Exercise 6 Pattern formation

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1 The Brusselator

$$u_t = D_u u_{xx} + A - (B+1)u + u^2 v, \tag{1}$$
$$v_t = D_v v_{xx} + Bu - u^2. \tag{2}$$

The above equations describe molecule concentrations during a coupled reaction. They are known to exhibit patterns.

2 Stability of the steady state

At the steady state $u_t = v_t = u_{xx} = v_{xx} = 0$. Thus following equations remain:

$$0 = A - (B+1)u + u^2 v, \tag{3}$$
$$0 = u(B - uv). \tag{4}$$
$$\tag{5}$$

From which $u_0 = A$ and $v_0 = B/A$ is deduced.