Liouville's Theorem (Differential algebra)

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Sber

January 14, 2025

Basic definitions

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Let F be the differential field. Then

- b is called the logarithm of a if $b' = \frac{a'}{a}$

What is integrability in elementary functions