Assignment 5

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Download python code from

https://github.com/v−samyuktha/AI1103/blob/main/Assignment5/Assignment−5.py

and latex-tikz code from

https://github.com/v−samyuktha/AI1103/blob/main/Assignment5/Assignment−5.tex

1 QUESTION

Let X be a binomial random variable with parameters $\left(11, \frac{1}{3}\right)$. At which value(s) of k is $\Pr(X = k)$ maximized?

1) $k = 2$
2) $k = 3$
3) $k = 4$
4) $k = 5$

2 SOLUTION

The binomial distribution is given by:

$$\Pr(X = k) = {}^{n}\mathrm{C}_k \times p^k \times q^{n-k} \tag{2.0.1}$$

We are given $n = 11$, $p = \frac{1}{3}$ and hence $q = \frac{2}{3}$

$$\Pr(X = k) = {}^{11}\mathrm{C}_k \left(\frac{2}{3}\right)^{11-k} \left(\frac{1}{3}\right)^{k} \tag{2.0.2}$$

To maximise $\Pr(X = k)$,

$$\Pr(X = k) \geq \Pr(X = k + 1) \tag{2.0.3}$$

$$\frac{\Pr(X = k)}{\Pr(X = k + 1)} = \frac{{}^{11}\mathrm{C}_k \left(\frac{2}{3}\right)^{11-k} \left(\frac{1}{3}\right)^{k}}{{}^{11}\mathrm{C}_{k+1} \left(\frac{2}{3}\right)^{10-k} \left(\frac{1}{3}\right)^{k+1}} \geq 1 \tag{2.0.4}$$

$$\frac{2(k + 1)}{11 - k} \geq 1 \tag{2.0.5}$$

$$k \geq 3 \tag{2.0.6}$$

$$\Pr(X = k) \geq \Pr(X = k - 1) \tag{2.0.7}$$

$$\frac{\Pr(X = k)}{\Pr(X = k - 1)} = \frac{{}^{11}\mathrm{C}_k \left(\frac{2}{3}\right)^{11-k} \left(\frac{1}{3}\right)^{k}}{{}^{11}\mathrm{C}_{k-1} \left(\frac{2}{3}\right)^{12-k} \left(\frac{1}{3}\right)^{k-1}} \geq 1 \tag{2.0.8}$$

$$\frac{12 - k}{2k} \geq 1 \tag{2.0.9}$$

$$k \leq 4 \tag{2.0.10}$$

From equations (2.0.6) and (2.0.10) we see that $\Pr(X = k)$ is maximized for $k = 3$, $k = 4$. Hence, options 2 and 3 are correct.

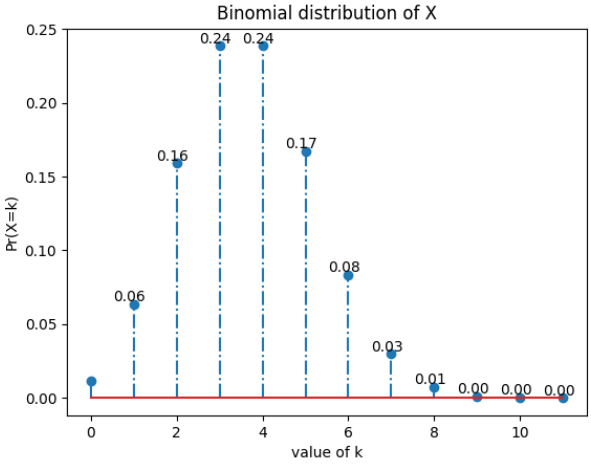

Fig. 1: Binomial Distribution of X