

EE5609-Matrix Theory Assignment 2

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Download python and latex codes from

<https://github.com/v-squared99/EE5609/tree/master/Assignment2>

Abstract—This document contains the solution to problem 26 from 3.9 Matrix Exercises

1 PROBLEM

If

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}, \quad (1.0.1)$$

Prove that

$$\mathbf{A}^3 - 6\mathbf{A}^2 + 7\mathbf{A} + 2\mathbf{I} = \mathbf{0} \quad (1.0.2)$$

2 SOLUTION

\mathbf{A} with 3 linearly independent eigenvectors can be factorised as:

$$\mathbf{A} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^{-1} \quad (2.0.1)$$

where $\mathbf{\Lambda}$ is a diagonal matrix with the diagonal elements corresponding to each eigenvalue.

\mathbf{A}^k , where k is a positive integer, can hence be written as:

$$\mathbf{A}^k = \mathbf{Q}\mathbf{\Lambda}^k\mathbf{Q}^{-1} \quad (2.0.2)$$

The characteristic equation is

$$|\mathbf{A} - \lambda\mathbf{I}| = 0 \implies \begin{vmatrix} 1-\lambda & 0 & 2 \\ 0 & 2-\lambda & 1 \\ 2 & 0 & 3-\lambda \end{vmatrix} = 0 \quad (2.0.3)$$

$$\implies (1-\lambda)(2-\lambda)(3-\lambda) + 2(-2(2-\lambda)) = 0 \quad (2.0.4)$$

$$\implies \lambda^3 - 6\lambda^2 + 7\lambda + 2 = 0 \quad (2.0.5)$$

Hence proved that $\mathbf{A}^3 - 6\mathbf{A}^2 + 7\mathbf{A} + 2\mathbf{I} = \mathbf{0}$.