Brief Article

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1 True Solution of Transport Equation

As a solution for the transport equation of form

$$\frac{\delta u}{\delta t} = \frac{\delta u}{\delta x} + \frac{\delta u}{\delta y} \tag{1}$$

with a given initial conditions, $u_0(x, y)$, and homogenous dirichet boundary conditions we propose a solution of the following form:

$$u(x, y, t) = u_0(x - vt, y - vt) \tag{2}$$

Calculating the partial derivatives found in the transport equation we get

$$\frac{\delta u}{\delta t} = (-v)\frac{\delta u_0(x, y)}{\delta x} + (-v)\frac{\delta u_0(x, y)}{\delta y} \tag{3}$$

$$\frac{\delta u}{\delta x} = \frac{\delta u_0(x, y)}{\delta x}, \frac{\delta u}{\delta y} = \frac{\delta u_0(x, y)}{\delta y} \tag{4}$$

Filling this then in the transport equation gives

$$-v\frac{\delta u_0}{\delta x} - v\frac{\delta u_0}{\delta y} = \frac{\delta u_0}{\delta x} + \frac{\delta u_0}{\delta y} \tag{5}$$

which fits when $v = -1$ and so our solution is

$$u(x, y, t) = u_0(x + t, y + t) \tag{6}$$

2 error analysis of the transport problem

The truncation error for the upwind scheme with $a = 1$ can be found to be

$$T_j^n = -\frac{1}{2}(1 - \nu)\Delta x u_{xx} + ... \tag{7}$$

Which is forst order in Δx, and therefor, under constant ν, also first order in Δt.

We then now that the maximum error E^n at a point in time n is bound by a function of the same order. And this is what we see in figure 1

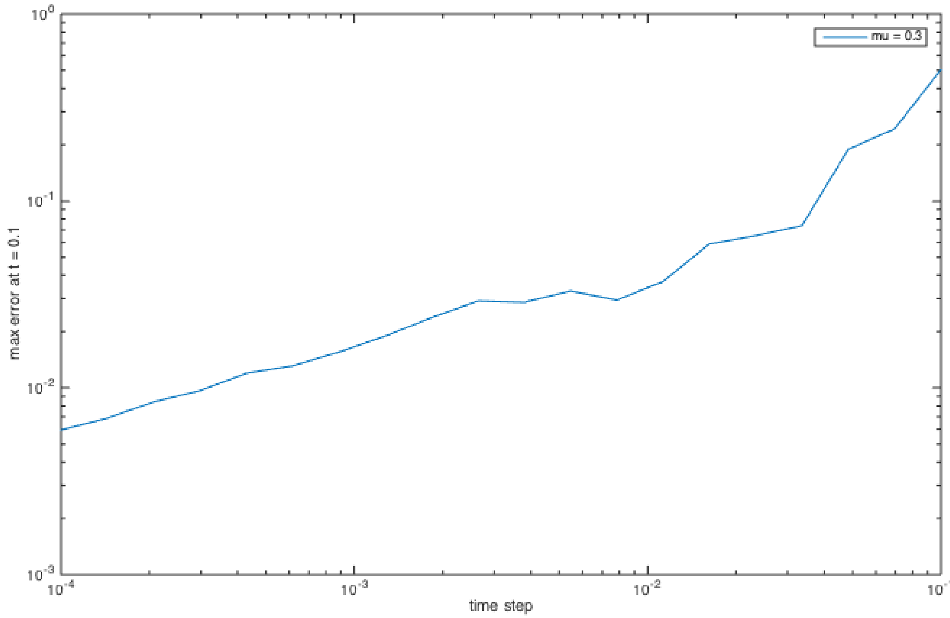

Figure 1: The maximum error at time = 0.1 in function of the tilmestep at a constant ν of 0.3