EE5609-Matrix Theory Assignment 1

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***Abstract*—This document contains the solution to problem 30 from Points and Vectors**

1 Problem

To find the torque about the origin by the force $\mathbf{F} = \begin{pmatrix} 7 \\ 3 \\ -5 \end{pmatrix}$ acting at the point $\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$.

2 Theory

The torque $\mathbf{T}$ is given by the cross product (vector product) of the position (or distance) vector $\mathbf{r}$ and the force vector $\mathbf{F}$.

$$\mathbf{T} = \mathbf{r} \times \mathbf{F} \tag{2.0.1}$$

And the vector cross product of vectors

$$\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \tag{2.0.3}$$

can be expressed as the product of a skew-symmetric matrix and a vector:

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \tag{2.0.4}$$

3 Solution

Torque at the origin is given by,

$$\mathbf{F} \times \mathbf{r} = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 7 \\ 3 \\ -5 \end{pmatrix} \tag{3.0.1}$$

$$\mathbf{F} \times \mathbf{r} = \begin{pmatrix} (0 \times 7) + (-1 \times 3) + (-1 \times -5) \\ (1 \times 7) + (0 \times 3) + (-1 \times -5) \\ (1 \times 7) + (1 \times 3) + (0 \times -5) \end{pmatrix} \tag{3.0.2}$$

$$\implies \mathbf{T} = \begin{pmatrix} 2 \\ 12 \\ 10 \end{pmatrix} \tag{3.0.3}$$