

Exercise 3 Imperfect bifurcations

Moritz Wolter

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1 The equation

$$-\frac{1}{2}u^3 + ru + h = 0. \quad (1)$$

2 Bifurcation analysis

2.1 Manual analysis

Bifurcation analysis means looking at fixed point movement and the evolution of their stability. In this report equation 1 will be analyzed thoroughly. It contains the information of the fixed point location for the system:

$$\dot{u} = -\frac{1}{2}u^3 + ru + h. \quad (2)$$

`matcont` may be used to do this kind of analysis. However to put the results into perspective. A manual analysis has been performed beforehand ¹. Figures 1,2,3 show each a graphical approach to find the zeros of 1 in the first column. The second column shows the evolution of the position of the zeros. For different values for the imperfection parameter $h = 5, 0$ and 1 as well as $r \in [-30, 30]$ with a step size of 0.1 . As 2 is a first order system linear stability analysis may be employed to learn more about the system dynamics. The first derivative of $f(u) = -\frac{1}{2}u^3 + ru + h$.

$$f'(u) = -\frac{3}{2}u^2 + r. \quad (3)$$

At the extreme points the derivative is zero, which leads to $u_{max} = \sqrt{\frac{2}{3}r}$. Thus leading to the maximum values $f(u_{max}) = \frac{1}{3}\sqrt{\frac{2}{3}}r^{3/2}$. As bifurcations occur when h hits the local maximum values one obtains $h_c = \pm f(u_{max})$. In the second column of the figures 1,2,3 the effect of the imperfection parameter may be observed. For $h = 0$ a textbook pitchfork bifurcation is observed. However if $h \neq 0$ the fork separates which one part moving up or down with respect to x depending on the sign.

¹see pages 70-73 in Strogatz' book

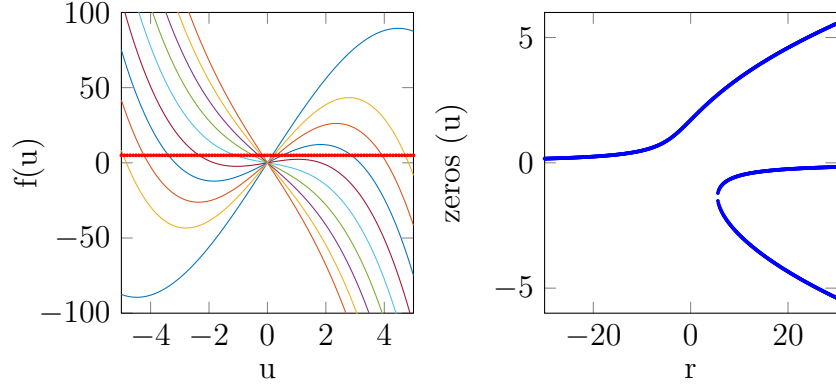


Figure 1: Solution branches with the function $-\frac{1}{2}u^3 + ru$ numerous colors for different $r \in [-30, 30]$ and the constant function $h = 5$ in red (left). Root locus plot for the same r values (right).

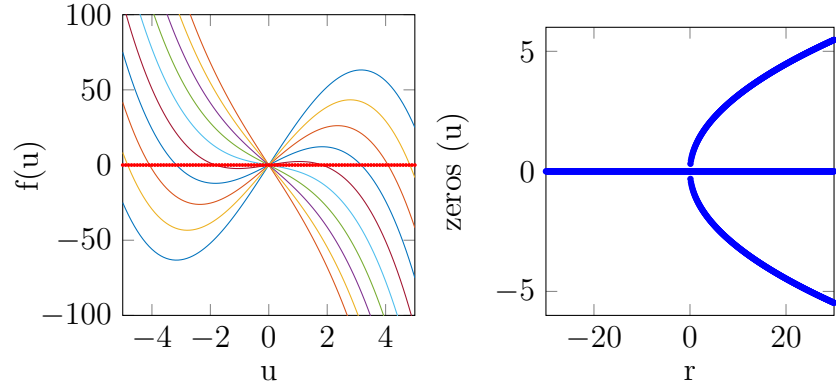


Figure 2: Solution branches with the function $-\frac{1}{2}u^3 + ru$ numerous colors for different $r \in [-30, 30]$ and the constant function $h = 0$ in red (left). Root locus plot for the same r values (right).

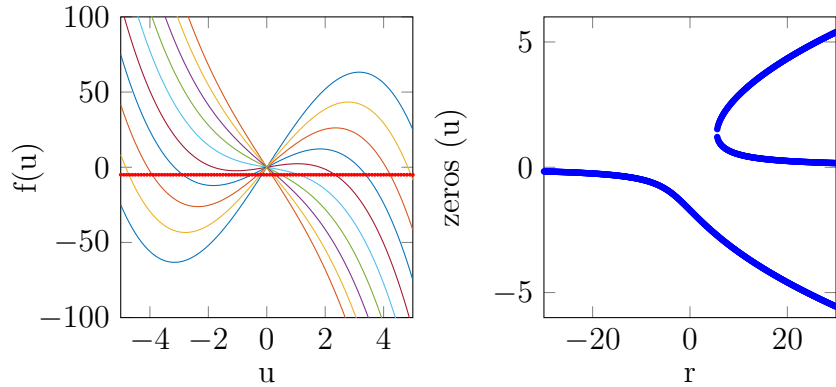


Figure 3: Solution branches with the function $-\frac{1}{2}u^3 + ru$ numerous colors for different $r \in [-30, 30]$ and the constant function $h = -5$ in red (left). Root locus plot for the same r values (right).

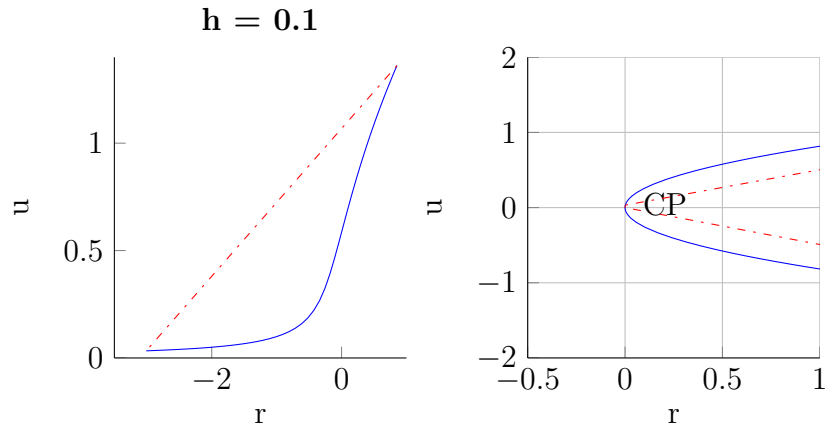


Figure 4: `matcont` output for $h = 0.1$

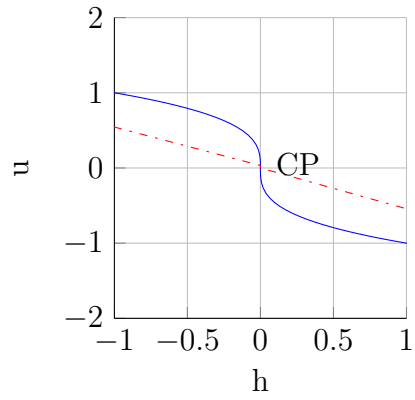


Figure 5: `matcont` output for $h = 0.1$

2.2 `matcont` analysis

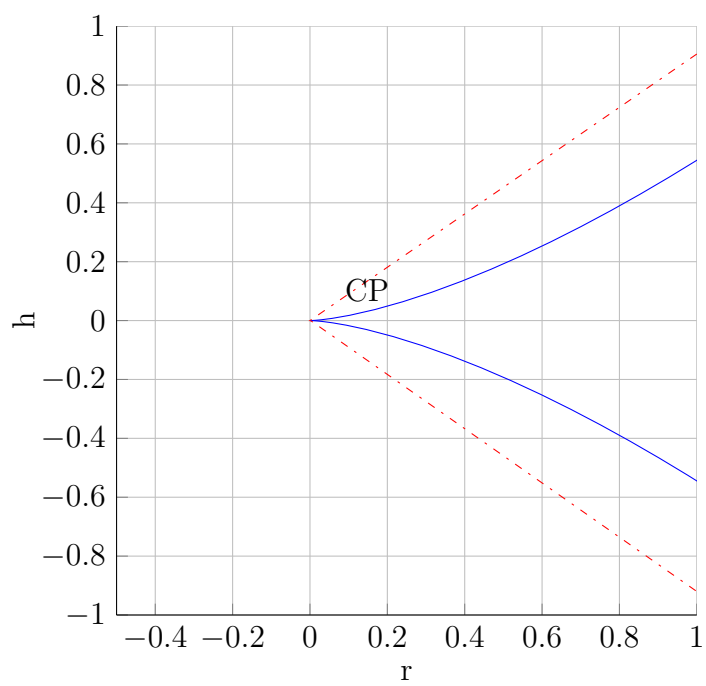


Figure 6: `matcont` output for $h = 0.1$