinear system of equations  $F(y) = 0$  for the unknowns  $y_1, y_2, y_3, y_4$ .

2. What is the Jacobian for the problem above?
3. Once you have the Jacobian, how do you perform one Newton iteration to solve  $F(y) = 0$ ?
4. Consider the two-point boundary value problem

$$y'' = x^2 + y^2 \quad y(0) = 0, y'(1) = 0.$$

Approximate  $y''$  by  $\frac{y_{n-1} - 2y_n + y_{n+1}}{\Delta x^2}$  and write the corresponding discretization for this BVP. Take  $N = 4$ ; write the nonlinear system of equations  $F(y) = 0$  for the unknowns  $y_1, y_2, y_3, y_4$ . Discretize the Neumann boundary condition so that the resulting method is of second order.

5. What changes in Question 1 and 4 if we instead consider the equation  $y'' + y' = x^2 + y^2$ ?
6. What changes in Question 4 if we instead consider the Dirichlet + Robin boundary conditions  $y(0) = \alpha, y(1) + y'(1) = \beta$ ?