NUMERICAL SIMULATION OF PARTIAL DIFFERENTIAL EQUATIONS IN TWO DIMENSIONS

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Project Report

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1 IMPLEMENTATION OF EXPLICIT METHODS

In this report we are going to implement explicit methods to solve three different partial differential equations in two dimensions.

1.1 HEAT EQUATION

We will begin with the numerical solution of the heat equation:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \tag{1}$$

From the lecture we know that this problem may be solved by extension of the one dimensional explicit scheme:

$$\frac{U^{n+1} - U^n}{\triangle t} = b[\frac{\delta_x^2 U^n}{(\triangle x)^2} + \frac{\delta_y^2 U^n}{(\triangle y)^2}] \tag{2}$$

with $b = 1$ in our case. By expanding the central differences we arrive at:

$$U^{n+1}_{r,x} = U^n_{r,s}(1 - 2\mu_x - 2\mu_y) + \mu_x U^n_{r+1,s} + \mu_x U^n_{r-1,s} + \mu_y U^n_{r,s+1} + \mu_y U^n_{r,s-1}. \tag{3}$$

Equation 3 my be implemented in matlab. As we are using a symmetric grid we have $\mu_x = \mu_y$ which leads to the implementation in listing 3:

```
tend = 3;
dt = 0.0001;
J = 30;
dx = 1/J;
dy = 1/J;
mu = dt/dx^2;

%Set up a mesh.
[x,y] = meshgrid(linspace(0,1,J));
%Initial solution.
U = sin(pi*x).*sin(pi*y);
for t = 1:(tend/dt)
    elements = 2:J-1;
    for i = 1:1:J
        %compute the columns where x is const.
        U1(elements,i) = mu*U(elements+1,i) + mu*U(elements-1,i);
        %compute the columns where y is const.
        U2(i,elements) = mu*U(i,elements+1) + mu*U(i,elements-1);
    end
```

```
    Unew = (1 - 4*mu) .* U + U1 + U2;
    U = Unew;
end
surf(x,y,U); axis([0 1 0 1 -1 1 -1 1]);
```

Listing 1: explicit solution of the heat equation in two dimensions

1.2 WAVE EQUATION

1.3 TRANSPORT EQUATION

2 ANALYSIS

2.1 ACCURACY

2.2 ANOTHER INITIAL SOLUTION