

Liouville's Theorem (Differential algebra)

Vanya Vorobiov

Sber

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Field F is differential if it's equipped with the unary function $'$ such that:

- ▶ $(a + b)' = a' + b'$
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Differential extension of the differential field F is field E such that $E \supseteq F$ and there is the same differentiation $'$ on E .

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Let F be the differential field. Then

- ▶ b is called the logarithm of a if $b' = \frac{a'}{a}$
- ▶ b is called the exponent of a if $a' = \frac{b'}{b}$

What is integrability in elementary functions

Liouville's
Theorem

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Basic definitions

What is
integrability in
elementary
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Definition

The extension E of F is called elementary if it can be presented as $E = F(t_1, \dots, t_n)$ and for all i t_i is logarithm or exponent or algebraic over $F(t_1, \dots, t_{i-1})$.

Remark

Common sense say us that any function f :