Formelblad för Funktionsteori

Matematik LTH vt 2021

Fourierserier

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{ik\Omega t} = c_0 + \sum_{k=1}^{\infty} a_k \cos k\Omega t + b_k \sin k\Omega t \qquad \Omega T = 2\pi$$

$$c_k = \frac{1}{T}\int_{\text{period}} e^{-ik\Omega t} f(t)\,dt \qquad \begin{cases} a_k = \dfrac{2}{T}\displaystyle\int_{\text{period}} \cos(k\Omega t) f(t)\,dt \\ b_k = \dfrac{2}{T}\displaystyle\int_{\text{period}} \sin(k\Omega t) f(t)\,dt \end{cases}$$

$$\begin{cases} a_k = c_k + c_{-k} \\ b_k = i\big(c_k - c_{-k}\big) \end{cases} \qquad \begin{cases} c_k = \frac{1}{2}\big(a_k - ib_k\big) \\ c_{-k} = \frac{1}{2}\big(a_k + ib_k\big) \end{cases}$$

Parsevals formel

$$\frac{1}{T}\int_{\text{period}} \overline{f(t)} g(t)\,dt = \sum_{k=-\infty}^{\infty} \overline{c_k(f)} c_k(g)$$

$$\frac{1}{T}\int_{\text{period}} |f(t)|^2\,dt = \sum_{k=-\infty}^{\infty} |c_k(f)|^2$$

$$\frac{1}{T}\int_{\text{period}} |f(t)|^2\,dt = |c_0|^2 + \frac{1}{2}\sum_{k=1}^{\infty} (|a_k|^2 + |b_k|^2)$$

Halvperiodutvecklingar

Cosinusserie

$$f(x) = c_0 + \sum_{k=1}^{\infty} \alpha_k \cos(\frac{k\pi}{L}x)$$

$$\alpha_k = \frac{2}{L}\int_0^L f(x)\cos(\frac{k\pi}{L}x)\,dx$$

$$c_0 = \frac{1}{L}\int_0^L f(x)\,dx$$

Sinusserie

$$f(x) = \sum_{k=1}^{\infty} \beta_k \sin(\frac{k\pi}{L}x)$$

$$\beta_k = \frac{2}{L}\int_0^L f(x)\sin(\frac{k\pi}{L}x)\,dx$$

Kvot- och rotkriteriet

$$\kappa = \lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| \qquad \kappa < 1 \Rightarrow \sum_k a_k \text{ konvergent} \qquad \kappa > 1 \Rightarrow \sum_k a_k \text{ divergent}$$

$$\rho = \lim_{k\to\infty}\sqrt[k]{|a_k|} \qquad \rho < 1 \Rightarrow \sum_k a_k \text{ konvergent} \qquad \rho > 1 \Rightarrow \sum_k a_k \text{ divergent}$$

Potensserier

$$(1+x)^\alpha = \sum_{k=0}^{\infty} \binom{\alpha}{k} x^k \qquad \frac{1}{1-x} = \sum_{k=0}^{\infty} x^k \qquad e^x = \sum_{k=0}^{\infty} \frac{1}{k!} x^k$$

$$\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k} \qquad \sin x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1} \qquad \ln(1+x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} x^k$$

$$\arctan x = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} x^{2k+1} \qquad f(z) = \sum_{k=0}^{\infty} c_k (z-a)^k, \text{ där } c_k = \frac{f^{(k)}(a)}{k!}$$

Cauchys integralformel

$$f^{(k)}(a) = \frac{k!}{2\pi i} \int_C \frac{f(z)}{(z-a)^{k+1}}\, dz$$

Residyregler

1. Om $f(z) = (z-a)^{-N} g(z)$ så är $\operatorname*{Res}_{z=a} f(z) = \dfrac{g^{(N-1)}(a)}{(N-1)!}$
2. Om $f(z) = (z-a)^{-N} g(z)$ och $g(z) = \sum_{k=0}^{\infty} c_k (z-a)^k$ så är $\operatorname*{Res}_{z=a} f(z) = c_{N-1}$
3. $\operatorname*{Res}_{z=a} f(z) = \lim_{z \to a} (z-a) f(z)$
4. $\operatorname*{Res}_{z=a} \dfrac{f_1(z)}{f_2(z)} = \dfrac{f_1(a)}{f_2'(a)}$

Funktionsserier

$$\left.\begin{array}{r} |u_k(t)| \le m_k\,,\ t \in I \\ \sum_k m_k \text{ konvergent} \end{array}\right\} \Rightarrow \sum_k u_k(t) \text{ likformigt konvergent på } I$$

$$\left.\begin{array}{l} \sum_k u_k(t) \text{ likformigt konvergent} \\ u_k(t) \text{ kontinuerliga} \end{array}\right\} \Rightarrow \sum_k u_k(t) \text{ kontinuerlig}$$

$$\left.\begin{array}{l} \sum_k u_k(t) \text{ konvergent} \\ \sum_k u_k'(t) \text{ likformigt konvergent} \end{array}\right\} \Rightarrow \frac{d}{dt}\Big(\sum_k u_k(t)\Big) = \sum_k u_k'(t)$$

$$\left.\begin{array}{l} \sum_k u_k(t) \text{ likformigt konvergent på } I \\ u_k \text{ kontinuerliga, } I \text{ begränsad} \end{array}\right\} \Rightarrow \int_I \Big(\sum_k u_k(t)\Big)\, dt = \sum_k \Big(\int_I u_k(t)\, dt\Big)$$