On Cybersecurity
Science and Engineering[*]

Francesco Beltramini[1], Marco Rocchetto[1], and Luca Viganò[2]

[1] *V-Research, Verona, Italy*
[2] *King's College London, UK*

Abstract

The objective of this research is to develop of a theory that defines (all and only) the possible insecurity and security configurations of any abstract system. The theory is structured upon other theories that defines how a component of a system can be abstracted into an agent, defining how agents can be formalized (both syntactically and semantically) to describe an abstract system, such as a graph. Some of these theories (e.g. used for the semantic definition of the abstract system) are the epistemological definition of knowledge, the Belief-Desire-Intent and the Assertion-Belief-Fact framework of reference, mereology, and topological structure. We argue that a mereology is the most appropriate abstract underlying structure, do to its generality, for defining the expressiveness of the system abstraction. Furthermore, a mereology allows us to define an ontology rather than a taxonomy. We also correlate different abstractions of the system to the TRL and the engineering V-model.

We implemented a formal theory (of axioms) of a mereotopology, and of the Region Connection Calculus (RCC3 and RCC5) in a Python program that uses the Z3 SMT solver. The results show that a single component (i.e. agent) of an abstract system has a definite number of different insecurity configurations (e.g. 53 using RCC5 over a topological structure) and only 1 secure (i.e. expected) configurations. The configurations are reported as models satisfying the abstract system semantics.

We considered the philosophical definition of truth behind our approach, rejecting "proof" by induction from partial empirical evidences. Our theory can be applied to system engineering and we show a concrete application of our theory to the risk assessment of an ad-hoc system. Finally, we provide a number of ideas to support the engineering of secure systems (e.g. purely cyber or cyber-physical).

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1 Introduction

Humanum est errare

Seneca the Elder

The European Commission states in[18] that: "Cybersecurity is one of the priority areas [...] of the Commission initiative on ICT Standards, which is part of the Digitising European Industry[19] strategy launched on 19 April 2016. The aim is to identify the essential ICT standards and present measures to accelerate their development in support of digital innovations across the economy". The same document (i.e.[18]) states that "The EU will invest up to €450 million [...], under its research and innovation programme Horizon 2020". The EU, in 2016 published a press release[20] in which they present a strategy to invest €1.8 *billion* to "increase measures to address cyber threats". The EU is not the only investor in cybersecurity, most of the developed countries and several companies are investing enormous amount of money towards various aspects of cybersecurity (e.g. The US vulnerability databases[57] maintained by the National Institute of Standards and Technologies, i.e. NIST, of the US Department of Commerce).

The cybersecurity industry is growing fast, e.g. as reported in[16]. For example, in[62], published by the Forbes, is stated that €5.3 billion of funding where poured by venture capitalist into cybersecurity companies in 2018. The Forbes, in the same article, also highlights another peculiar (as seemly contradictory) trend: "[...] during the same time period, the number of cybersecurity breaches increased exponentially". The data reported by the NIST through the official CPE (Common Platform Enumeration) Dictionary Statistics on the NVD websites in [58], show that in 2016 the number of reported vulnerabilities reported where around 6000 while in 2019 the number of vulnerabilities was above 16000. The scientific community also reports similar findings. In fact, in[30], Cormac Herley (Microsoft Research) shows how basic cybersecurity principles (such as the confidentiality benefit over the clear text for passwords typed into forms, e.g. for logins in websites) are not fully understood or shared between the cybersecurity research community[44]. The lack of understanding of basic security principle, the inverse proportionality between investments in cybersecurity and the number of reported vulnerabilities year after year, can be linked to the lack of a foundational theory on cybersecurity, as already highlighted by Cormac Herley in[32].

In this article, we give the first scientific theory (to the best of our knowledge) on security.

Structure. In Section 2 we define and formalize the problem statement. In Section 3 we outline our security theory, and in Section 4 we describe the implementation of the theory and some empirical tests of the theory. Finally, in Section 5 we conclude the paper with an overview of the related work.

Figure 1: Abstraction of an ad-hoc esemplificative protocol execution

2 Problem Statement

In[32], Cormac Herley explores what he calls "an asymmetry in computer security", which he defines as follows: "Things can be declared insecure by observation, but not the reverse. There is no observation that allows us to declare an arbitrary system or technique secure". Herley then uses this argument to show that "claims that any measure is necessary for security are empirically unfalsifiable". Given that, any theory which is not falsifiable by an empirical experiment is well known[1] to be nonscientific (i.e. unfalsifiability is a fallacy of a theory), Herley concludes that there is no scientific theory on cybersecurity; which means that cybersecurity lays in the realm of pseudo-sciences[31]. Herley, e.g. in[29], discusses the implications of a nonscientific approach to cybersecurity, and highlights the tremendous impact on all the scientific research and engineering of systems; leading often to terrorism and wars, and wasting of resources in useless protections or overspending. While the criticism is investigated in[32], no solution is provided. On the contrary, the goal of this work *is* to lay the foundations of a scientific cybersecurity theory. Furthermore, in Section 2.1, we consider the problem raised by Herley not confined to "computer security" but to any abstract system (so that our theory may hold for any sound implementation such as networks, mechanical, cyber, or cyber-physical system, or even a single computer or a single device such as an hard-drive). There is also an apparent inconsistency in[32] that we seek to clarify before following (as we agree) the scientific path draw by Herley: cybersecurity is defined as an abstract property in many formal approaches to the investigation of the security of systems, and the security of the design of a formally verified protocol is indeed falsifiable. For example, in the protocol verification community, security is often defined as a formalization of the high-level properties confidentiality, integrity, and availability. The problem in such approaches is not the definition of what cybersecurity is, but the use of theories (such as the

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[1] "A theory which is not refutable by any conceivable event is nonscientific. Irrefutability is not a virtue of a theory (as people often think) but a vice." – Karl Popper, Conjectures and Refutations[46]

Dolev-Yao attacker model[2][21]) that only applies to specific instances (often called scenarios) and abstraction of the protocol. This, in turn, creates a false sense of security since requires assumptions on the abstraction of the system of which security is verified. As an example, for the formal security verification of the system in Figure 1, a formalized scenario needs to be defined by a modeler who chooses (among others): (i) a scope of the formalization (e.g. excluding the server that distributes the public key is often done when verifying the security of authentication protocols), (ii) the number of sessions (even tough some approaches do reason on an infinite number of sessions such as[23]), (iii) honesty/dishonesty of the peers (e.g. in the ASLan++ language[63]), and (iv) the abstraction of the cryptographic primitives (e.g. ProVerif vs CryptoVerif[5]). Some of the choices will completely change the results of the formal verification of the system. For example, under the perfect cryptography assumption[3] and assuming that no violation to any security property is done after message I); in Figure 1, the freedom of choosing the scope determines that the flaws related to the dishonest impersonation of the Server may or may not be considered in the verification process. This choice has tremendous impact on the focus and findings of the verification of the security of the protocol. While this may seem to turn upon minutiae and foreseeable, this highlights the false sense of security that may derive from a non-scientific theory of system security[4].

2.1 Sicurezza: Safety and Security

In most of the natural languages, and in Italian too, the concepts of safety and security are not syntactically differentiated and both terms (safety and security) are expressed by the same word, e.g. sicurezza in Italian. A semantic distinction between safety and security is correlated to a belief[5] that safety deals with *accidents* (i.e. an unfortunate incident) posed by the natural environment (e.g. natural events such as wearing of hardware components) while security deals with *incidents* posed by mankind (e.g. attackers and bugs). The fundamental difference between nature and mankind (and, in turn, between safety and cybersecurity) is believed to be on the different intents[6] (accidents are unfortunate

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[2]For the sake of simplicity, the Dolev-Yao attacker can be considered as an abstraction of an active attacker who controls the network but cannot break cryptography.

[3]As defined in[51]: "In the so called perfect cryptography assumption, the security encryption scheme is suppose to be perfect, without any exploitable flaw, and so the only way for the attacker to decrypt a message is by using the proper key. That assumption is widely accepted in the security protocol community, and most of the formal reasoning tools for the analysis of security protocols abstract away the mathematical and implementation details of the encryption scheme[60, 3, 2, 52]"

[4]"To the superficial observer, the analysis of these forms seems to turn upon minutiae. It does in fact deal with minutiae, but they are of the same order as those dealt with in microscopic anatomy." – Karl Marx, Capital Volume 1, 1867

[5]A belief has to be intended as a proposition which is supposed to be true by the majority of humans in our society without a scientific underlying theory but based on partial empirical evidences or inductive proofs.

[6]"The belief–desire–intention software model (BDI) is a software model developed for programming intelligent agents."[36]. In the BDI model, the intents represents the deliberative

while incidents are not) of the causes that generates the threat; namely, nature is believed not to have malicious intents (but unfortunate causes-effects) while threats generated by mankind are believed to be malicious[7]. An overview on the aforementioned aspects of safety and security is depicted in Figure 2 and is used as a baseline for a definition of the terms that structure our current understanding of safety and security.

- *Mankind* "refers collectively to humans" [39], while the concept of *Nature* is related "to the intrinsic characteristics that plants, animals, and other features of the world develop of their own accord" (e.g. the physical universe)[40].

  - So far, we have used several terms to refer to an *attacker*, i.e. threat agent or threat source, considering those terms to be semantically equivalent. This "shallowness" raise form the necessity of properly citing the different sources, but, in the reminder of this paper, we consider the Causality principle to be the *threat source*, Nature or Mankind to be the *threat agents* and an *attacker* as a specific malicious threat agent which materialize a threat.

- *Vulnerability*[8], as defined in[43] (and adopted in[4]), is "weakness in an information system, system security procedures, internal controls, or implementation that could be exploited by a threat source". On the one hand, the definition is broad to enclose as much causes (that generates a vulnerability) as possible; on the other hand, it derives from empirical evidences (which should be considered beliefs[9] since they are partial results in nature) while a vulnerability should be defined in a way that is empirically falsifiable. This means that the term vulnerability should have a complete and sound definition, so that no other causes (e.g. other sources) but the ones in the definition are responsible for a vulnerability. Furthermore, the term "threat sources" used in the definition in[43] may be identified with both Nature and Mankind, not differentiating between safety and security. In Definition 2, we provide a formal theory of vulnerability (so that the scientific community can identify tests for the completeness and soundness of the definition itself).

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state of an agent which determines the choice of that agent on what to do.

[7]Of course, logical flaws or bugs may be introduced by other means (e.g. ignorance) without explicit malicious intents, but the exploitation of those flaws is considered (for now, and detailed afterwards in the article) malicious, and then we consider any vulnerability to be malicious (without loss of generality) even if due to the lack of skills.

[8]The term vulnerability is not present in the Encyclopedia of Cryptography and Security, while it is used in 12 entries (such as in the definition of "penetration testing"[6]) highlighting how commonly this word is used without a proper supporting semantics

[9]"For this view, that *That Which Is Not* exists, can never predominate. You must debar your thought from this way of search, nor let ordinary experience in its variety force you along this way, (namely, that of allowing) the eye, sightless as it is, and the ear, full of sound, and the tongue, to rule; but (you must) judge by means of the Reason (Logos) the much-contested proof which is expounded by me." – Parmenides of Elea, On Nature (circa 500 B.C.), fragments B7.1–8.2 [28]

Figure 2: Overview security and safety keywords

Most of the safety-preserving principles in the field of engineering of safety-critical cyber-physical systems (such as elevators and aircraft), upon which safety requirements (e.g. in standards such as the IEC 61508 or 61511[1]) are defined, have been defined following empirical tests and measurements. While reasoning by induction based on the empirical observation should be avoided, since it may easily lead to false beliefs instead of scientific theories, this approach is often justified by the supposed impossibility of defining a theory that correctly predicts failures which, in turn, pose hazards to a system. To the best of our knowledge, and supported by[32], the correlation between predictability of environment and believed unpredictability of attackers (i.e. a malicious environment) has not been correlated to a theory on cybersecurity. Therefore, inductive research efforts in predicting malicious effects are accepted (and published) in scientific conferences (e.g. [50]). A failure of a wire due to environment (e.g. due to humidity, dust, heat &c) is defined from empirical evidences and processes have been standardized to test qualities of hardware components This process completely breaks down when a malicious environment (i.e. an attacker) is considered instead of the (supposedly honest and predictable) natural environment. Therefore, the same approach that is in use for safety, seems not to be applicable to test security.

Going back to Figure 2, a vulnerability does not necessarily become a threat for the system, unless exploited "through a channel that allows the violation of the security policy [. . .]"[43] (e.g. a software or procedure) that takes advantage of the vulnerability causing an *attack* to the system, which may result in several correlated incidents and threats. The process of exploitation of a defect as a vulnerability is reported in Figure 2 such that the difference between exploit and failure, and attack and accident is to be found just in the maliciousness of the intents that causes this process (i.e. excluding the intent, the terms are just syntactic transformation from a vulnerability to defect, from accident to incident). In the following, we conclude the informal definition of the terms that we used in this section and in Figure 2.

- *Causality* refers to the causality principle; defined in[17] as "Causality is a genetic connection of phenomena through which one thing (the cause) under certain conditions gives rise to, causes something else (the effect).

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The essence of causality is the generation and determination of one phenomenon by another. In this respect causality differs from various other kinds of connection, for example, the simple temporal sequence of phenomena, of the regularities of accompanying processes".

- An *Exploit*[10] is "An exploit (from the English verb to exploit, meaning to use something to one's own advantage) is a piece of software, a chunk of data, or a sequence of commands that takes advantage of a bug or vulnerability to cause unintended or unanticipated behavior to occur on computer software, hardware, or something electronic (usually computerized)."[38].

- An *Attack*, as defined by the International Standard ISO/IEC 27000 is an "attempt to destroy, expose, alter, disable, steal or gain unauthorized access to or make unauthorized use of an asset"; where an *Asset* is "anything that has value to the organization". We note that for the purpose of this article, we do not want to focus on a specific organization or business to define asset but, in general, on any abstract organization (e.g. a company or a society). We do not consider ethical hackers as attacking a system. In fact, we consider the term *hack* as non-malicious (see Hacker[55]).

- A *Threat*, as defined in[43], is "Any circumstance or event with the potential to adversely impact organizational operations (including mission, functions, image, or reputation), organizational assets, individuals, other organizations, or the Nation through an information system via unauthorized access, destruction, disclosure, modification of information, and/or denial of service".

- *Defect*, "anything that renders the product not reasonably safe"[49] (i.e. a characteristic of an object which hinders its proper usability).

- *Failure*, as defined in[35] as "a state of inability to perform a normal function". The term is structured and detailed in [43, 22] but relying on an abstract notion of failure without a specific definition.

- *Hazard*, "a potential source of harm"[22].

2.2   Glossary – A Formalization

We now define a formalizations of the concepts described in Section 2 and depicted in Figure 2. We base our formalization on first order logic (FOL) with the standard truth-value semantics. The choice of this logic is required by the semantics of the concepts formalized afterwards in this section.

We consider a first-order language $\mathcal{L}$ over a signature Σ_P where $P, P', \ldots, P^n$ represent terms, $\varphi, \varphi', \ldots, \varphi^n$ represent formulas. The syntax is defined as follows.

$$\varphi := P \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \varphi \Rightarrow \varphi' \mid \forall P.\ \varphi \mid \exists P.\ \varphi$$

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[10]We note that the term exploit is only used as a verb in[56]

where $\wedge$, $\neg$, $\vee$, and $\Rightarrow$ are connectives representing conjunction, negation, disjunction, and (material) implication respectively; while $\forall$ and $\exists$ represent the standard universal and existential quantifiers (resp.). Finally, the symbol "." is just syntactic sugar. We consider $\sigma \subset \Phi \times \{\top, \bot\}$ as the interpretation function, where Φ is any collection of sentences in $\mathcal{L}$ and $\top$ and $\bot$ represent the concepts of "Tautology/True" and "Contradiction/False" respectively.

Following Figure 2, we start by formalizing the outermost term: Causality.

2.2.1 Causality Principle

We formalize the Causality Principle starting from a K modal logic[25] (i.e. without restrictions on the causality relation between worlds). The standard definition of K modal logic is given in Definition 2.1 in terms of an interpretation function (which we named σ with a slight abuse of notation) defined in Definition 2.2.

Definition 2.1. K Modal Logic – A K-frame is a frame $\mathbf{K} = \langle G, R \rangle$ in the K modal logic where R is the binary relation (i.e. a set of ordered pairs) between possible worlds $R \subseteq G \times G$, where G represents the possible worlds, and $G \neq \emptyset$. An actual world $\omega* \in G$ is assumed. For any proposition P, an interpretation function $\sigma(\omega, P)$ returns the truth value of P; e.g. $\sigma(\omega, P) = \top$ means that P holds in ω. A model is defined as the tuple $M = \langle G, R, \sigma \rangle$.

Definition 2.2. Causality as K Modal Interpretation Function – Causality is (recursively) defined as the modal interpretation function σ, as follows.

($\sigma 0$) if $\sigma(\omega, P) = \top$ then $\omega \models P$
($\sigma 1$) $\omega \models \neg P$ iff $\omega \not\models P$
($\sigma 2$) $\omega \models P \wedge Q$ iff $\omega \models P$ and $\omega \models Q$
($\sigma 3$) $\omega \models \Box P$ iff for any world $\omega' \in G$ if $\omega R \omega'$ then $\omega' \models P$
($\sigma 4$) $\omega \models \Diamond P$ iff there exists a set of worlds $\Omega' \subset G$ such that for any $\omega' \in \Omega'$, if $\omega R \omega'$ then $\omega' \models P$
($\sigma 5$) $\models P$ iff $\omega* \models P$

where truth is defined as necessary with $\Box$ and possible with $\Diamond$.

The causality principle has been defined in its generic form. In fact, the accessibility relation R is free from any axiomatic restriction (e.g. it's not reflexive nor anti-reflexive). We will focus in Section 2.4 (for our tests) on the application of our theory to CPS system engineering. More detailed case studies (i.e. **tofix:** a CPS as a smart power-grid), will be defined in Section 4, where the accessibility relation defines in details how the system itself evolves, due to causal relation (i.e. by restricting the causality principle only to those cause-effect that defines the system). Therefore, the definition of R will be specialized in a more strict way based on the application domains and case study.

tofix: We note that we have, so far, defined the Causality principle without considering it a *threat source* since we lack the concept of intent and maliciousness. We introduce those concepts in Section 2.2.2 and then formally define a threat agent in the same section.

Table 1: RCC3, RCC5, and RCC8 relations between Regions X, Y and Z

RCC3	RCC5	RCC8	Name	Notation	Definition
			Connects with	$C(X,Y)$	$X \subseteq Y$
			Disconnected from	$\neg C(X,Y)$	$X \nsubseteq Y$
			Part of	$P(X,Y)$	$\forall Z\ C(Z,X) \rightarrow C(Z,Y)$
			Overlaps	$O(X,Y)$	$\exists Z\ P(Z,X) \wedge P(Z,Y)$
●			**Overlaps Not Equal**	$ONE(X,Y)$	$O(X,Y) \wedge \neg EQ(X,Y)$
●	●	●	**Equal to**	$EQ(X,Y)$	$P(X,Y) \wedge P(Y,X)$
●	●	●	**DiscRete from**	$DR(X,Y)$	$\neg O(X,Y)$
	●	●	**Partial-Overlap**	$PO(X,Y)$	$O(X,Y) \wedge \neg P(X,Y) \wedge \neg P(Y,X)$
	●		**Proper-Part-of**	$PP(X,Y)$	$P(X,Y) \wedge \neg P(Y,X)$
	●		**Proper-Part-of-*inverse***	$PPi(X,Y)$	$P(Y,X) \wedge \neg P(X,Y)$
		●	**Externally Connected**	$EC(X,Y)$	$C(X,Y) \wedge \neg O(X,Y)$
		●	**Tangential PP**	$TPP(X,Y)$	$PP(X,Y) \wedge \exists Z\ [EC(Z,X), EC(Z,Y)]$
		●	**Tangential PPi**	$TPPi(X,Y)$	$TPP(Y,X)$
		●	**Non-Tangential PP**	$NTPP(X,Y)$	$PP(X,Y) \wedge \neg\exists Z\ [EC(Z,X), EC(Z,Y)]$
		●	**Non-Tangential PPi**	$NTPPi(X,Y)$	$NTPP(Y,X)$

2.2.2 Agents: Mankind and Nature

Mankind and Nature are defined in Section 2 as two abstract agents, both as collections (i.e. an abstract type that does not imply a specific implementation) of their sub-agents (i.e. humans for Mankind and plants, animals, &c. for Nature). Similarly to[53], we define Mankind and Nature, and any other agent in the reminder of this article, as a meronomy (an hierarchy of Part-Whole relations) based on a standard definition of mereology, i.e. based on the definition of Parthood relation between *Parts*. However, we will need to consider different types of Part; so, we extend the mereology to a mereo-topology[54, 61, 47], to increase the number of different types considered and to generalize the relations between Parts (as in Table 1). For the sake of readability, we use the term *Region* both to refer to a mereological Part and to a topological Region. The choice of mereotopology is also correlated to the objective of defining a formal ontology, which we use to define the (formal) semantics of the terms (Parts) in Section 2, and of the concepts of safety and security (whole). We aim at creating a meronomy instead of the taxonomies such as the one provided in[57, 48] or instead of the dogmatic CVSS[42] scoring system.

A mereotopology, as defined e.g. in[47], is an ordered mathematical structure where the basic relation between Regions is the reflexive and symmetric**FIX**[11] *Parthood* relation $\subseteq$.

Definition 2.3. Parthood – Given any pair of mereotopological Regions X and Y,

1. Reflexivity: $\forall X.\ (X \subseteq X)$
2. Symmetry: $\forall X, Y.\ (X \subseteq Y \Rightarrow Y \subseteq X)$

As later defined in Definition 2.5, the Parthood relation orders a universe of agents $\mathcal{A}$ by defining the so called *Connects with* (see in Table 1) relation

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[11]**mr:** I guess it must be monotonic as defined in[47] but I don't find it consistently in other papers.

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between Regions. We want this universe $\mathcal{A}$ to be expressible in FOL. In this way, we can reason both on the constituent of security (i.e. its terms and agents defining a system where security needs to be considered), and on the evolution of those constituent w.r.t. cause-effects relations according to the modal structure of causality we defined in Section 2.2.1. This will allow us (in Section 2.4 and Section 4) a better**FIX**[12] positioning w.r.t. risk assessment technologies (which most often reason on the constituent of a system design), and protocol verification tools (which requires some formalization of a cause-effect relation, e.g. the Linear Temporal Logic).

As we argued in Section 2, we must correlate the definition of agent (i.e. Mankind and Nature) in Definition 2.11 to the mathematical structure of the logic that defines them. We express Mankind and Nature as formulas over the theory of mereology and then in terms of mereotopological Regions, extending the interpretation function σ, to include a formal theory of mereotopology. We use the Region Connection Calculus (RCC), as defined in [41, 26], to provide an axiomatization of the spatial concepts and relations in first-order logic to correlate the algebraic structure to mereology. In its broader definition, the RCC theory is composed by eight axioms, and is known as RCC8, but here we restrict to RCC5 (without loss of generality) by not considering tangential connections between spatial Regions. We discuss the choice of RCC5 in more detail in Appendix A. In Table 1, we summarize the axioms of the Region Connection Calculus (see, e.g., [27]).

Definition 2.4. RCC axiomatization – For any X, Y pair of Regions in a mereotopology:

($\sigma6$) $\sigma(X \subseteq Y)$ iff $[\sigma(X \subseteq X) = \top$, and $\sigma(X \subseteq Y) = \bot$ or $\sigma(Y \subseteq X) = \top]$**FIX**[13]

($\sigma7$) $\omega \models C(X,Y)$ iff $\omega \models X \subseteq Y$

($\sigma8$) $\omega \models \neg C(X,Y)$ iff $\omega \not\models X \subseteq Y$

($\sigma9$) $\omega \models P(X,Y)$ iff $\omega \models \forall Z.\ C(Z,X) \Rightarrow C(Z,Y)$

($\sigma10$) $\omega \models O(X,Y)$ iff $\omega \models \exists Z.\ P(Z,X) \wedge P(Z,Y)$

($\sigma11$) $\omega \models EQ(X,Y)$ iff $\omega \models P(X,Y) \wedge P(Y,X)$

($\sigma12$) $\omega \models DR(X,Y)$ iff $\omega \models \neg O(X,Y)$

($\sigma13$) $\omega \models PO(X,Y)$ iff $\omega \models O(X,Y) \wedge \neg P(X,Y) \wedge \neg P(Y,X)$

($\sigma14$) $\omega \models PP(X,Y)$ iff $\omega \models P(X,Y) \wedge \neg P(Y,X)$

($\sigma15$) $\omega \models PPi(X,Y)$ iff $\omega \models P(Y,X) \wedge \neg P(X,Y)$

where Z is a mereotopological Region.

Definition 2.5. Agent: Mankind or Nature – An agent $a \in \mathcal{A}$ is a tuple $\langle rcc(\chi', \chi''), \ldots, rcc(\chi^{n-1}, \chi^n) \rangle$ of RCC relations rcc over mereotopological Regions $\chi', \ldots, \chi^n \subseteq X$.

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[12]**mr:** Can we prove this by construction?

[13]**mr:** would it be better/clearer or just correct to write $\sigma(X \subseteq Y)$ iff $\sigma(X \subseteq X \wedge [X \subseteq Y \vee Y \subseteq X]) = \top$

Currently, we do not distinguish between Mankind and Nature (since we still lack of the definition of "malicious intent", which is defined in Section 3) and we have defined them as two generic agents.**FIX**[14]

As depicted in Figure 2, Causality, Mankind, and Nature have a dashed border representing their correlation in terms of cause-effect and then in terms of formal structure which defines them: Modal Logic. Vulnerability, Defect, Incident, Accident, and Threat, similarly, are correlated (depicted as a solid border) in terms of underlying formal structure: Mereotopology.

2.2.3   Regions: Vulnerability and Defect (and Weakness)

As defined in Section 2, a Vulnerability or a Defect is a *Weakness*. As an example, a categorization of weaknesses is given in[15] with 808 weaknesses categorized as "Research Concepts", distributed as follows:

- Incorrect Calculation - (682)
- Incorrect Access of Indexable Resource ("Range Error") - (118)
- Use of Insufficiently Random Values - (330)
- Improper Interaction Between Multiple Correctly-Behaving Entities - (435)
- Improper Control of a Resource Through its Lifetime - (664)
- Insufficient Control Flow Management - (691)
- Protection Mechanism Failure - (693)
- Incorrect Comparison - (697)
- Improper Check or Handling of Exceptional Conditions - (703)
- Improper Enforcement of Message or Data Structure - (707)
- Improper Adherence to Coding Standards - (710)

The definition given by the MITRE in[24] of weakness is: "Software weaknesses are errors that can lead to software vulnerabilities. A software vulnerability, such as those enumerated on the Common Vulnerabilities and Exposures (CVE) List, is a mistake in software that can be directly used by a hacker to gain access to a system or network". The definition is circular if we interpret the word "error" and "mistake" with the same semantics: a weakness is an error that leads to a vulnerability and a vulnerability is a mistake which, in turn, is a weakness. The only difference (between weakness and vulnerability) seems to be that one can consider weakness as a ground term and state that a vulnerability is caused by a weakness, i.e. $\Omega, W \models \Diamond V$ where W, V are Regions of Weaknesses and Vulnerabilities (resp.); accepting the hierarchy in the CWE[7] as a ground truth. Similarly, we consider the CVE[8] (a database of Vulnerability) or the CVE reported in the NVD as a ground truth.

Definition 2.6. Region: Weakness and Vulnerability – A Region $\chi \subseteq a$ of an agent $a \in \mathcal{A}$ is defined as Weakness W (i.e. representing a weakness introduced by the agent into any phase of production, e.g. of the secure process development life-cycle, of any system or subsystem) iff there exists $\omega \in G$ such

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[14]**mr:** check if it's a repetition and if links to sections are consistent

that $\Omega, W \models \Diamond \exists W', V'. \; PO(W', V')$, where $W' \subseteq W$, and $V' \subseteq V$ is a Region of an agent $a' \in \mathcal{A}$.

Example 1. *CWE-116: Improper Encoding or Escaping of Output[34] –*

- *Description: The software prepares a structured message for communication with another component, but encoding or escaping of the data is either missing or done incorrectly. As a result, the intended structure of the message is not preserved.*

- *Example: This code displays an email address that was submitted as part of a form. Example language JSP.*

  ```
  <% String email = request.getParameter("email"); %>
  ...
  Email Address: <%= email %>
  ```

  The value read from the form parameter is reflected back to the client browser without having been encoded prior to output, allowing various XSS attacks (CWE-79).

- *Observed Examples*

  - *CVE-2008-4636[13]: OS command injection in backup software using shell metacharacters in a filename; correct behavior would require that this filename could not be changed.*

  - *CVE-2008-0769[11]: Web application does not set the charset when sending a page to a browser, allowing for XSS exploitation when a browser chooses an unexpected encoding.*

  - *CVE-2008-0005[9]: Program does not set the charset when sending a page to a browser, allowing for XSS exploitation when a browser chooses an unexpected encoding.*

  - *CVE-2008-5573[14]: SQL injection via password parameter; a strong password might contain &*

  - *CVE-2008-3773[12]: Cross-site scripting in chat application via a message subject, which normally might contain & and other XSS-related characters.*

  - *CVE-2008-0757[10]: Cross-site scripting in chat application via a message, which normally might be allowed to contain arbitrary content.*

Definition 2.7. Region: Vulnerability and Defect – A region $\chi \subseteq a$ of an agent $a \in \mathcal{A}$ is called *Vulnerability* if the agent a is referred to as Mankind, *Defect* if the agent is referred to as Nature.

Property of V-Research S.r.l.

(a) Representation with RCC5

(b) Representation with RCC8
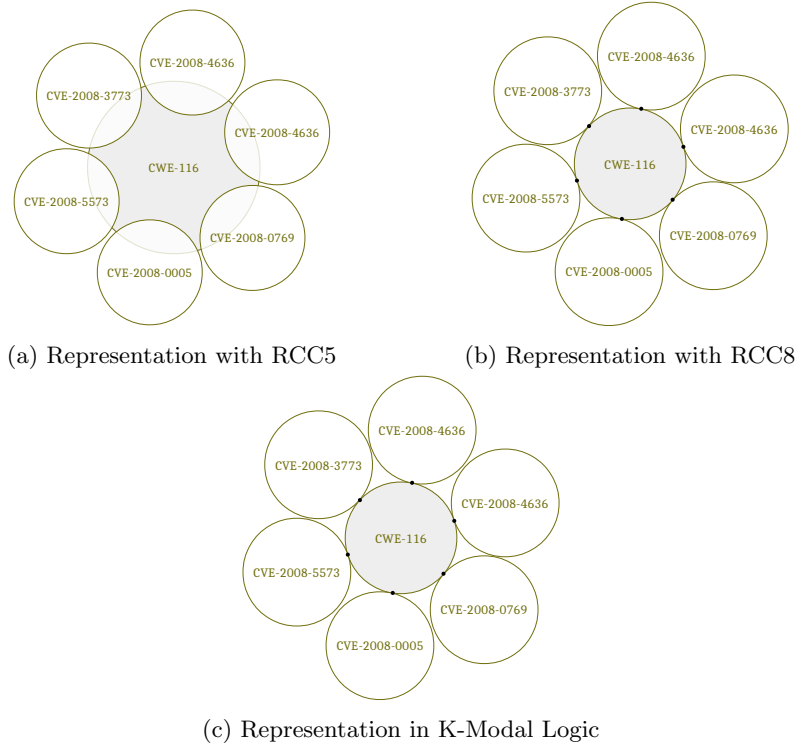
(c) Representation in K-Modal Logic

Figure 3: Relations between CWE-116 and correlated CVEs

In our informal definition depicted in Figure 2, a Vulnerability is seen as a process through which an agent (i.e. Mankind) generates an Incident, i.e. Agent→Vulnerability→Exploit→Attack→Incident. We describe processes, in our current formalization, in terms of the K-Modal Structure in Definition 2.2 (i.e. in terms of causality principle).

Definition 2.8. Vulnerability-Incident Process – Given two

The whole-part relation **tofix:** (i.e. HAS-A[15]) is given, in the following, over the K Modal logic described beforehand. To do so, we first need to formally define the concept of *agent* in its abstract form. We build it following the same structure of[53] but over the terms used in Section 2, i.e. knowledge, and belief as epistemic concepts. We'll then introduce the concept of *intent* to define a *threat agent*.

The difference between Knowledge and Belief is depicted in Figure 5 (see [37]). However, according to[59], Knowledge[16] as an epistemological concept is difficult (and sometimes believed impossible[**citation**]) to formally define. In

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[15]HAS-A is just an hint for the mechanization of this theory. We'll come back to this in Section 4.1

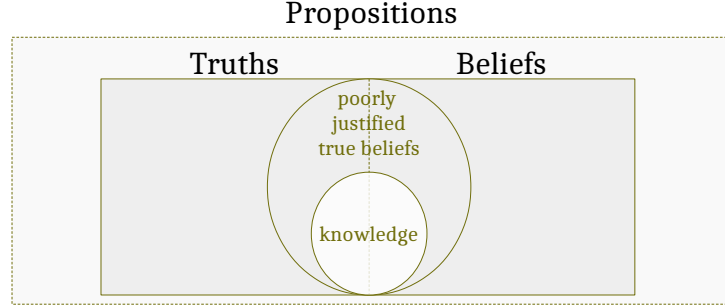[16]"*Theaetetus*: [. . .] He said that knowledge was true opinion accompanied by reason, but

Figure 4: Informal representation of Knowledge and Belief

this work, we are not interested in how knowledge can be precisely formalized from an epistemic standpoint. We assume that a semantic of a correct definition of epistemic knowledge exists, for example the one given in[33] by Hintikka, and we then define knowledge in terms of the Kripke structure defined in Definition **??**.

Definition 2.9. Knowledge – Given an abstract collection of Agents Ag, and the modal operator K_a (where $a \in Ag$), Knowledge is defined as a collection of predicates known by an agent $\mathbb{K}_a = \bigcup_\Phi K_a \varphi$ (where Φ is the collection of all the propositions known by a). Given a proposition P, we extend the semantics of the Causality structure with:

(σ6) $\omega \models K_a P$ iff $\omega' \models P$ for all ω' such that $\omega R \omega'$.

Definition 2.10. Belief – **tofix:** we should help the reader in understanding the upgrades of the logic. We used propositional with modal operators with a K Kripke semantics and we are now introducing Belief as in Doxastic logic? and we have epistemological operators as knowledge Given an abstract collection of Agents Ag, and the modal operator B_a (where $a \in Ag$), Belief is defined as a collection of predicates believed by an agent $\mathbb{B}_a = \bigcup_\Phi B_a \varphi$ (where Φ is the collection of all the propositions believed by a). Given a proposition P, we extend the semantics of the Causality structure with:

(σ7) $\omega \models B_a P$ iff $\omega \models \neg K_a \neg P$ (i.e. the agent a considers P possible) and $\omega' \models P$ for all ω' such that $\omega R \omega'$.

Definition 2.11. Mankind/Nature – Mankind/Nature is represented as an agent.

that unreasoning true opinion was outside of the sphere of knowledge; and matters of which there is not a rational explanation are unknowable – yes, that is what he called them – and those of which there is are knowable. [. . .] *Socrates*: [. . .] the primary elements of which we and all else are composed admit of no rational explanation; for each alone by itself can only be named, and no qualification can be added, neither that it is nor that it is not, for that would at once be adding to it existence or non-existence, whereas we must add nothing to it, if we are to speak of that itself alone. [. . .]" Plato – Theaetetus 201 [45]

Property of V-Research S.r.l.

2.2.4 Mereotopological Representation of Safety and Security

The objectives of this section are:

1. formalize the abstract algebraic structure (as a mereotopology) on top of which we have formalized the terms in Figure 2 so far, i.e. Mankind, Nature, Truths, Beliefs, and Knowledge.
2. correlate mereological structure to Causality and then Kripke structure by defining the relation R over the Kripke structure as the mereotopological basic relation connects with, i.e. reflexive and symmetric
3. we map signature to mereotopological Regions to express the logical formulation of terms (e.g. Mankind, Nature, Truths, Beliefs, Knowledge, Vulnerabilities) over the mereotopological structure
4. we formalize the relations between terms using the RCC(5)

tofix: change propositions in propositional language http://logic.stanford.edu/intrologic/lectures/lecture_02.pdf

Given that the concepts of Truths, Beliefs, and Knowledge are positioned over an underlying structure of Propositions, the first concept to formalize is that of Propositions. We do not restrict Propositions to a specific structure but, as for Mankind and Nature, we refer to them as a collection. We express the generality of a collection as a (propositional) signature over an abstract algebraic structure. In our formalization, we use mereotopology as algebraic structure (due to its generality, as discussed in Section 2) and the RCC to formalize concepts over a mereotopology.

We can now define the possible mereotopological relations between Belief and Knowledge (informally depicted in Figure 5) and then define the peculiarity of the believed difference between Mankind and Nature (i.e. maliciousness as intent). We first start from the formalization of Figure 5, as follows.

Definition 2.12. Information

Definition 2.13. Desire

Definition 2.14. Intent

2.3 Vulnerability and Defect

In order to formalize the semantics of the concepts (informally) described in Section 2,
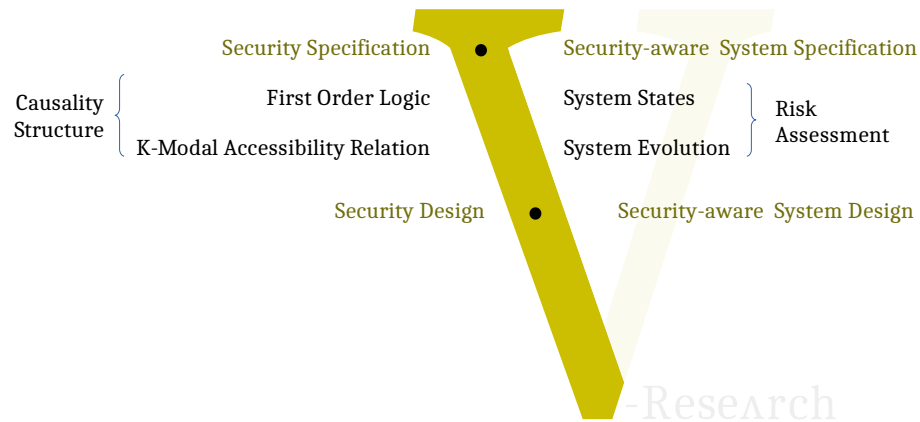
Definition 2.15. Vulnerability

Property of V-Research S.r.l.

Figure 5: Cybersecurity Engineering Life-cycle

2.4   Cybersecurity Engineering Process

> The difference between ideas and reality is the difference between philosophy and engineering. The work to transform one into the other is scientific research
>
> *V-Research*

2.4.1   Relevant Standards

1. DO-326A

3   Honesty and Maliciousness – A Cybersecurity Theory

1. cybersecurity as a requirement
2. vulnerability

4   Prototype and Empirical Tests

4.1   Prototype Implementation

4.2   Empirical Tests

5   Conclusion and Related Work

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Appendices

A   RCC3, RCC5, or RCC8

B   Wrong idea 1

In order to apply consistently the mereotopological structure not just to reason the agents but also on their evolution, we extend the properties of the Parthood relation (reflexivity and symmetry) to the modal relation R of the K Modal logic, which in turns defines the causality principle. This updates the K Modal logic to a B (Brouwer) Modal Logic[25].

Definition B.1. B Modal Logic – The B Modal Logic (defined, with a slight abuse of notation as $\mathbf{B_K} = \langle G, R \rangle$) is an extension of the K modal logic with

the following axioms

(σ8) $\omega \models P \Rightarrow Q$ iff $\omega \not\models P$ or $\omega \models Q$ (logical implication) [17]

(σ9) $\Box P \Rightarrow P$ (which "claims that whatever is necessary is the case"[25]), and

(σ10) $P \Rightarrow \Box\Diamond P$ (which says that if P is the case, then P is necessarily possible[25])

Theorem B.1. Causality, Parthood, and B Modal Logic

1. *(Reflexivity)* $\mathbf{B_K} \models \Box P \Rightarrow P$ *(σ8) iff R is reflexive*
2. *(Symmetry)* $\mathbf{B_K} \models P \Rightarrow \Box\Diamond$ *(σ9) iff R is symmetric*
3. *(Consistency)* $\omega R \omega'$ *iff* $C(\chi, \chi')$

whenever ω and ω', and χ and χ' are logically equivalent ($\equiv$).

Proof. The proof is exhaustive over all possible cases.

1. (Reflexivity) proven in[]
2. (Symmetry) proven in[]
3. (Equivalence) **tofix:** todo If $\omega R \omega'$ and $\omega \equiv \chi$, then if it doesn't hold that $C(\chi, \chi',)$ it follows that there is no reflexive and transitive relation that correlates χ to χ'. Given that Regions are defined as signatures

$\square$

---

[17]In Definition 2.1 we didn't specify the semantics in the case of the connective $\Rightarrow$ (i.e. logical implication), so we state it here. Given that $P \Rightarrow Q$ is equisatisfiable to $\neg(P \wedge \neg Q)$, for any proposition P and Q, $\omega \models P \Rightarrow Q$ is equisatisfiable to $\omega \models \neg(P \wedge \neg Q)$. If $\omega \models \neg(P \wedge \neg Q)$ then $\omega \not\models P \wedge \neg Q$ and then $\omega \models \neg P$ or $\omega \models Q$.