Exercise 3 Imperfect bifurcations

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1 The equation

$$\dot{x} = x(x-a)(1-x) - bxy \tag{1}$$
$$\dot{y} = xy - cy - d. \tag{2}$$

With $a = 0.4,\ b = 0.3,$ and $c \in [0.650.75]$.

2 Analysis of a simplified model $d = 0$

Setting d and y equal to zero the system turns into:

$$\dot{x} = x(x-a)(1-x). \tag{3}$$

For this simplified case the fixed points may be read off easily. $\dot{x} = 0$ yields $x_1 = 0,\ x_2 = a,\ x_3 = 1$. Linear analysis will lead to further insight in the nature of these fixed points reading of $f(x) = x(x-a)(1-x)$ and computing $f'(x)$ leads to:

$$f'(x) = -3x^2 + 2x + 2xa - a. \tag{4}$$

Substituting the fixed points leads to:

$$f'(x_1) = -a \tag{5}$$
$$f'(x_2) = -a^2 + a = -0.4^2 + 0.4 > 0 \tag{6}$$
$$f'(x_3) = -3 + 2 + 2a - a = -1 + a = -0.6 < 0 \tag{7}$$

Thus it may be concluded, that x_2 is unstable and $x_3 \wedge x_1$ are stable. Figure 1 shows simulation results produced by a Runge-Kutta type numerical integration routine. The fixed point positions that where read off from the simplified system equation are confirmed by the results to be at $x_1 = 0,\ x_2 = a = 0.4,\ x_3 = 1$. Furthermore the fixed points show the predicted characteristics.

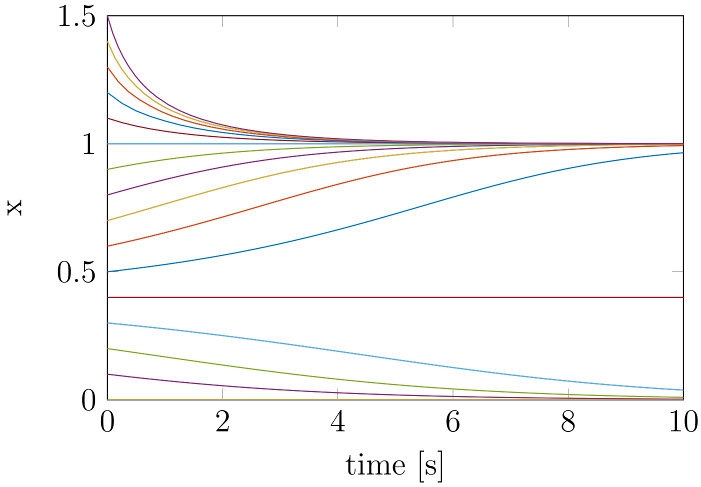

Figure 1: Simulation of the simplified system described by equation 3.