EE2101-Control Systems Assignment 1

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Download codes from

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https://github.com/v−squared99/EE2101/tree/
    master/Assignment1
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***Abstract*—This document contains the solution to problem 56 from chapter 2 of Control Systems Engineering by Norman Nise**

1 Problem

For a given differential equation model:

$$\frac{dC(t)}{dt} = -(\lambda + \mu + \gamma + \delta + \nu)C(t) + \lambda N(t) \tag{1.0.1}$$

$$\frac{dN(t)}{dt} = -(\nu + \delta)C(t) - \mu N(t) + I(t) \tag{1.0.2}$$

where

$$\nu = \delta = 0.05, \mu = 0.02, \gamma = 0.08, \lambda = 0.07 \tag{1.0.3}$$

$$C(0) = C_o = 47,000,500 \tag{1.0.4}$$

$$N(0) = N_o = 61,100,500 \tag{1.0.5}$$

$$I(t) = I = 6x10^6 \tag{1.0.6}$$

(a) A block diagram is to be drawn showing the output N(s), the input I(s), the transfer function, and the initial conditions.
(b) Analytic expression for N(t) for $t \geq 0$ is to be found using any method.

2 Solution

2.1 Part(a)

After substituting the values in (1.0.1) and (1.0.2), we get:

$$\frac{dC(t)}{dt} = -0.9C(t) + 0.7N(t) \tag{2.1.1}$$

$$\frac{dN(t)}{dt} = -0.1C(t) - 0.02N(t) + I(t) \tag{2.1.2}$$

Finding the Laplace Transform of the above equations:

$$sC(s) - C(0) = -0.9C(s) + 0.7N(s) \tag{2.1.3}$$

$$sN(s) - N(0) = -0.1C(s) - 0.02N(s) + I(s) \tag{2.1.4}$$

Substituting C(0) and N(0) gives us:

$$C(s)(s + 0.9) = 0.7N(s) + 47000500 \tag{2.1.5}$$

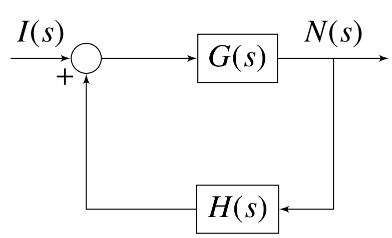
$$N(s)(s + 0.02) = -0.1C(s) + I(s) + 61100500 \tag{2.1.6}$$

Substituting (2.0.5) in (2.0.6):

$$N(s) = \frac{\frac{I(s)+61100500}{s+0.02} + \frac{-47000500}{(s+0.02)(s+0.9)}}{1 + \frac{0.07}{(s+0.02)(s+0.9)}} \tag{2.1.7}$$

$$= \frac{(I(s) + 61100500)(s + 0.9) - 47000500}{s^2 + 0.92s + 0.088} \tag{2.1.8}$$

$$\implies N(s) = \frac{I(s) + \frac{61100500s+7989950}{s+0.9}}{s^3 + 1.82s^2 + 0.916s + 0.0792} \tag{2.1.9}$$



2.2 Part(b)

We know that

$$I(s) = \frac{6 \times 10^6}{s} \tag{2.2.1}$$

Substituting (2.2.1) in (2.1.8):

$$N(s) = \frac{(\frac{6\times10^6}{s} + 61100500)(s + 0.9) - 47000500}{s^2 + 0.92s + 0.088} \tag{2.2.2}$$

$$\implies N(s) = \frac{61100500s^2 + 56290400s + 54 \times 10^5}{s(s + 0.8116)(s + 0.1084)} \tag{2.2.3}$$

Solving this using Partial Fractions:

$$N(s) = \frac{A}{s} + \frac{B}{s + 0.8116} + \frac{C}{s + 0.1084} \tag{2.2.4}$$

Solving for A, B and C, we get:

$$A = 61.364 \times 10^6 \tag{2.2.5}$$

$$B = -7 \times 10^4 \tag{2.2.6}$$

$$C = -19.3 \times 10^4 \tag{2.2.7}$$

Inverse Laplace Transform of N(s) equation is:

$$N(t) = 6.1364 \times 10^7 - 7 \times 10^4 e^{-0.8116t} - 19.3 \times 10^4 e^{-0.1084t} \tag{2.2.8}$$