**Problem**

Imagine a deck of 500 cards numbered from 1 to 500. If all the cards are shuffled randomly and you are asked to pick three cards, one at a time, what's the probability of each subsequent card being larger than the previous drawn card?

**Solution**

Imagine this as a sample space problem ignoring all other distracting details. If you have to draw three different numbered cards without replacement, and they are all unique, then we are assuming that there will be effectively a lowest card, a middle card, and a high card.

Let's make it easy and assume we drew the numbers 1,2, and 3. In our scenario, if we drew (1,2,3), then that would be the winning scenario. But what's the full range of outcomes we could draw? Let's map out all of the possibilities.

(3,2,1)  
(3,1,2)  
(2,1,3)  
(2,3,1)  
(1,3,2)  
(1,2,3)

So six possibilities in the total sample space. And only one of them is the partition that we want. Given this, the answer is 1/6.

The trick is to not be distracted by the size of the population. The population does not matter if you are looking at the order within the sample.

**Problem**

Amy and Brad take turns in rolling a fair six-sided die. Whoever rolls a "6" first wins the game. Amy starts by rolling first.

What's the probability that Amy wins?

**Solution**

***Approach 1***

Let's set some definitions.

**pA** = Probability that Amy wins = P[win if go first]  
**pB** = Probability that Brad wins = P[win if go second]

So we can then deduce that Brad's probability of winning then becomes the probability of going first after Amy loses the first rol. We can represent that with this equation of:

 pB = P[Amy loses first roll] \* P[win if go first].

We also know that the probabilities of either Amy or Brad winning should add up to 1. So mathematically we can create two equations:

**pB = 5/6 \* pA (eq. #1)**

**pA + pB = 1** **(eq. #2)**

This is now a linear algebra question. Two equations and two unknowns.

From **(eq. #1) to (eq. #2):**

**pA + 5/6 \* pA = 1 -> 11/6 pA = 1 -> pA = 6/11**

***Approach 2***

pA = 1/6 + P[Amy lose her first try] \* P[Brad lose his first try] \* pA = 1/6 + 5/6 \* 5/6 \* pA

11/36 \* pA = 1/6

pA = 6/11

**Problem**

There are four people on the ground floor of a building that has five levels not including the ground floor. They all get into the same elevator.

If each person is equally likely to get on any floor and they leave independently of each other, what is the probability that no two passengers will get off at the same floor?

**Solution**

The number of ways to assigning five floors to four different people is to get the total sample space. In this case it would be 5 \* 5 \* 5 \* 5. For each person, they can choose one of five floors, which happens four times for four people. So the total number of combinations is **5^4**.

The number of ways to assign five floors to four people without repetition of floors is 5 \* 4 \* 3 \* 2 because for the first passenger you have five different options. The second person has four, and so on. Note that this number counts all possible orders betwen passengers as well.

The result is then 5/5 \* 4/5 \* 3/5 \* 2/5 = **0.192**