EE5609-Matrix Theory Assignment 2

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Download python and latex codes from

https://github.com/v−squared99/EE5609/tree/master/Assignment2

Abstract—**This document contains the solution to problem 26 from 3.9 Matrix Exercises**

1 Problem

If

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}, \tag{1.0.1}$$

Prove that

$$\mathbf{A^3} - \mathbf{6A^2} + \mathbf{7A} + \mathbf{2I} = \mathbf{0} \tag{1.0.2}$$

2 Theory

A n X n square matrix $\mathbf{A}$ with n linearly independent eigenvectors can be factorised as:

$$\mathbf{A} = \mathbf{Q\Lambda Q^{-1}} \tag{2.0.1}$$

where $\mathbf{\Lambda}$ is a diagonal matrix with the diagonal elements corresponding to each eigenvalue.

A^k, where k is a positive integer, can hence be written as:

$$\mathbf{A^k} = \mathbf{Q\Lambda^k Q^{-1}} \tag{2.0.2}$$

NOTE: A diagonal matrix can be raised to a power by simply raising the diagonal entries to that power.

3 Solution

Finding eigenvalues of $\mathbf{A}$: The characteristic equation is

$$|\mathbf{A} - \lambda\mathbf{I}| = 0 \implies \begin{vmatrix} 1-\lambda & 0 & 2 \\ 0 & 2-\lambda & 1 \\ 2 & 0 & 3-\lambda \end{vmatrix} = 0 \tag{3.0.1}$$

$$\implies (1-)(2-\lambda)(3-\lambda) + 2(-2(2-\lambda)) = 0 \tag{3.0.2}$$

$$\implies \lambda^3 - 6\lambda^2 + 7\lambda + 2 = 0 \tag{3.0.3}$$

$$\implies \mathbf{\Lambda} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \tag{3.0.4}$$

where $\lambda_1, \lambda_2, \lambda_3$ are the solutions of the characteristic equation.
Substituting in $\mathbf{A^3} - \mathbf{6A^2} + \mathbf{7A} + \mathbf{2I}$:

$$LHS = \mathbf{Q}(\mathbf{\Lambda^3} - \mathbf{6\Lambda^2} + \mathbf{7\Lambda} + \mathbf{2I})\mathbf{Q^{-1}} = 0 \tag{3.0.5}$$

Hence proved that $\mathbf{A^3} - \mathbf{6A^2} + \mathbf{7A} + \mathbf{2I} = \mathbf{0}$.