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Nonlinear Optimization Assignment 1

The (.mat) files with function inputs for all the problems are given in the folder.

Problem 1: Interval Scan

Interval scan is implemented to calculate the interval that contains the local minima in a unimodal and univariate function. The function call is

```
[a,b] = interval_scan(f,x,h)
```

a: lower interval
b: upper interval
f: function
x: initial guess
h: step size

In the example given below, the function is $f(x) = x^2 - 3x + 4$. The initial guess is 5 and step size is 0.01. The result obtained is $(a, b) = (-0.11, 3.73)$. The model took 9 iterations to converge to the solution. The graph of the function and the interval is given in Figure 1.

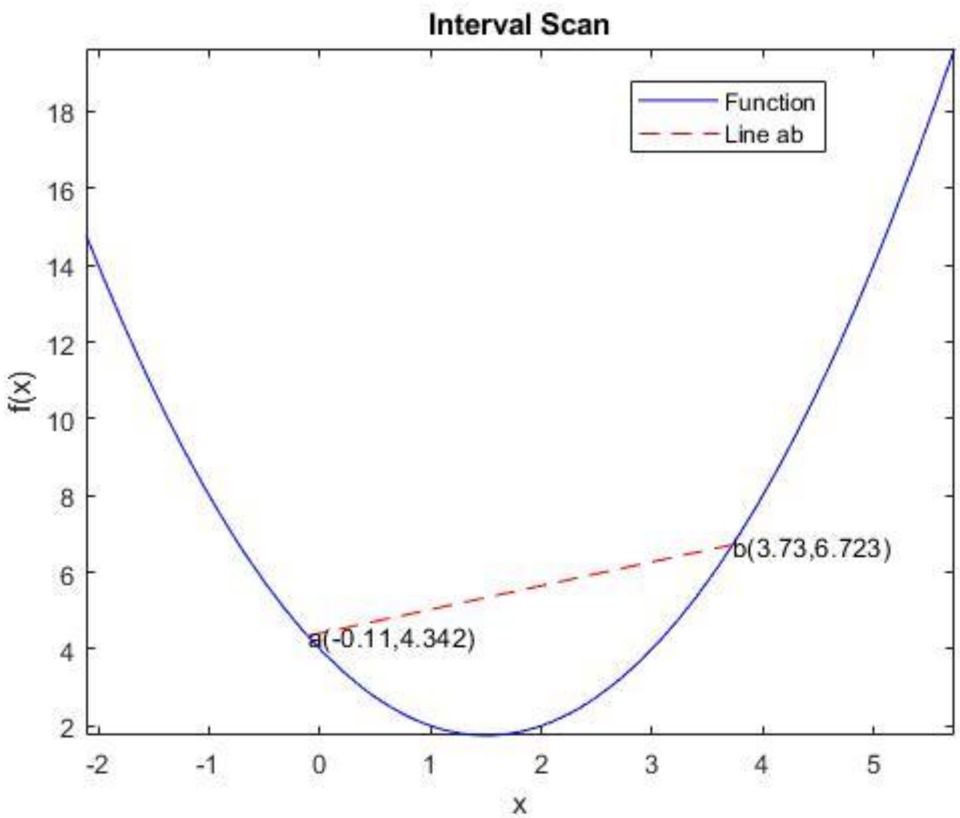

Figure 1: Interval Scan

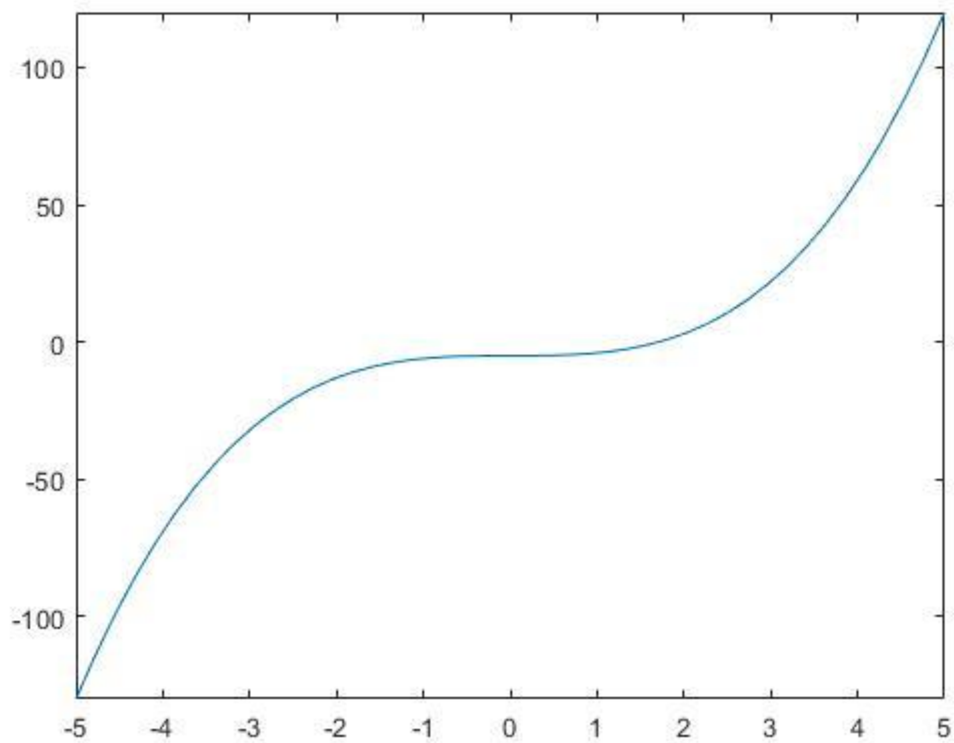

Figure 2: Graph of $f(x) = x^3 - 5$

This algorithm was tested on a cubic function $f(x) = x^3 - 5$
The algorithm did not work for this function because this function is not unimodal.

Problem 2: Bisection Method

Bisection method calculates the local minima of a function given the interval (a,b) within required tolerance. The function call is

```
[c] = bisection(f,a,b,tol)
```

c: local minima
tol: tolerance

For the same function in Problem 1 and intervals obtained using interval scan method, the obtained solution, i.e., the local minima is 1.5005. The value of the function is 1.75. The

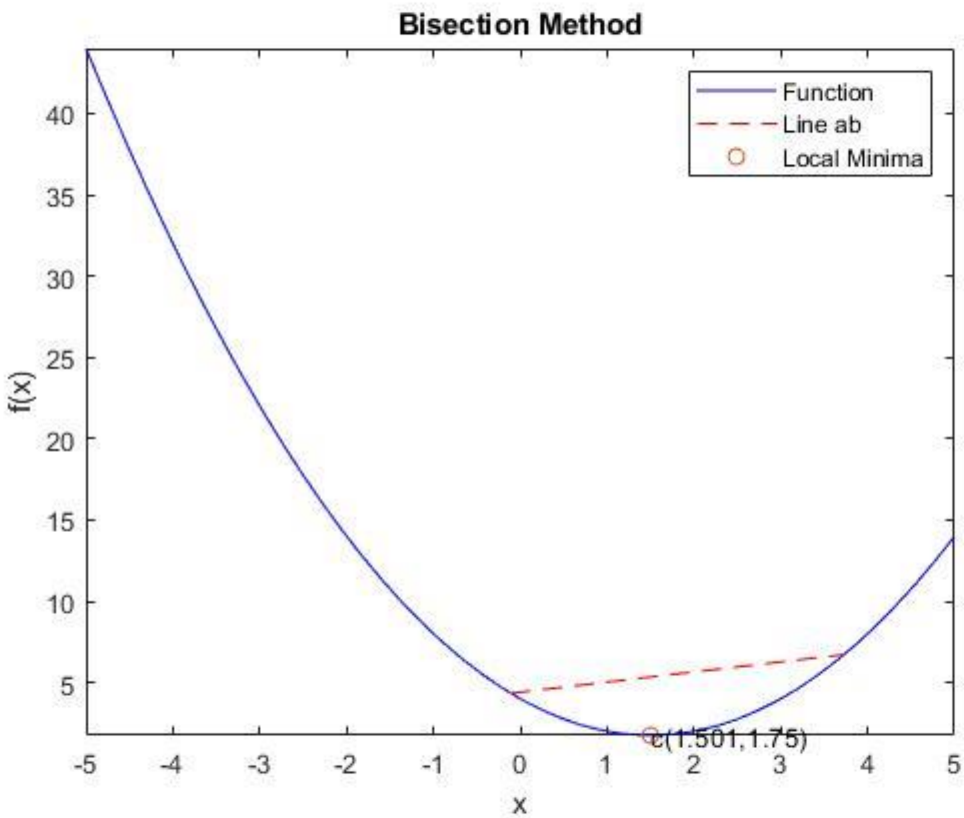

Figure 3: Bisection Method

tolerance is set to 0.001. The plot of the function and the minima is given in Figure 3.

The bisection method is a slow method for optimization. To converge to the solution given above, the algorithm was iterated 25 times.

```
n = log(tol)/log(0.75);
```

#iteration = ceil(n)

Problem 3: Newton-Raphson Method

Newton-Raphson method is implemented to find the local minima of a function using the initial guess. The function call is

```
[x] = newton_raphson(f,x0)
```

x: Local minima
f: function
x0: Initial guess

For the same function as the first problem and initial guess of 0.5, the Newton-Raphson method converged within 2 iterations for tolerance of 0.001. The minima obtained in 1.4950 and the value of the function at the minima is 1.75. This method is faster compared to the bisection method. Also, there is a small difference in the local minima due to the calculation of derivatives based on finite differences method. The graph of the function and the local minima is given in Figure 4.

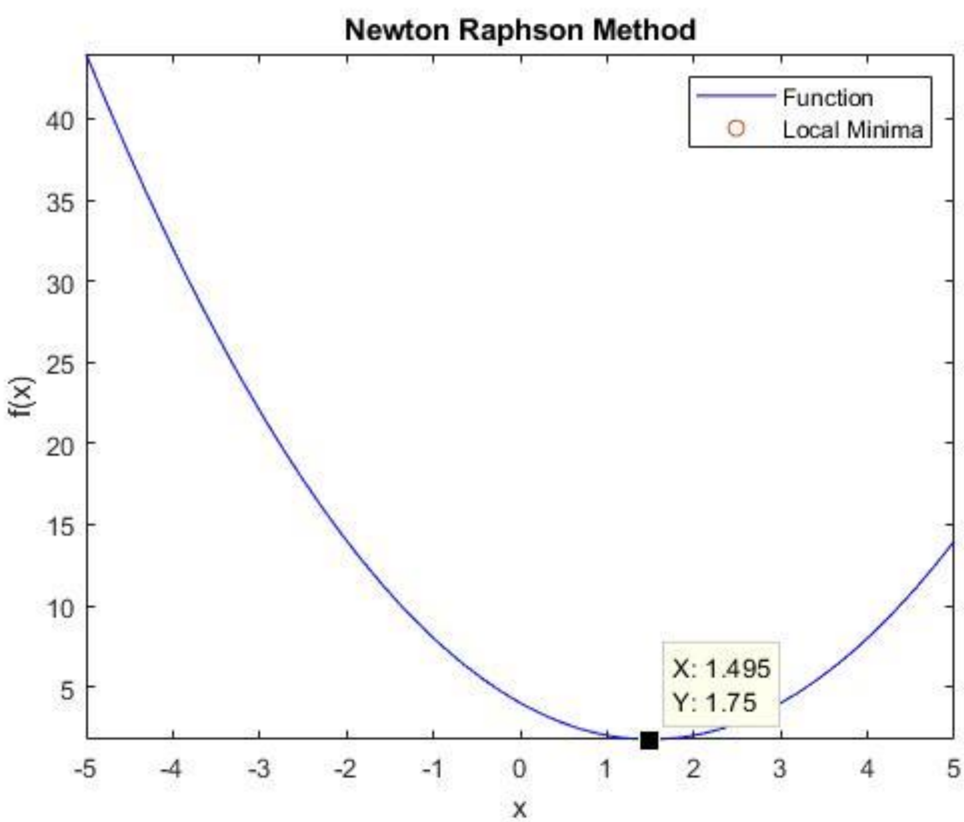

Figure 4: Newton-Raphson Method

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Problem 4: Total Least Squares Method

The total least squares method is used to approximate a best fit line passing through the origin for a set of data formed using a linear equation with added noise. The objective function is given in the assignment paper.
To form the data using the equation given in the question, slope (*m*) is used as 5, intercept(*b*) is set to zero, the number of data points (*N*) is 40, and the magnitude of noise is taken as 0.1. The optimizer used in this problem is the Newton-Raphson method due to its nature of faster convergence when compared to bisection method. The optimal value of the slope for this set of input is 4.996. The value of the objective function at this value is 0.026. Since the data points are given random noises, the function value is slightly different every time. The graph plotting the objective function and the best fit line are given in Figures 5a, and 5b, respectively.

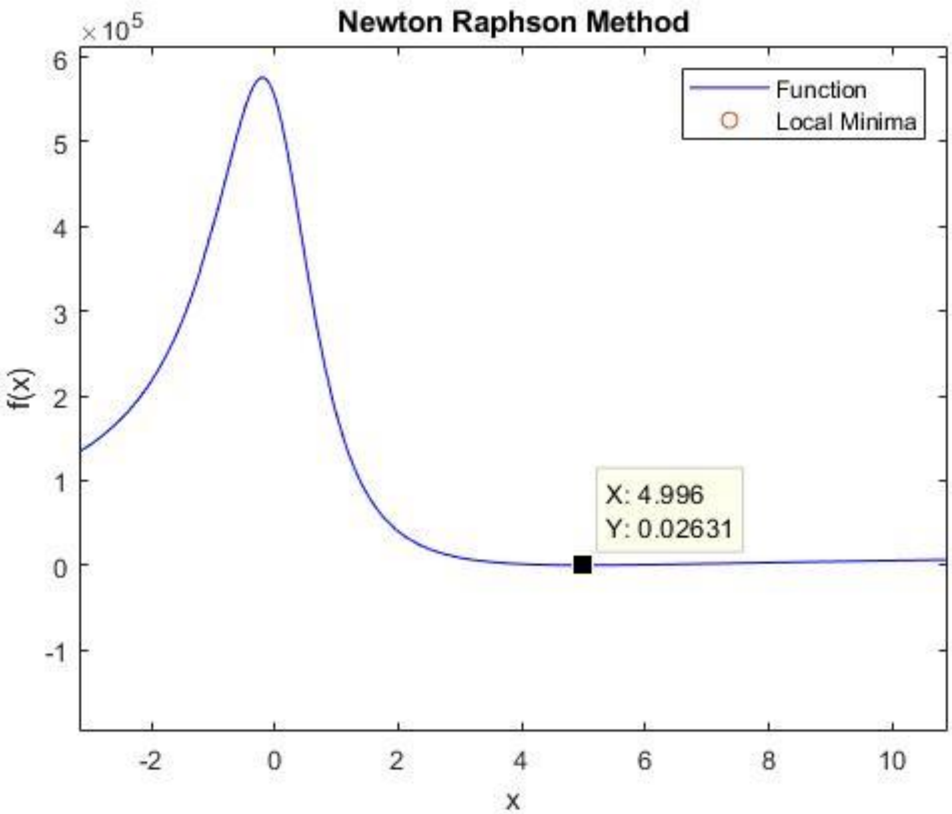


Figure 5a: Objective Function and minima

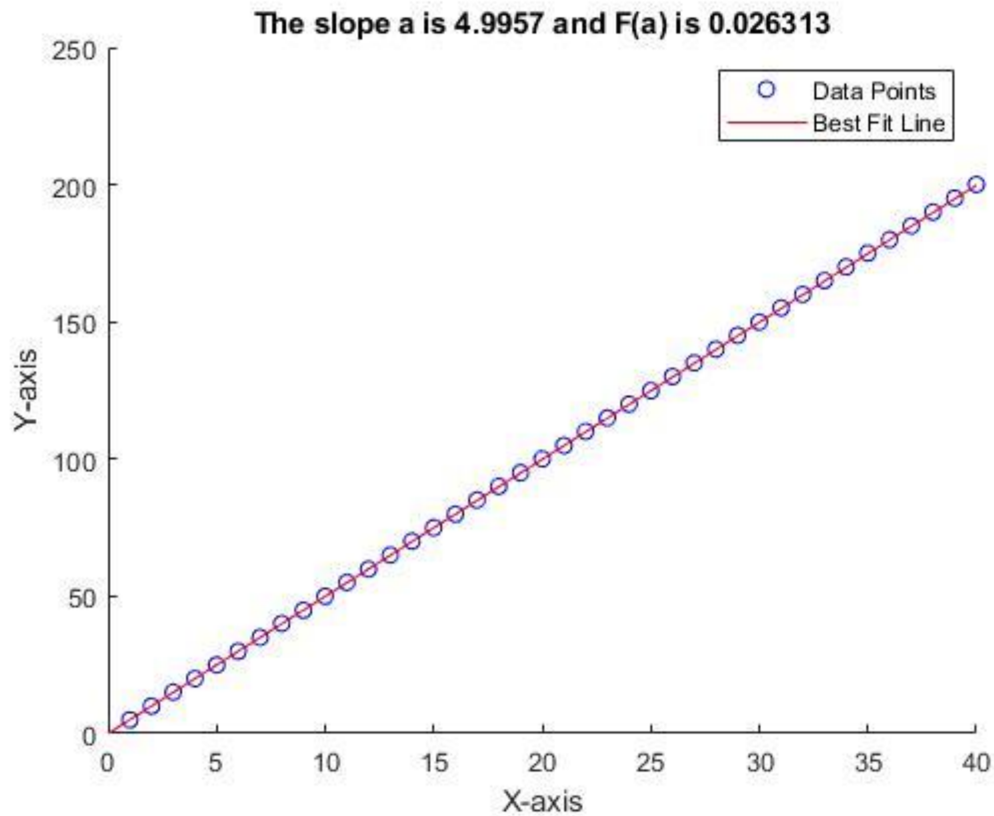


Figure 5b: Best fit line

Problem 5: Routing

(a) The objective function whose minimum would describe an optimal route to target P (x_p, y_p) from origin without crossing the circle at C (x_c, y_c) of radius (r) is given by

$$A(x) = (x - x_p)^2 + (y_p)^2$$

$$B(x) = 2[(x_p - x)(x - x_c) - y_p y_c]$$

$$C(x) = (x - x_c)^2 + (y_c)^2 - r^2$$

$$\Delta = B^2 - 4AC$$

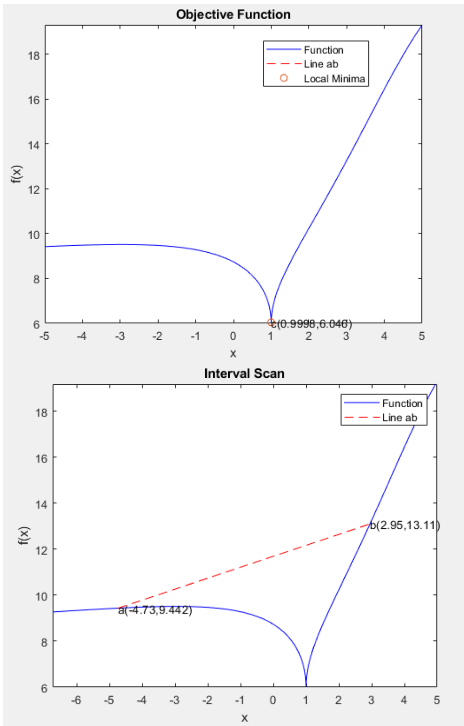
$$F(x) = x + \sqrt{A}\left(1 + \rho\left|\frac{\sqrt{\Delta}}{A}\right|\right)\ (\rho\ is\ penalty)$$

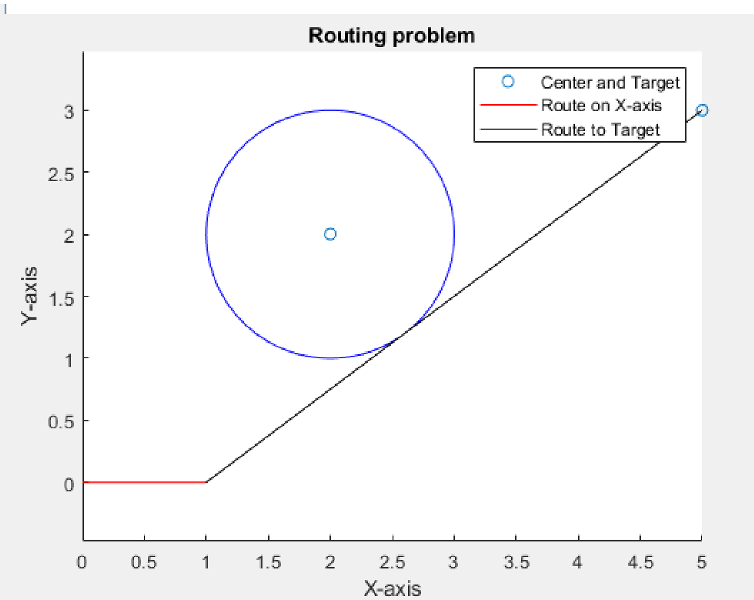
(b) The function call is `[dist] = routing(p,c,r,rho)`
p: target
c: center of the circle
r: radius
rho: penalty (2)

(c) P (5,3) C (2,2), and R = 1.

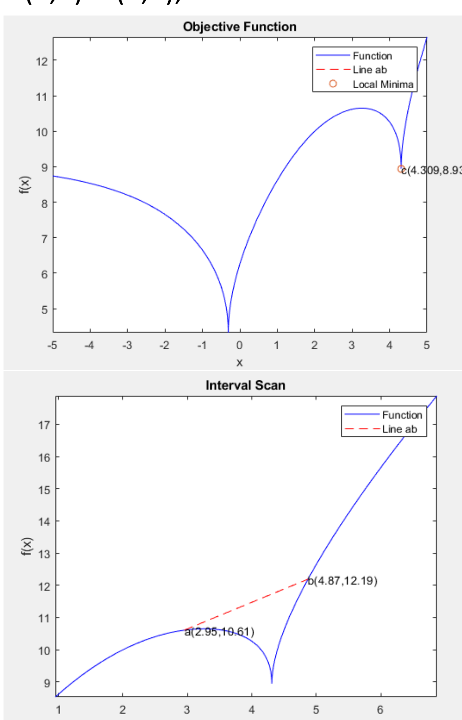
The problem is optimized using the interval scan and bisection method. To ensure the model does not consider the negative kink, the initial guess for the interval scan is taken as 5.5

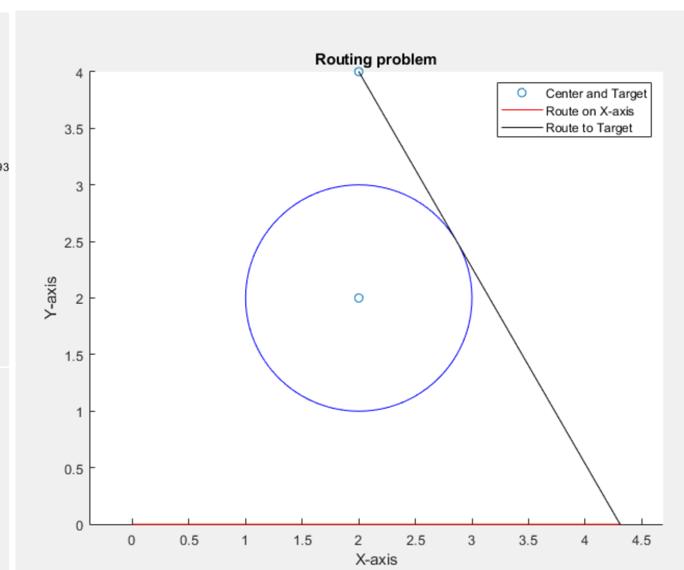
Distance in x axis is 0.9998 and value of function is 6.046.





P (2,4) C (2,2), and R = 1. The distance is 4.3094 and function value id 8.939





P (2,4) C (2,2), and R = 1. The distance is 0.2333 and function value is 5.66