Brief Article

The Author

December 20, 2014

1 True Solution of Transport Equation

As a solution for the transport equation of form

$$\frac{\delta u}{\delta t} = \frac{\delta u}{\delta x} + \frac{\delta u}{\delta y} \tag{1}$$

with a given initial conditions, $u_0(x, y)$, and homogenous dirichet boundary conditions we propose a solution of the following form:

$$u(x, y, t) = u_0(x - vt, y - vt) \tag{2}$$

Calculating the partial derivatives found in the transport equation we get

$$\frac{\delta u}{\delta t} = (-v)\frac{\delta u_0(x, y)}{\delta x} + (-v)\frac{\delta u_0(x, y)}{\delta y} \tag{3}$$

$$\frac{\delta u}{\delta x} = \frac{\delta u_0(x, y)}{\delta x}, \frac{\delta u}{\delta y} = \frac{\delta u_0(x, y)}{\delta y} \tag{4}$$

Filling this then in the transport equation gives

$$-v\frac{\delta u_0}{\delta x} - v\frac{\delta u_0}{\delta y} = \frac{\delta u_0}{\delta x} + \frac{\delta u_0}{\delta y} \tag{5}$$

which fits when $v = -1$ and so our solution is

$$u(x, y, t) = u_0(x + t, y + t) \tag{6}$$