Exercise 3 Imperfect bifurcations

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1 The equation

$$-\frac{1}{2}u^3 + ru + h = 0. \tag{1}$$

2 Bifurcation analysis

2.1 Manual analysis

Bifurcation analysis means looking at fixed point movement and the evolution of their stability. In this report equation 1 will be analyzed thoroughly. It contains the information of the fixed point location for the system:

$$\dot{u} = -\frac{1}{2}u^3 + ru + h. \tag{2}$$

`matcont` may be used to do this kind of analysis. However to put the results into perspective. A manual analysis has been performed beforehand [1]. Figures 1,2 3 show each a graphical approach to find the zeros of 1 in the first column. The second column shows the evolution of the position of the zeros. For different values for the imperfection parameter $h = 5, 0$ and 1 as well as $r \in [-30, 30]$ with a step size of 0.1. As 2 is a first order system linear stability analysis may be employed to learn more about the system dynamics. The first derivative of $f(u) = -\frac{1}{2}u^3 + ru + h$.

$$f'(u) = -\frac{3}{2}u^2 + r. \tag{3}$$

At the extreme points the derivative is zero, which leads to $u_{max} = \sqrt{\frac{2}{3}r}$. Thus leading to the maximum values $f(u_{max}) = \frac{1}{3}\sqrt{\frac{2}{3}}r^{3/2}$. As bifurcations occur when h hits the local maximum values one obtains $h_c = \pm f(u_{max})$. In the second column of the figures 1,2 3 the effect of the imperfection parameter may be observed. For $h = 0$ a textbook pitchfork bifurcation is observed. However if $h \neq 0$ the fork separates which one part moving up or down with respect to x depending on the sign.

[1] see pages 70-73 in Strogatz' book

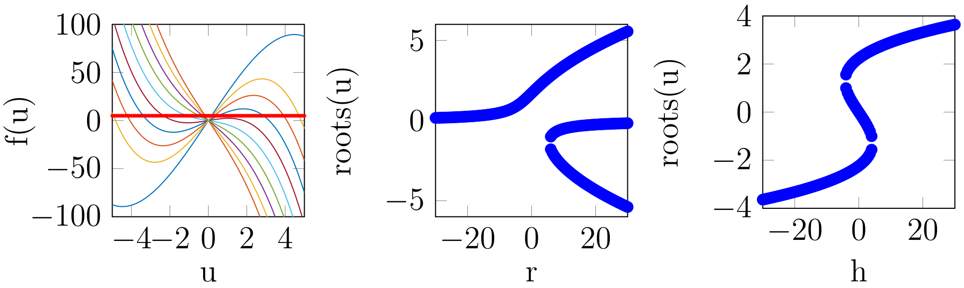


Figure 1: Solution branches with the function $-\frac{1}{2}u^3 + ru$ numerous colors for different $r \in [-30, 30]$ and the constant function $h = 5$ in red (left). Root locus plot for the same r values (right).

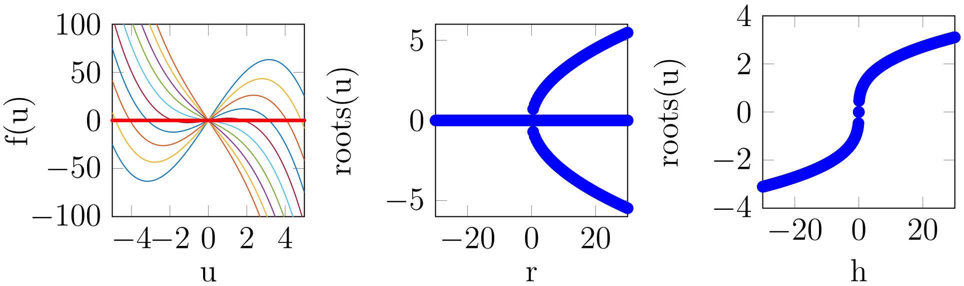


Figure 2: Solution branches with the function $-\frac{1}{2}u^3 + ru$ numerous colors for different $r \in [-30, 30]$ and the constant function $h = 0$ in red (left). Root locus plot for the same r values (right).

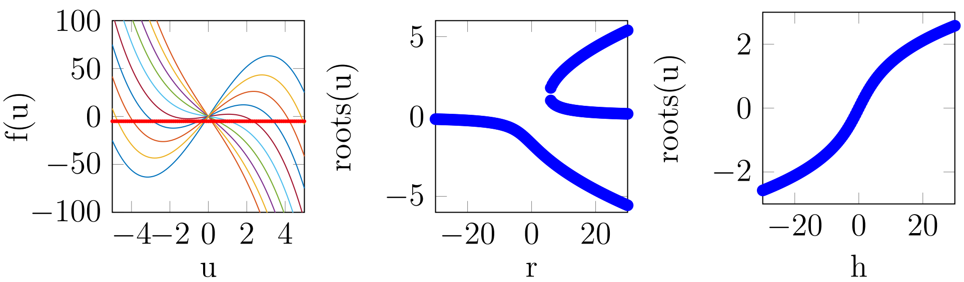


Figure 3: Solution branches with the function $-\frac{1}{2}u^3 + ru$ numerous colors for different $r \in [-30, 30]$ and the constant function $h = -5$ in red (left). Root locus plot for the same r values (right).

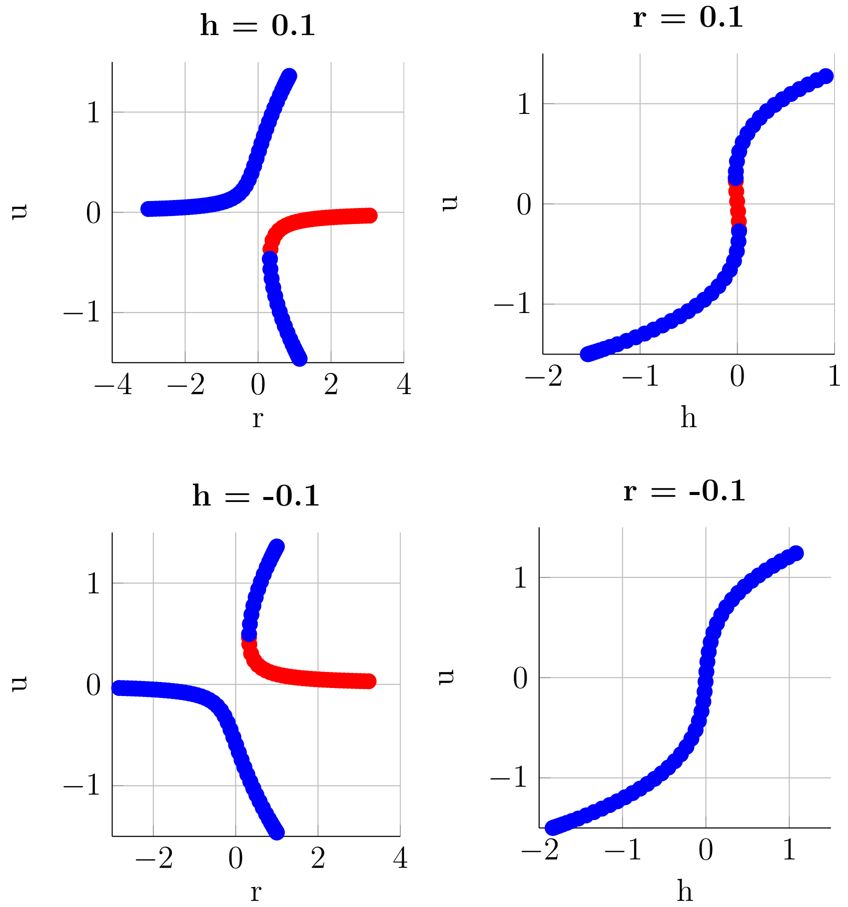
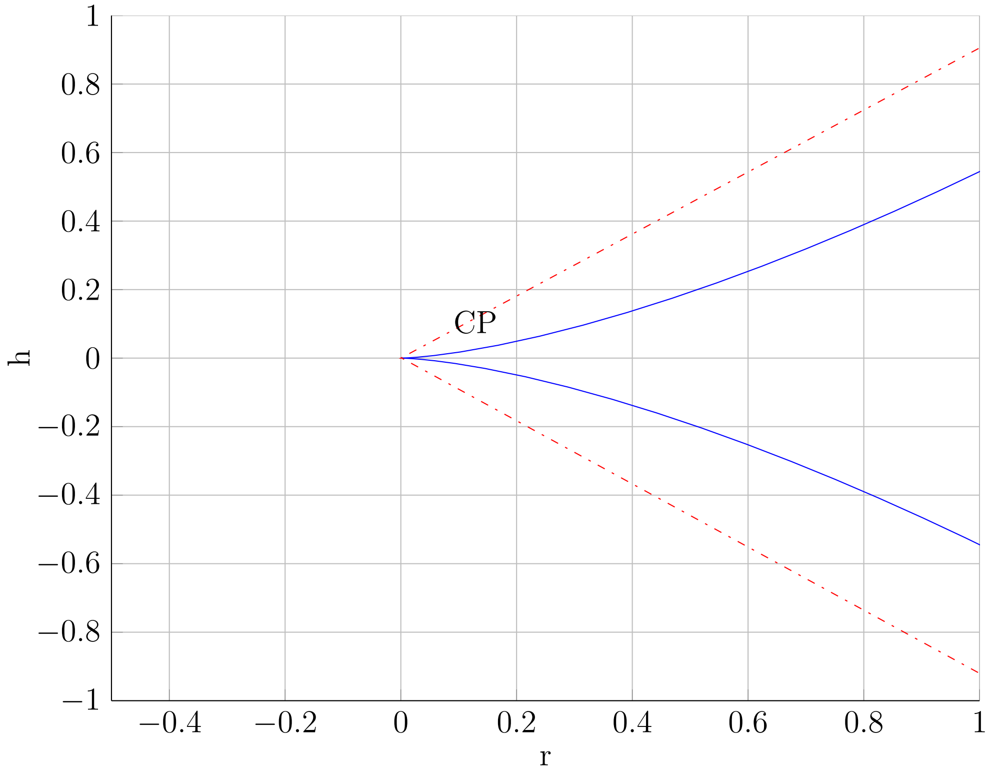


Figure 4: `matcont output for` $h = 0.1$

2.2 `matcont` analysis



CP
h
r
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1