

Exercise 2 Bridge Oscillations

Moritz Wolter

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In this report the effects of Wind on a poorly designed bridge will be explored.

1 Equation of Motion

The model for the structure leads to the following equation of motion:

$$0 = -F_I - F_d - F_e + F_{dr} \quad (1)$$

$$0 = -m\ddot{y} - r\dot{y} - ky + \frac{1}{2}\rho V^2 a C(\alpha). \quad (2)$$

Where $C(\alpha)$ is a nonlinear function.

2 Linear Analysis

$C(\alpha)$ is defined as a sum of several odd powers of α :

$$C(\alpha) = A_1\alpha - \underbrace{A_3\alpha^3 + A_5\alpha^5 - A_7\alpha^7}_{\approx 0 \text{ for small } \alpha}. \quad (3)$$

for small α additionally the approximation $\alpha = \frac{\dot{y}}{V}$ is given. Plugging into (2) yields:

$$0 = -m\ddot{y} + \left(\frac{1}{2}\rho V a A_1 - r\right)\dot{y} - ky. \quad (4)$$

Which may be rewritten in terms of two first order equations:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \frac{\frac{1}{2}\rho V a A_1 - r}{m} & -\frac{k}{m} \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad (5)$$

Setting the derivatives to zero the fixed point $\mathbf{x}_1^* = (0 \ 0)^T$ is obtained. As a linear approximation as already taken place the Jacobi-matrix is identical to the system matrix given above. Thus for the trace τ_1 and determinant Δ_1 at the fixed point the following equations are obtained:

$$\tau_1 = \frac{\frac{1}{2}\rho V a A_1 - r}{m} - \frac{k}{m} \quad (6)$$

$$\Delta_1 = \frac{k}{m}. \quad (7)$$

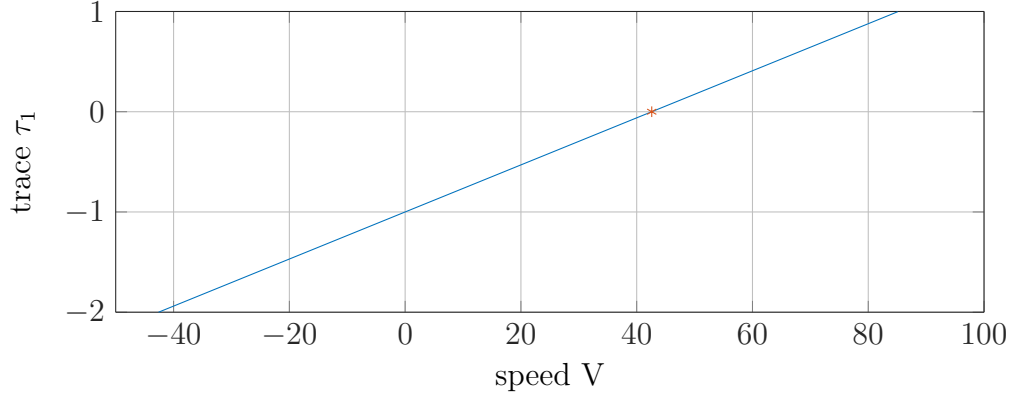


Figure 1: Plot of τ_1 for different speed values. The critical wind speed V_c is marked with an red asterisk.

Assuming $(k \wedge m) > 0$ the nature of the fixed point is determined by the trace. The critical value will occur for $\tau_1 = 0$, therefore it may be found from:

$$0 = \frac{1}{2} \frac{\rho V^2 a A_1}{m V_c} - \frac{r}{m}. \quad (8)$$

Solving for V_c leads to:

$$V_c = \frac{2r}{\rho a A_1} = 42.5985. \quad (9)$$

When $m = 1$, $\rho = 1$, $r = 1$, $k = 100$, $a = 1$ and $A_1 = 100$. A plot for τ_1 with respect for different values for V is given in figure 1. As the determinant remains positive at all times the fixed point at the center changes from a stable to an unstable spiral at the critical wind speed V_c .

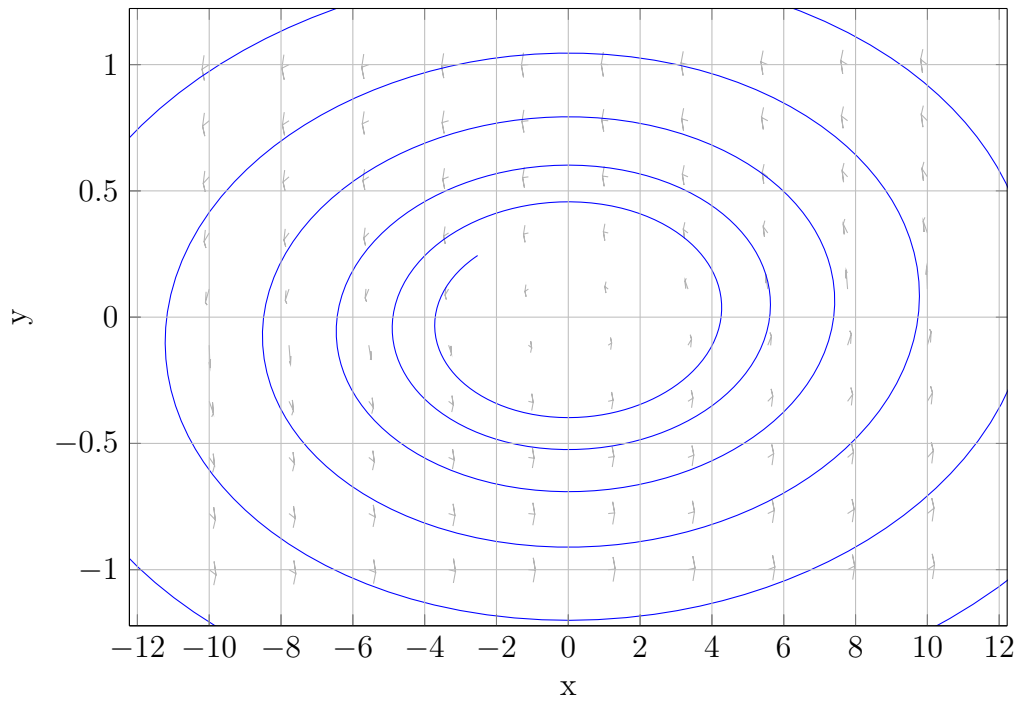


Figure 2: Plots of the two dimensional linearized systems with $V = 80 > V_c$.