

# Liouville's Theorem (Differential algebra)

Vanya Vorobiov

Sber

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# Basic definitions

Through the all of presentation we will suppose that all fields have 0 characteristic.

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## Definition

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- ▶  $(a + b)' = a' + b'$
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Let  $F$  be the differential field. Then

- ▶  $b$  is called the logarithm of  $a$  if  $b' = \frac{a'}{a}$
- ▶  $b$  is called the exponent of  $a$  if  $a' = \frac{b'}{b}$

# What is integrability in elementary functions

Liouville's  
Theorem

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