Machine Learning in the Frequency Domain

Moritz Wolter

wolter@cs.uni-bonn.de

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UNIVERSITÄT BONN

The Fourier transform

An analog dial pad

	1209 Hz	1336 Hz	1477 Hz
697 Hz	1	2	3
770 Hz	4	5	6
852 Hz	7	8	9
941 Hz	*	0	#

dial

The time domain signal

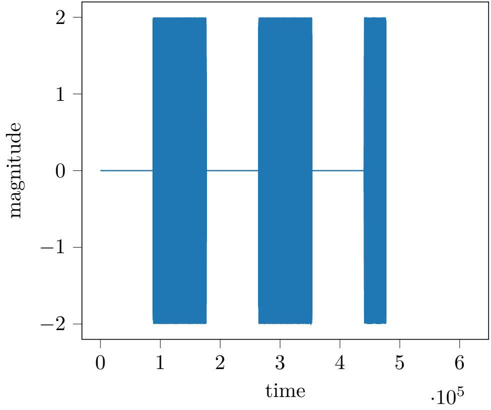


Figure: The time domain signal sampled at 44.1khz

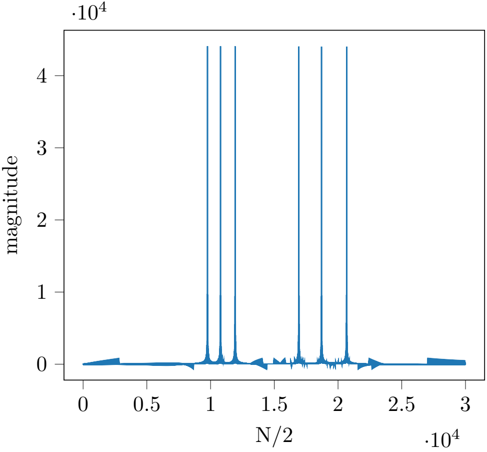
The Fourier Transform

$$\mathcal{F}(\mathbf{x}[l]) = \sum_{l=-\infty}^{\infty} \mathbf{x}[l]e^{-j\omega l}, \tag{1}$$

Euler's formula:

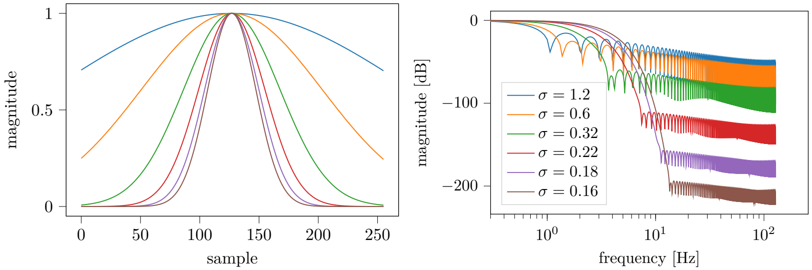
$$e^{j\omega} = \cos(\omega) + j\sin(\omega) \tag{2}$$

The fourier transfrom applied

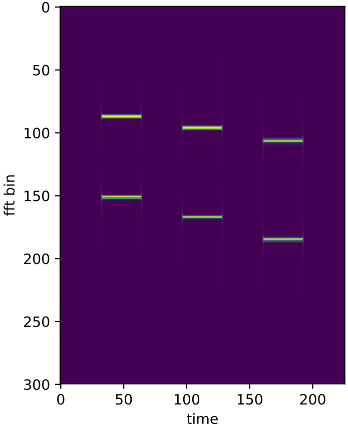


The Short time Fourier transform

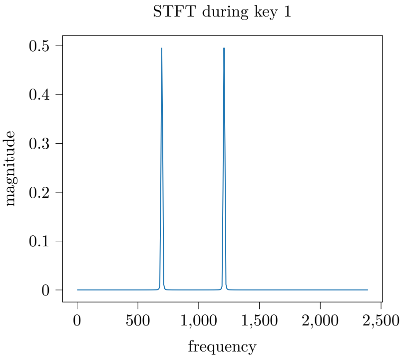
$$\mathbf{X}[\omega, Sm] = \mathcal{F}_s(\mathbf{x}) = \mathcal{F}(\mathbf{w}[Sm-l]\mathbf{x}[l]) = \sum_{l=-\infty}^{\infty} \mathbf{w}[Sm-l]\mathbf{x}[l]e^{-j\omega l}, \tag{3}$$



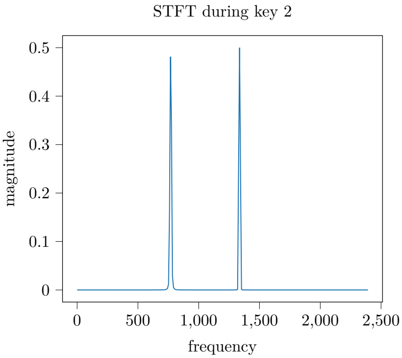
Short Time Fourier Transform Magnitude



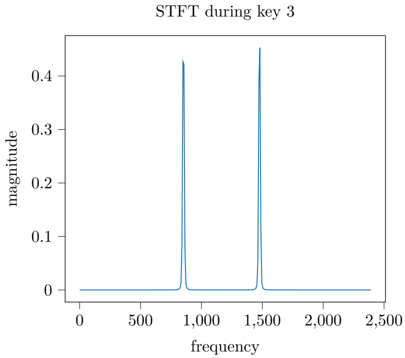
Key 1


STFT during key 1
magnitude
0.5
0.4
0.3
0.2
0.1
0
0
500
1,000
1,500
2,000
2,500
frequency

Key 2


STFT during key 2
magnitude
0.5
0.4
0.3
0.2
0.1
0
0
500
1,000
1,500
2,000
2,500
frequency

Key 3


STFT during key 3
magnitude
0.4
0.3
0.2
0.1
0
0
500
1,000
1,500
2,000
2,500
frequency

The uncertainty principle [1]

Example 1:

- 80ms
- 170ms
- 330ms
- 670ms
- 1s
- 2s
- 5s

[1] http://newt.phys.unsw.edu.au/jw/uncertainty.html

The uncertainty principle 2[2]

Example 2:

- 80ms
- 170ms
- 330ms
- 670ms
- 1s
- 2s
- 5s

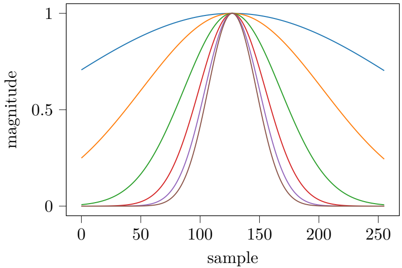
[2] http://newt.phys.unsw.edu.au/jw/uncertainty.html

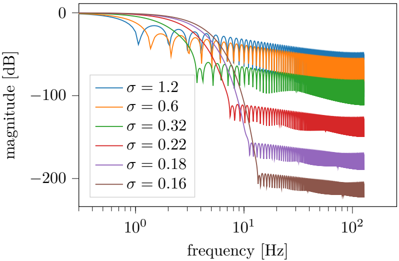
Learning trough the STFT

- The first example consisted a 400 and 403Hz Sine wave.
- The second of a 400 and 401Hz Sine wave.
- Time and frequency resolution are coupled trough the uncertainty principle.
- Working in the time domain can overlaods RNNs for long sequences.
- Transfer a signal into the Frequency domain do the prediction and compute the inverse transfrom.
- IDEA: Larn the window shape.

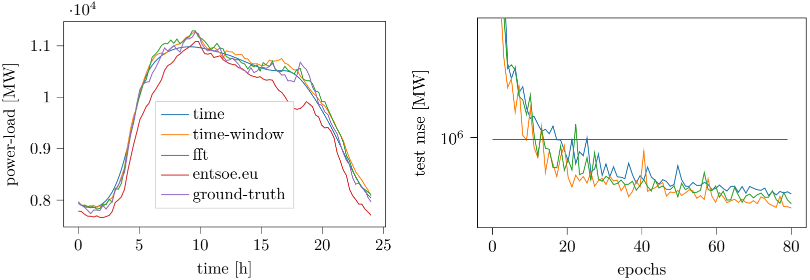
Learning trough the STFT

$$\mathbf{X}[\omega, Sm] = \mathcal{F}_s(\mathbf{x}) = \mathcal{F}(\mathbf{w}[Sm - l]\mathbf{x}[l]) = \sum_{l=-\infty}^{\infty} \mathbf{w}[Sm - l]\mathbf{x}[l]e^{-j\omega l}, \tag{4}$$



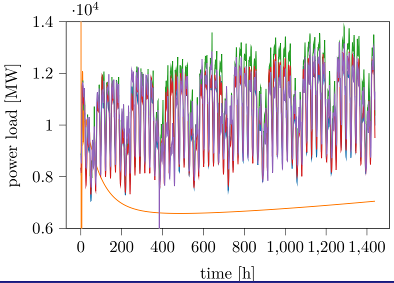


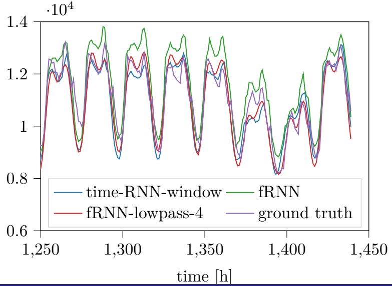
Learning trough the STFT [WY18b]



Learning trough the STFT [WY18b]

Network	mse [MW]	weights	run [min]
time-RNN	$1.3 \cdot 10^7$	13k	772
time-RNN-windowed	$8.8 \cdot 10^5$	28k	12
fRNN	$8.3 \cdot 10^5$	44k	13
fRNN-lowpass-1/4	$7.6 \cdot 10^5$	20k	13
fRNN-lowpass-1/8	$1.3 \cdot 10^6$	16k	13





Summary

- The fourier transforms finds frequency components.
- The short time fourier transform preserves time information.
- Heisenberg tells us that time and frequency resolution are coupled.
- Working in the frequency domain makes machine learning more efficient.
- The frequency domain is complex valued and required comples networks [WY18a].

References I

Moritz Wolter and Angela Yao, *Complex gated recurrent neural networks*, 32nd Conference on Neural Information Processing Systems, 2018.

———, *Fourier rnns for sequence prediction*, arXiv preprint arXiv:1812.05645, 2018.

Discussion

Thanks for your attention and feedback.
Feel free to contact me at: wolter@cs.uni-bonn.de