

Assignment 5

V.Samyuktha - MS20BTECH11024

Download python code from

<https://github.com/v-samyuktha/AI1103/blob/Assignment-5/Assignment-5.py>

and latex-tikz code from

<https://github.com/v-samyuktha/AI1103/blob/Assignment-5/Assignment-5.tex>

1 QUESTION

Let X be a binomial random variable with parameters $(11, \frac{1}{3})$. At which value(s) of k is $\Pr(X = k)$ maximized?

- 1) $k=2$
- 2) $k=3$
- 3) $k=4$
- 4) $k=5$

2 SOLUTION

The binomial distribution is given by:

$$\Pr(X = k) = {}^nC_k \times p^k \times q^{n-k} \quad (2.0.1)$$

We are given $n = 11$, $p = \frac{1}{3}$ and hence $q = \frac{2}{3}$

$$\Pr(X = k) = {}^{11}C_k \left(\frac{2}{3}\right)^{11-k} \left(\frac{1}{3}\right)^k \quad (2.0.2)$$

To maximise $\Pr(X = k)$,

$$\Pr(X = k) \geq \Pr(X = k + 1) \quad (2.0.3)$$

$$\frac{\Pr(X = k)}{\Pr(X = k + 1)} = \frac{{}^{11}C_k \left(\frac{2}{3}\right)^{11-k} \left(\frac{1}{3}\right)^k}{{}^{11}C_{k+1} \left(\frac{2}{3}\right)^{10-k} \left(\frac{1}{3}\right)^{k+1}} \geq 1 \quad (2.0.4)$$

$$\frac{2(k+1)}{11-k} \geq 1 \quad (2.0.5)$$

$$k \geq 3 \quad (2.0.6)$$

$$\Pr(X = k) \geq \Pr(X = k - 1) \quad (2.0.7)$$

$$\frac{\Pr(X = k)}{\Pr(X = k - 1)} = \frac{{}^{11}C_k \left(\frac{2}{3}\right)^{11-k} \left(\frac{1}{3}\right)^k}{{}^{11}C_{k-1} \left(\frac{2}{3}\right)^{12-k} \left(\frac{1}{3}\right)^{k-1}} \geq 1 \quad (2.0.8)$$

$$\frac{12-k}{2k} \geq 1 \quad (2.0.9)$$

$$k \leq 4 \quad (2.0.10)$$

From equations (2.0.6) and (2.0.10) we see that $\Pr(X = k)$ is maximized for $k=3, k=4$

Hence, options 2 and 3 are correct.

