EE5609-Matrix Theory Assignment 2

Vinjam Lakshmi Sai Vignatha
Es17btech11024

Download python and latex codes from

https://github.com/v−squared99/EE5609/tree/master/Assignment2

***Abstract*—This document contains the solution to problem 26 from 3.9 Matrix Exercises**

1 Problem

If

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}, \tag{1.0.1}$$

Prove that

$$\mathbf{A^3 - 6A^2 + 7A + 2I = 0} \tag{1.0.2}$$

2 Solution

The characteristic equation is

$$|\mathbf{A} - \lambda\mathbf{I}| = 0 \implies \det\begin{pmatrix} 1-\lambda & 0 & 2 \\ 0 & 2-\lambda & 1 \\ 2 & 0 & 3-\lambda \end{pmatrix} = 0 \tag{2.0.1}$$

$$\implies (1-)(2-\lambda)(3-\lambda) + 2(-2(2-\lambda)) = 0 \tag{2.0.2}$$

$$\implies \lambda^3 - 6\lambda^2 + 7\lambda + 2 = 0 \tag{2.0.3}$$

The above equation is similar to the equation to be proved.

By the Cayley-Hamilton theorem, every square matrix satisfies its own characteristic equation.

Hence proved that $\mathbf{A^3 - 6A^2 + 7A + 2I = 0}$.