LUNDS TEKNISKA HÖGSKOLA MATEMATIK

FORMELBLAD SYSTEM OCH TRANSFORMER

$$\theta(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

Fourierserier:

(1)
$$\begin{cases} f(t) = \sum_{k=-\infty}^{+\infty} c_k(f)e^{ik\Omega t} \\ c_k(f) = \frac{1}{T} \int_{\text{period}} f(t)e^{-ik\Omega t} dt \end{cases}, \quad \Omega T = 2\pi$$

(2)
$$c_k(f') = ik\Omega c_k(f)$$
 (distributions derivata)

Parsevals formel:

(3)
$$\frac{1}{T} \int_{\text{period}} \overline{f(t)} g(t) dt = \sum_{k=-\infty}^{+\infty} \overline{c_k(f)} c_k(g)$$

Fouriertransformer:

(4)
$$\mathcal{F}f(\omega) = \hat{f}(\omega) = F(\omega) = \int_{-\infty}^{+\infty} e^{-i\omega t} f(t) dt$$

(5)
$$f(t) = (\mathcal{F}^{-1}F)(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{i\omega t} F(\omega) d\omega$$

Parsevals formel:

(6)
$$\int_{-\infty}^{+\infty} \overline{f(t)} g(t) dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \overline{F(\omega)} G(\omega) d\omega$$

 $\stackrel{\mathcal{F}}{\longmapsto}$

(7)
$$\lambda f(t) + \mu g(t)$$
 $\lambda F(\omega) + \mu G(\omega)$

(8)
$$f(at) \qquad \frac{1}{|a|} F\left(\frac{\omega}{a}\right)$$

$$(9) f(t-t_0) e^{-it_0\omega}F(\omega)$$

(10)
$$e^{i\omega_0 t} f(t) \qquad \qquad F(\omega - \omega_0)$$

$$(11) f'(t) i\omega F(\omega)$$

(13)
$$f * g(t) F(\omega)G(\omega)$$

$$(14) \delta(t) 1$$

$$(15) 1 2\pi\delta(\omega)$$

(16)
$$e^{-t}\theta(t) \qquad \frac{1}{1+i\omega}$$

$$(17) e^{-|t|} \frac{2}{1+\omega^2}$$

$$\frac{1}{1+t^2} \qquad \qquad \pi e^{-|\omega|}$$

$$(19) e^{-t^2} \sqrt{\pi}e^{-\omega^2/4}$$

(20)
$$\theta(t+a) - \theta(t-a) \qquad \qquad 2\frac{\sin a\omega}{\omega}$$

Laplacetransformer:

(21)
$$\mathcal{L}f(s) = \mathcal{L}_{II}f(s) = \int_{-\infty}^{+\infty} e^{-st} f(t) dt, \quad \alpha < \operatorname{Re} s < \beta, \quad s = \sigma + i\omega$$

(22)
$$\mathcal{F}f(\omega) = \mathcal{L}_{II}f(i\omega)$$

(23)
$$\mathcal{L}_{I}f = \mathcal{L}_{II}(f\theta)$$

 $\stackrel{\mathcal{L}_{II}}{\longmapsto}$

(24) $\lambda f(t) + \mu g(t)$ $\lambda F(s) + \mu G(s)$ (25) $f(at)$ $\frac{1}{ a }F\left(\frac{s}{a}\right)$, a reellt (26) $f(t-t_0)$ $e^{-tos}F(s)$ (27) $e^{at}f(t)$ $F(s-a)$ (28) $f'(t)$ $sF(s)$ (29) $tf(t)$ $-\frac{d}{ds}F(s)$ (30) $f*g(t)$ $F(s)G(s)$ (31) $f'(t)\theta(t)$ $s\mathcal{L}_{II}(f\theta)(s) - f(0)$ (32) $\delta(t)$ 1 (33) $\theta(t)$ $\frac{1}{s}, \ \sigma > 0$ (34) $\theta(t) - 1 = -\theta(-t)$ $\frac{1}{s}, \ \sigma < 0$ (35) $t^k\theta(t)$ $\frac{k!}{s^{k+1}}, \ \sigma > 0$ (36) $\sin(bt)\theta(t)$ $\frac{s}{s^2 + b^2}, \ \sigma > 0, \ b \text{ reellt}$ (37) $\cos(bt)\theta(t)$ $\frac{s}{s^2 + b^2}, \ \sigma > 0, \ b \text{ reellt}$ (38) e^{-t^2}	\longmapsto		
(26) $f(t-t_0)$ $e^{-t_0s}F(s)$ (27) $e^{at}f(t)$ $F(s-a)$ (28) $f'(t)$ $sF(s)$ (29) $tf(t)$ $-\frac{d}{ds}F(s)$ (30) $f*g(t)$ $F(s)G(s)$ (31) $f'(t)\theta(t)$ $s\mathcal{L}_{II}(f\theta)(s) - f(0)$ (32) $\delta(t)$ 1 (33) $\theta(t)$ $\frac{1}{s}, \ \sigma > 0$ (34) $\theta(t) - 1 = -\theta(-t)$ $\frac{1}{s}, \ \sigma < 0$ (35) $t^k\theta(t)$ $\frac{k!}{s^{k+1}}, \ \sigma > 0$ (36) $\sin(bt)\theta(t)$ $\frac{b}{s^2+b^2}, \ \sigma > 0, \ b \text{ reellt}$ (37) $\cos(bt)\theta(t)$ $\frac{s}{s^2+b^2}, \ \sigma > 0, \ b \text{ reellt}$	(24)	$\lambda f(t) + \mu g(t)$	$\lambda F(s) + \mu G(s)$
(27) $e^{at}f(t)$ $F(s-a)$ (28) $f'(t)$ $sF(s)$ (29) $tf(t)$ $-\frac{d}{ds}F(s)$ (30) $f*g(t)$ $F(s)G(s)$ (31) $f'(t)\theta(t)$ $s\mathcal{L}_{II}(f\theta)(s) - f(0)$ (32) $\delta(t)$ 1 (33) $\theta(t)$ $\frac{1}{s}, \sigma > 0$ (34) $\theta(t) - 1 = -\theta(-t)$ $\frac{1}{s}, \sigma < 0$ (35) $t^k\theta(t)$ $\frac{k!}{s^{k+1}}, \sigma > 0$ (36) $\sin(bt)\theta(t)$ $\frac{s}{s^2 + b^2}, \sigma > 0, b \text{ reellt}$ (37) $\cos(bt)\theta(t)$ $\frac{s}{s^2 + b^2}, \sigma > 0, b \text{ reellt}$	(25)	f(at)	$\frac{1}{ a }F\left(\frac{s}{a}\right)$, a reellt
(28) $f'(t)$ $sF(s)$ (29) $tf(t)$ $-\frac{d}{ds}F(s)$ (30) $f * g(t)$ $F(s)G(s)$ (31) $f'(t)\theta(t)$ $s\mathcal{L}_{II}(f\theta)(s) - f(0)$ (32) $\delta(t)$ 1 (33) $\theta(t)$ $\frac{1}{s}, \sigma > 0$ (34) $\theta(t) - 1 = -\theta(-t)$ $\frac{1}{s}, \sigma < 0$ (35) $t^k\theta(t)$ $\frac{k!}{s^{k+1}}, \sigma > 0$ (36) $\sin(bt)\theta(t)$ $\frac{b}{s^2 + b^2}, \sigma > 0, b \text{ reellt}$ (37) $\cos(bt)\theta(t)$ $\frac{s}{s^2 + b^2}, \sigma > 0, b \text{ reellt}$	(26)	$f(t-t_0)$	$e^{-t_0s}F(s)$
$(29) \qquad tf(t) \qquad -\frac{d}{ds}F(s)$ $(30) \qquad f * g(t) \qquad F(s)G(s)$ $(31) \qquad f'(t)\theta(t) \qquad s\mathcal{L}_{II}(f\theta)(s) - f(0)$ $(32) \qquad \delta(t) \qquad 1$ $(33) \qquad \theta(t) \qquad \frac{1}{s}, \sigma > 0$ $(34) \qquad \theta(t) - 1 = -\theta(-t) \qquad \frac{1}{s}, \sigma < 0$ $(35) \qquad t^k\theta(t) \qquad \frac{k!}{s^{k+1}}, \sigma > 0$ $(36) \qquad \sin(bt)\theta(t) \qquad \frac{b}{s^2 + b^2}, \sigma > 0, b \text{ reellt}$ $(37) \qquad \cos(bt)\theta(t) \qquad \frac{s}{s^2 + b^2}, \sigma > 0, b \text{ reellt}$	(27)	$e^{at}f(t)$	F(s-a)
(30) $f * g(t)$ $F(s)G(s)$ (31) $f'(t)\theta(t)$ $s\mathcal{L}_{II}(f\theta)(s) - f(0)$ (32) $\delta(t)$ 1 (33) $\theta(t)$ $\frac{1}{s}, \sigma > 0$ (34) $\theta(t) - 1 = -\theta(-t)$ $\frac{1}{s}, \sigma < 0$ (35) $t^k\theta(t)$ $\frac{k!}{s^{k+1}}, \sigma > 0$ (36) $\sin(bt)\theta(t)$ $\frac{b}{s^2 + b^2}, \sigma > 0, b \text{ reellt}$ (37) $\cos(bt)\theta(t)$ $\frac{s}{s^2 + b^2}, \sigma > 0, b \text{ reellt}$	(28)	f'(t)	sF(s)
$(31) f'(t)\theta(t) s\mathcal{L}_{II}(f\theta)(s) - f(0)$ $(32) \delta(t) 1$ $(33) \theta(t) \frac{1}{s}, \sigma > 0$ $(34) \theta(t) - 1 = -\theta(-t) \frac{1}{s}, \sigma < 0$ $(35) t^k\theta(t) \frac{k!}{s^{k+1}}, \sigma > 0$ $(36) \sin(bt)\theta(t) \frac{b}{s^2 + b^2}, \sigma > 0, b \text{ reellt}$ $(37) \cos(bt)\theta(t) \frac{s}{s^2 + b^2}, \sigma > 0, b \text{ reellt}$	(29)	tf(t)	$-\frac{d}{ds}F(s)$
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(33) $\theta(t)$ $\frac{1}{s}, \sigma > 0$ (34) $\theta(t) - 1 = -\theta(-t)$ $\frac{1}{s}, \sigma < 0$ (35) $t^{k}\theta(t)$ $\frac{k!}{s^{k+1}}, \sigma > 0$ (36) $\sin(bt)\theta(t)$ $\frac{b}{s^{2} + b^{2}}, \sigma > 0, b \text{ reellt}$ (37) $\cos(bt)\theta(t)$ $\frac{s}{s^{2} + b^{2}}, \sigma > 0, b \text{ reellt}$	(31)	$f'(t)\theta(t)$	$s\mathcal{L}_{II}(f\theta)(s) - f(0)$
(34) $\theta(t) - 1 = -\theta(-t)$ $\frac{1}{s}, \sigma < 0$ $\frac{k!}{s^{k+1}}, \sigma > 0$ (36) $\sin(bt) \theta(t)$ $\frac{b}{s^2 + b^2}, \sigma > 0, b \text{ reellt}$ (37) $\cos(bt) \theta(t)$ $\frac{s}{s^2 + b^2}, \sigma > 0, b \text{ reellt}$	(32)	$\delta(t)$	1
(35) $t^{k}\theta(t) \qquad \frac{k!}{s^{k+1}}, \sigma > 0$ (36) $\sin(bt)\theta(t) \qquad \frac{b}{s^{2} + b^{2}}, \sigma > 0, b \text{ reellt}$ (37) $\cos(bt)\theta(t) \qquad \frac{s}{s^{2} + b^{2}}, \sigma > 0, b \text{ reellt}$	(33)	heta(t)	$\frac{1}{s}$, $\sigma > 0$
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	(36)	$\sin(bt)\theta(t)$	$\frac{b}{s^2 + b^2}$, $\sigma > 0$, b reellt
$(38) e^{-t^2} \sqrt{\pi}e^{s^2/4}$	(37)	$\cos(bt)\theta(t)$	$\frac{s}{s^2 + b^2}$, $\sigma > 0$, b reellt
	(38)	e^{-t^2}	$\sqrt{\pi}e^{s^2/4}$

(39)
$$h(t) = Ce^{tA}B\theta(t) + D\delta(t)$$

(40)
$$H(s) = C(sI - A)^{-1}B + D$$

Distributioner:

$$\frac{d\theta_a}{dt} = \delta_a$$

$$(42) f(t)\delta_a(t) = f(a)\delta_a(t)$$

(43)
$$f(t)\delta'_a(t) = f(a)\delta'_a(t) - f'(a)\delta_a(t)$$

Invers Laplacetransformation

Om f(t) har den rationella funktionen F(s) som Laplacetransform så är

(44)
$$f(t) = \begin{cases} \sum_{\text{Re } s < \sigma} \text{Res}(e^{st}F(s)), & t > 0 \\ -\sum_{\text{Re } s > \sigma} \text{Res}(e^{st}F(s)), & t < 0 \end{cases}$$

Residyregler

1. Om
$$f(z) = (z-a)^{-N}g(z)$$
 så är $\underset{z=a}{\text{Res }} f(z) = \frac{g^{(N-1)}(a)}{(N-1)!}$

2. Om
$$f(z) = (z-a)^{-N} g(z)$$
 och $g(z) = \sum_{k=0}^{\infty} c_k (z-a)^k$ så är $\underset{z=a}{\text{Res}} f(z) = c_{N-1}$

3.
$$\underset{z=a}{\text{Res}} f(z) = \lim_{z \to a} (z - a) f(z)$$

4. Res_{z=a}
$$\frac{f_1(z)}{f_2(z)} = \frac{f_1(a)}{f'_2(a)}$$