Diffusion Models for Time Series Testing on Synthetical Data

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Diffusion Models for Time Series: A Fundamental Approach

Key Idea

- Comparative analysis of two diffusion models for time series generation
- Novel evaluation approach: Instead of standard industrial datasets, we test on:
  - Stochastic processes (e.g., OU process, Gaussian RBF process)
  - Deterministic functions (where theoretical ground truth is known)

Why This Matters

- Current literature focuses exclusively on large industrial datasets
- Our approach provides:
  - Clear theoretical benchmarks
  - Better understanding of model fundamentals
  - More meaningful comparison in early-stage research

When the field is in its infancy, controlled experiments on theoretical cases may reveal more than benchmarks on complex real-world data.

Diffusion key idea – model

$$p_\theta(x_t|y_{obs}), \tag{1}$$

CSDI arhitecture

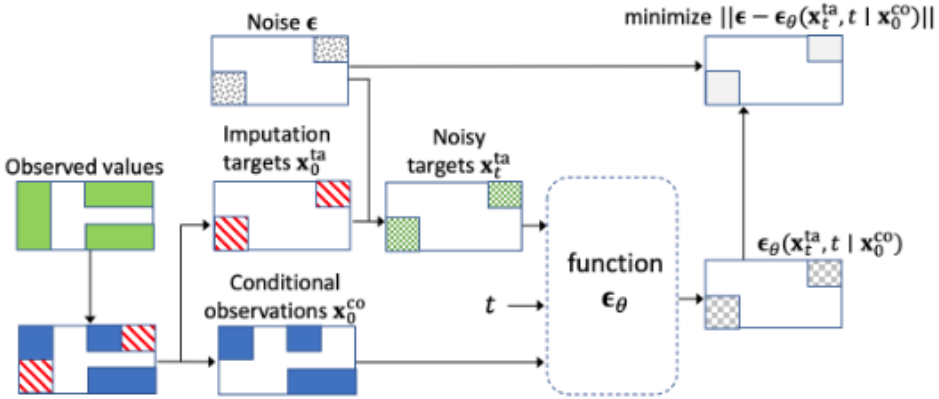


Figure 2: The self-supervised training procedure of CSDI. On the middle left rectangle, the green and white areas represent observed and missing values, respectively. The observed values are separated into red imputation targets x_0^{ta} and blue conditional observations x_0^{co}, and used for training of ϵ_θ. The colored areas in each rectangle mean the existence of values.

UNCONDITIONAL DIFFUSION KEY IDEA

Let $t \geq 0$ be an arbitrary diffusion step. Applying Bayes' rule, we have

$$p_\theta(x_t|y_{obs}) \propto p_\theta(y_{obs}|x_t)p_\theta(x_t), \tag{2}$$

which yields the following relation between the conditional and marginal score functions,

$$\nabla_{x_t} \log p_\theta(x_t|y_{obs}) = \nabla_{x_t} \log p_\theta(y_{obs}|x_t) + \nabla_{x_t} \log p_\theta(x_t). \tag{3}$$

Given access to the guidance distribution, $p_\theta(y_{obs}|x_t)$, we can draw samples from $p_\theta(x_t|y_{obs})$

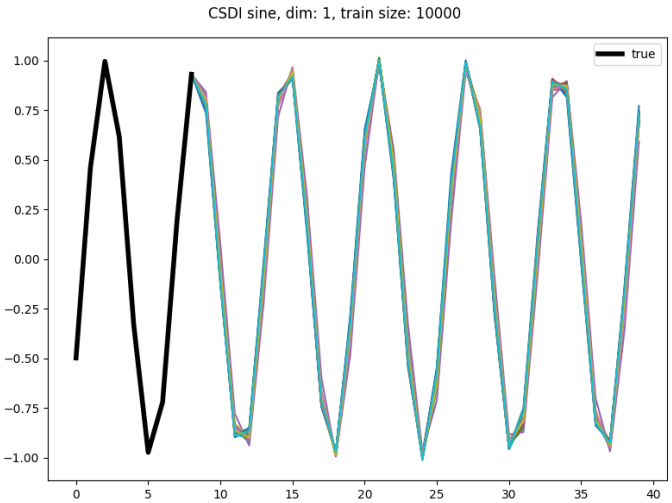
$$p_\theta(x_{t-1}|x_t, y_{obs}) = \mathcal{N}\left(x_{t-1}; \mu_\theta(x_t, t) + s\sigma_t^2 \nabla_{x_t} \log p_\theta(y_{obs}|x_t), \sigma_t^2 I\right). \tag{4}$$

$p_\theta(y_{obs}|x_t)$ modeled as a multivariate Gaussian distribution,

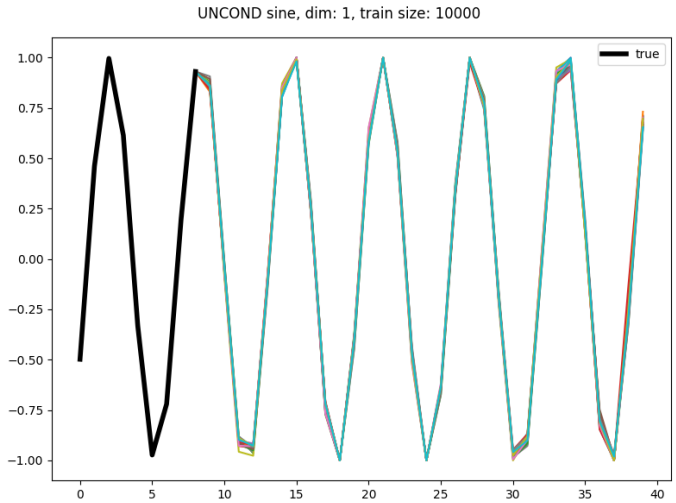
$$p_\theta(y_{obs}|x_t) = \mathcal{N}\left(y_{obs}|f_\theta(x_t, t), I\right), \tag{5}$$

Sine Waves (dim=1), Train Size=10,000

Samples

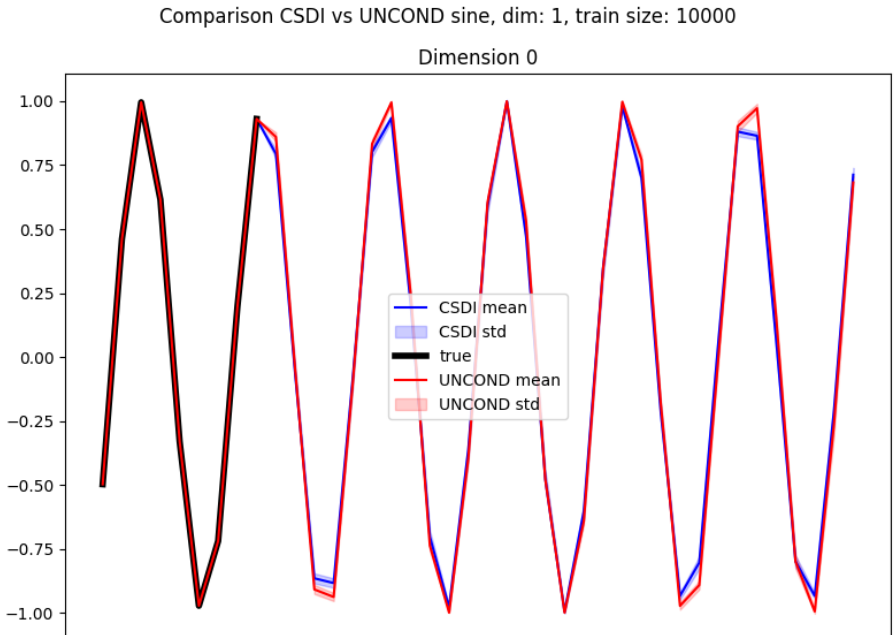


CSDI Sample



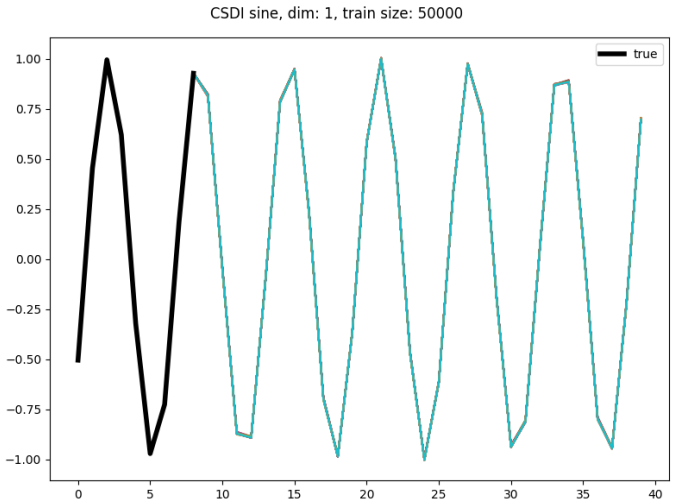
Unconditional Sample

Mean Comparison

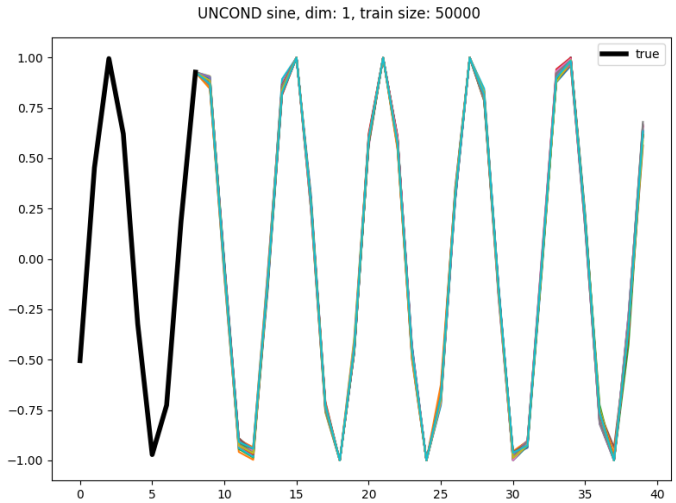


Sine Waves (dim=1), Train Size=50,000

Samples



CSDI Sample



Unconditional Sample

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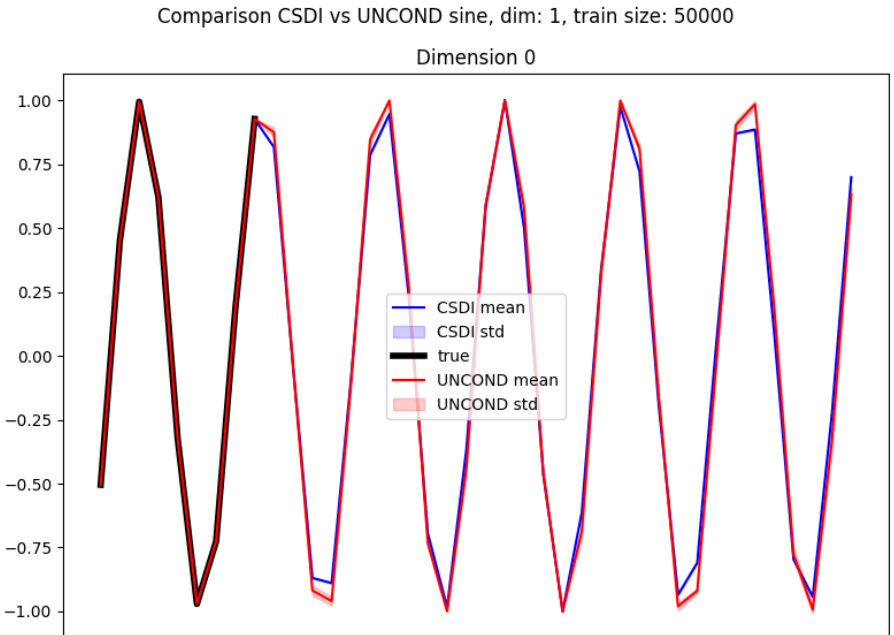
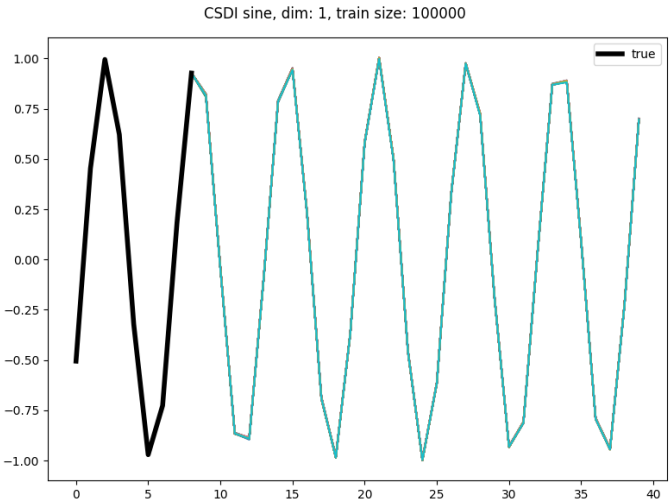


Sine Waves (dim=1), Train Size=100,000

Samples



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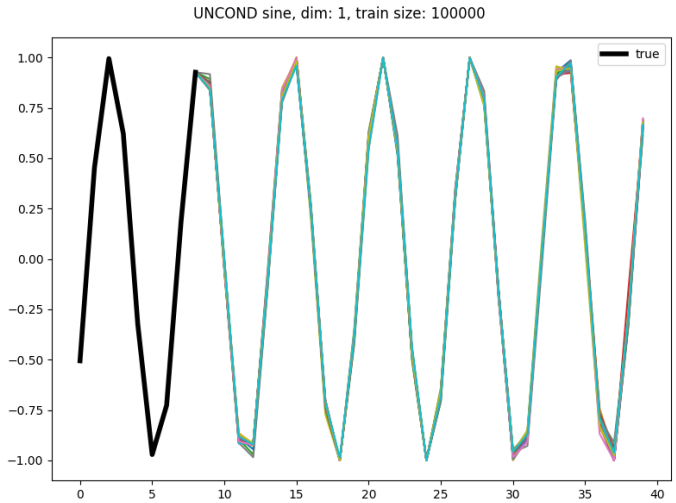

Unconditional Sample

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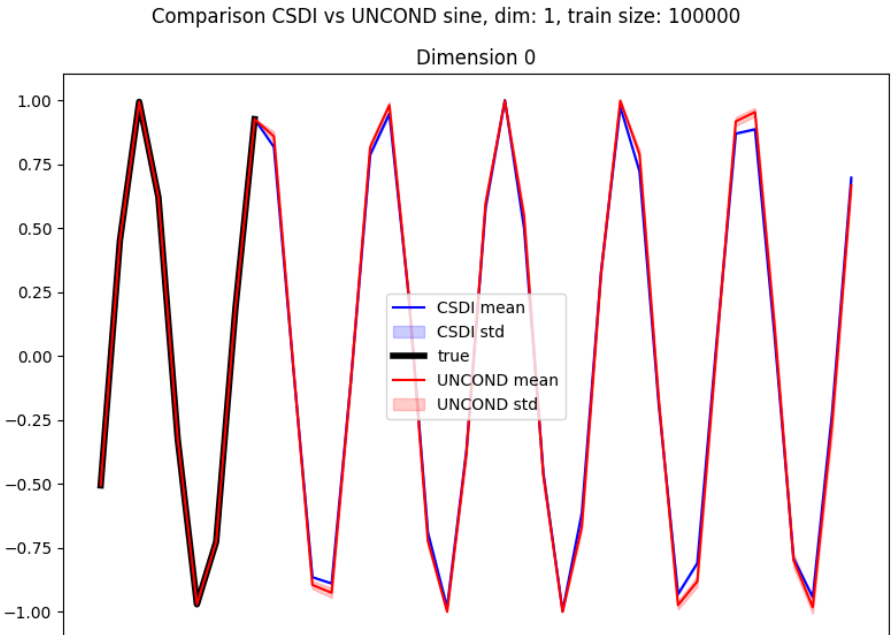
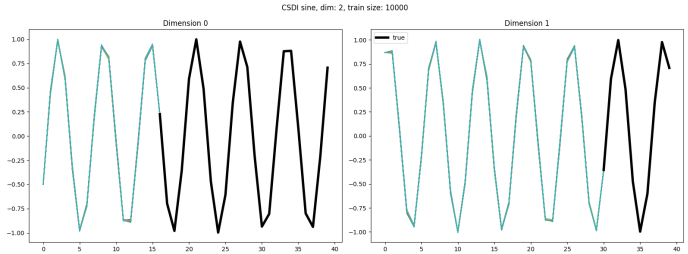
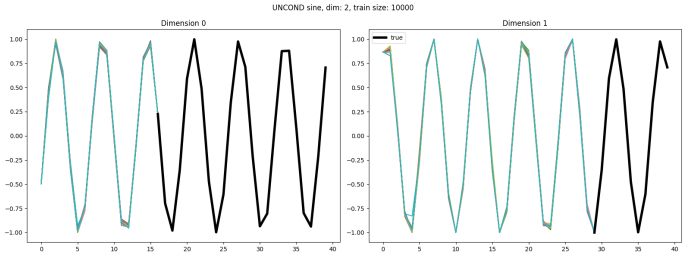


Sine Waves (dim=2), Train Size=10,000

Samples



CSDI Sample



Unconditional Sample

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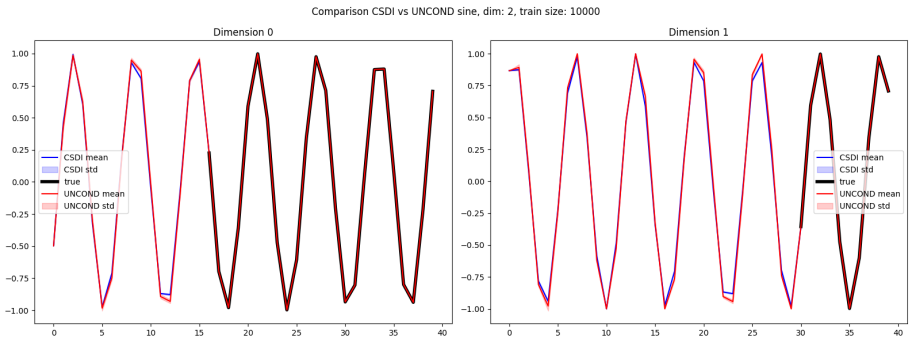
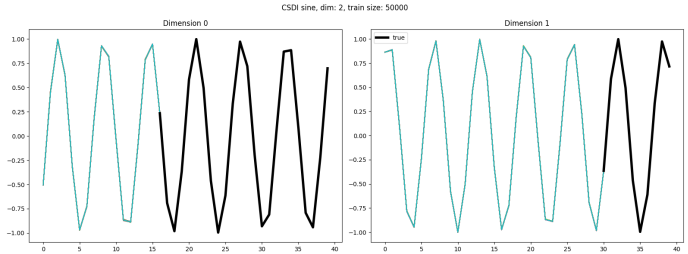


Sine Waves (dim=2), Train Size=50,000

Samples



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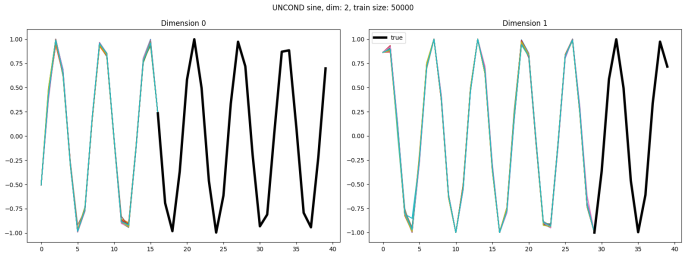
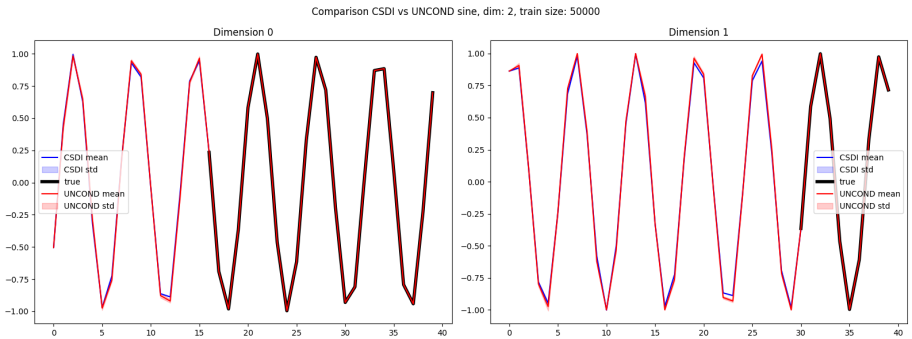


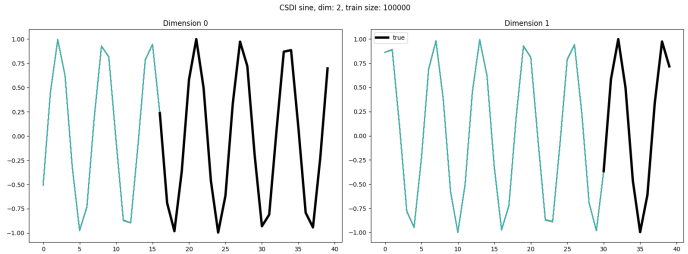
Unconditional Sample

Mean Comparison



Sine Waves (dim=2), Train Size=100,000

Samples



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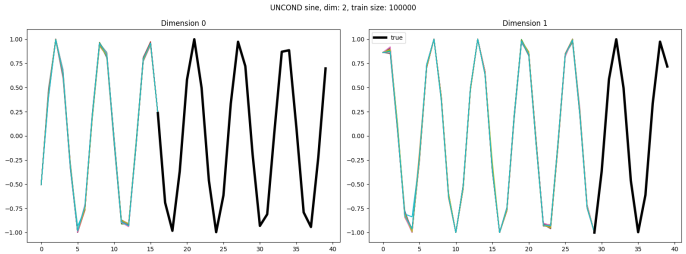
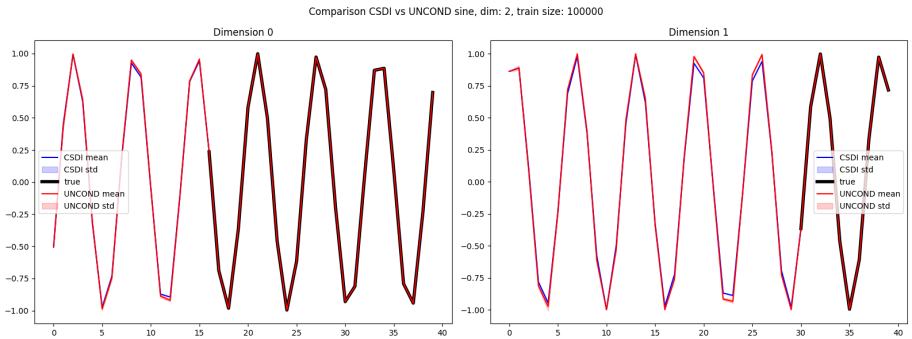


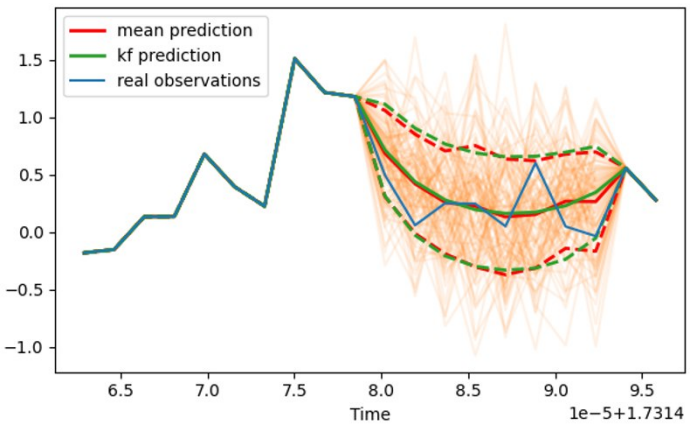
Unconditional Sample

Mean Comparison



Ornstein-Uhlenbeck (dim=1)

Samples



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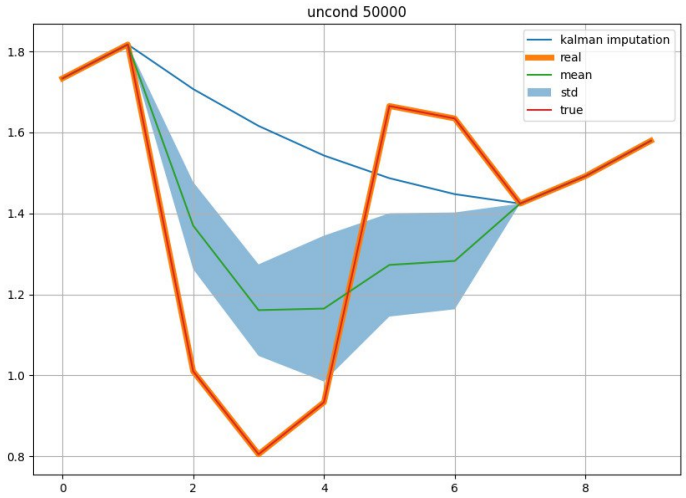
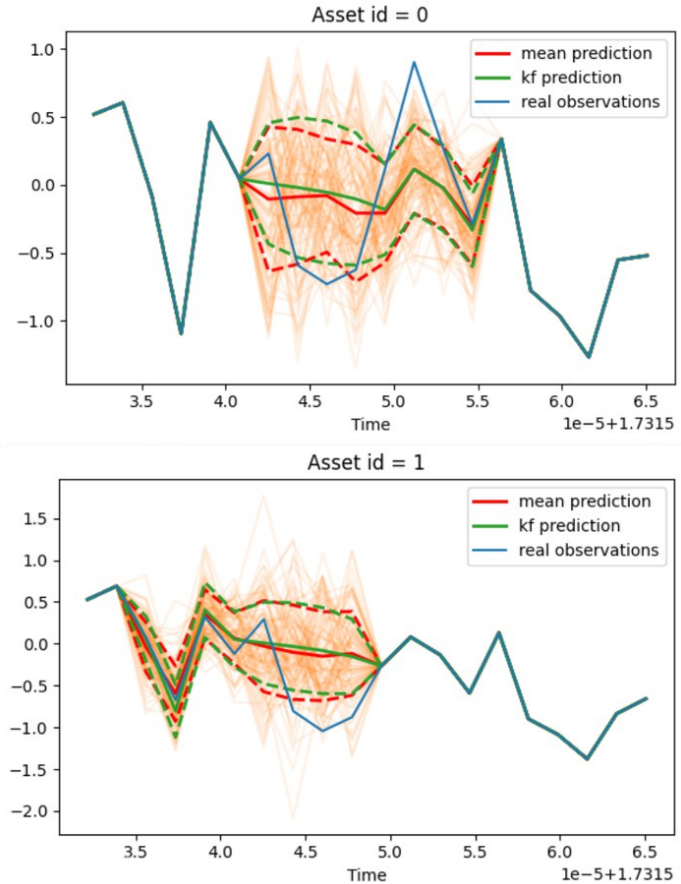


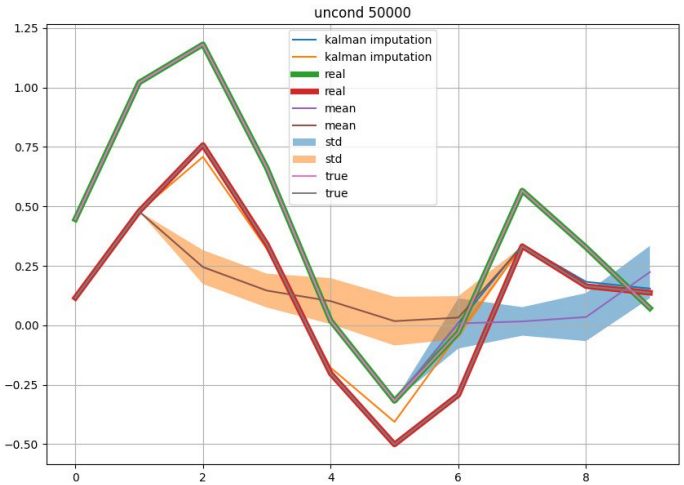
Unconditional Sample

Ornstein-Uhlenbeck (dim=2)

Samples



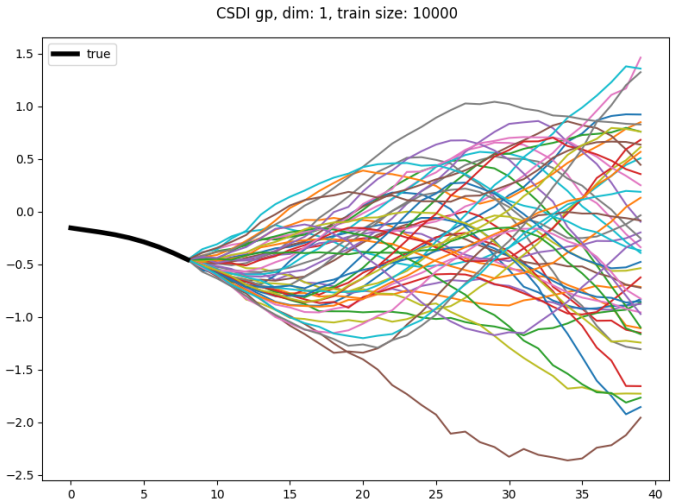
CSDI Sample



Unconditional Sample

Gaussian Process (dim=1), Train Size=10,000

Samples



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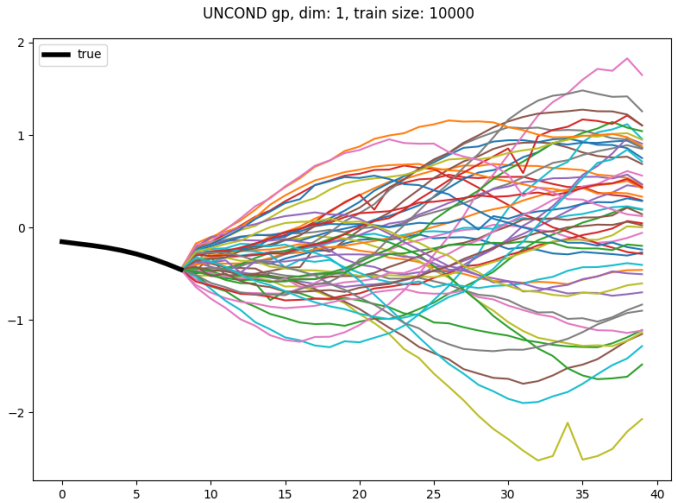
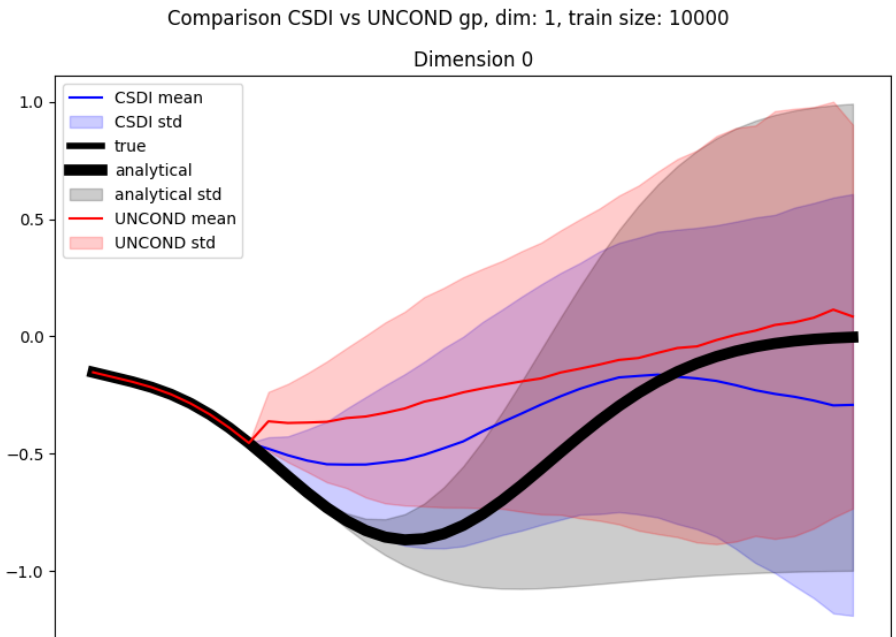


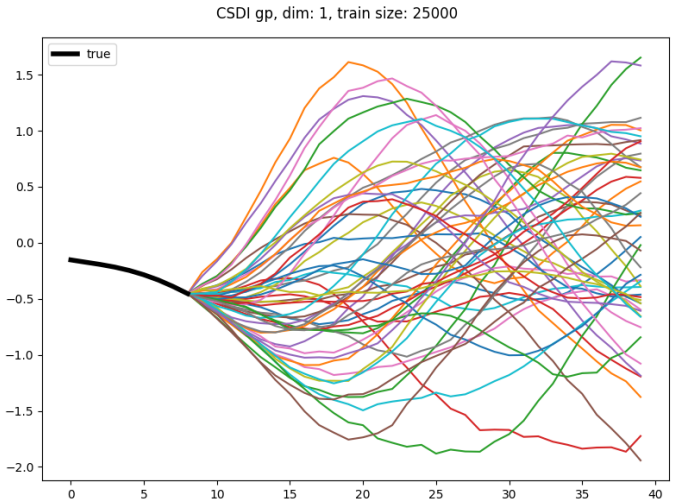
Unconditional Sample

Mean Comparison

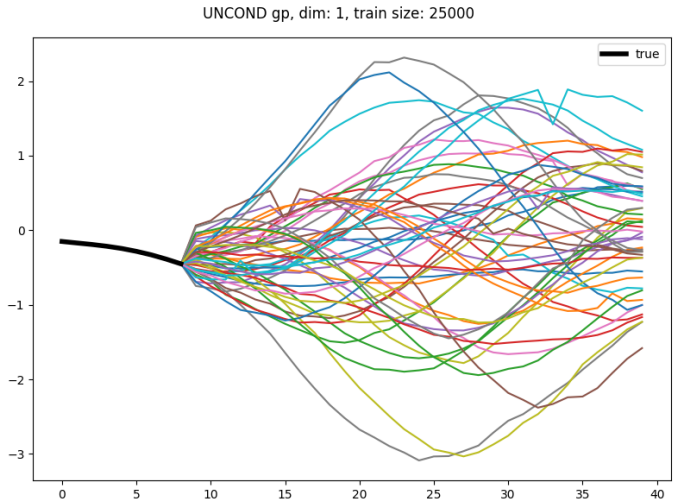


Gaussian Process (dim=1), Train Size=25,000

Samples

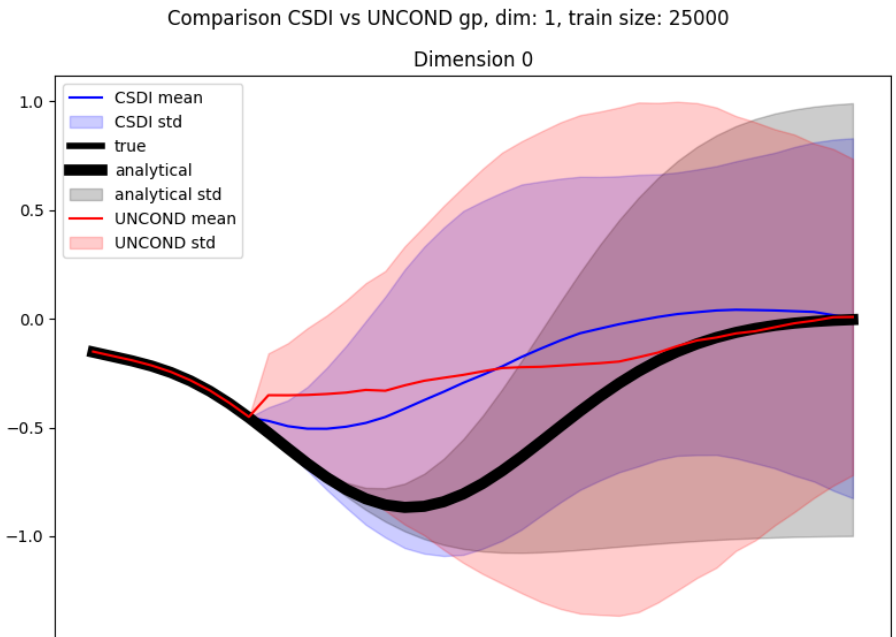


CSDI Sample



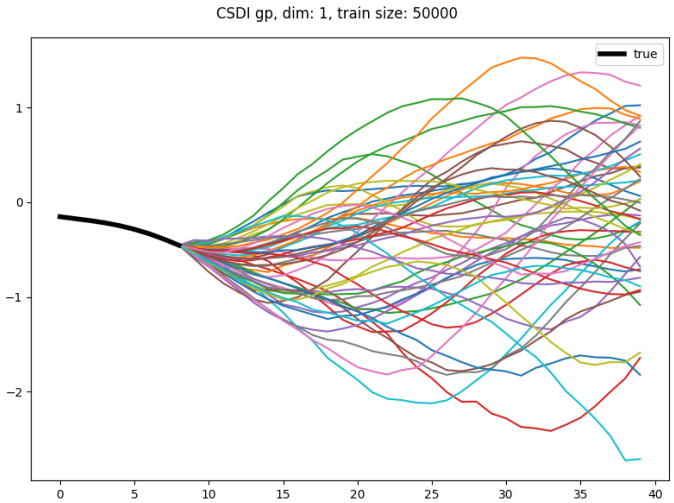
Unconditional Sample

Mean Comparison



Gaussian Process (dim=1), Train Size=50,000

Samples



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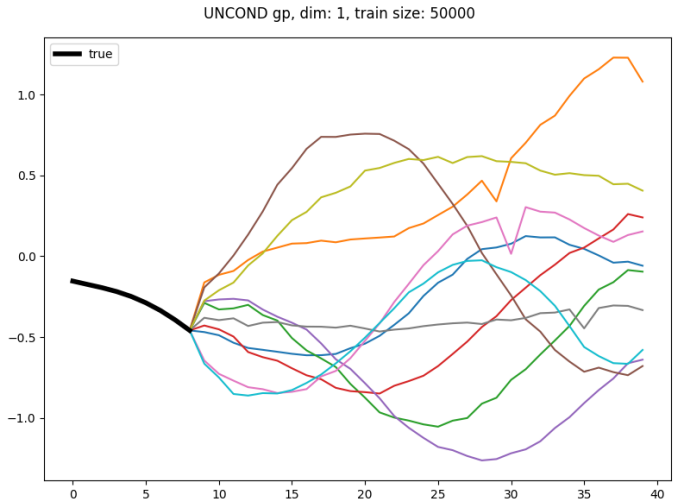
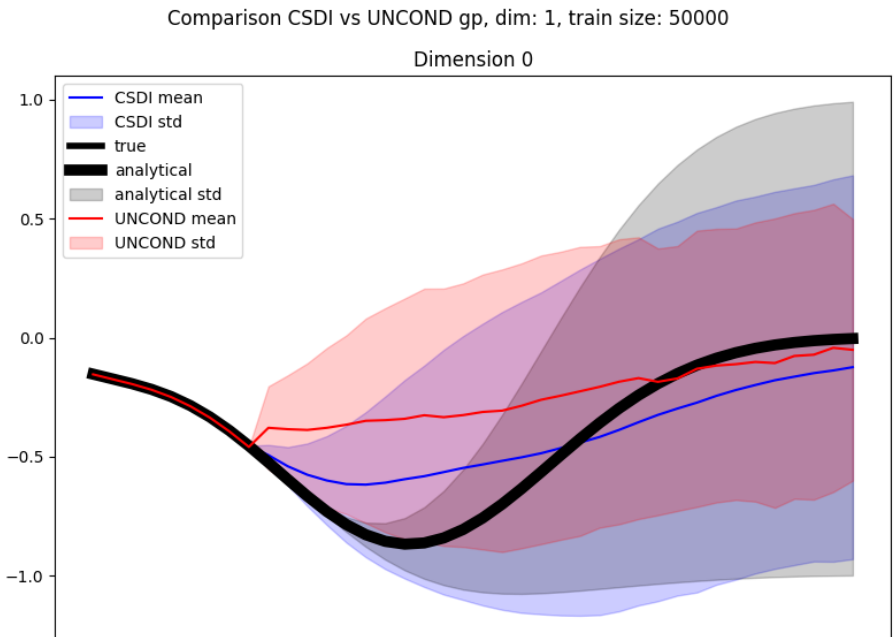


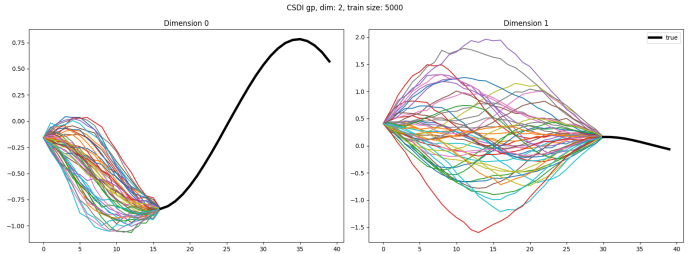
Unconditional Sample

Mean Comparison



Gaussian Process (dim=2), Train Size=5,000

Samples



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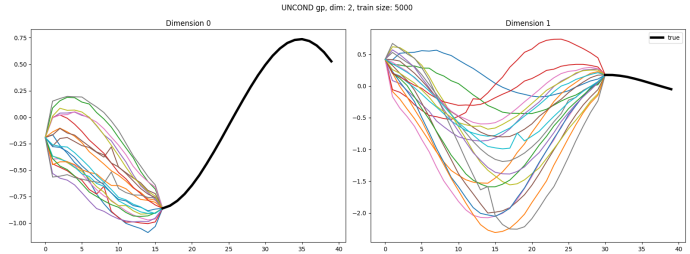
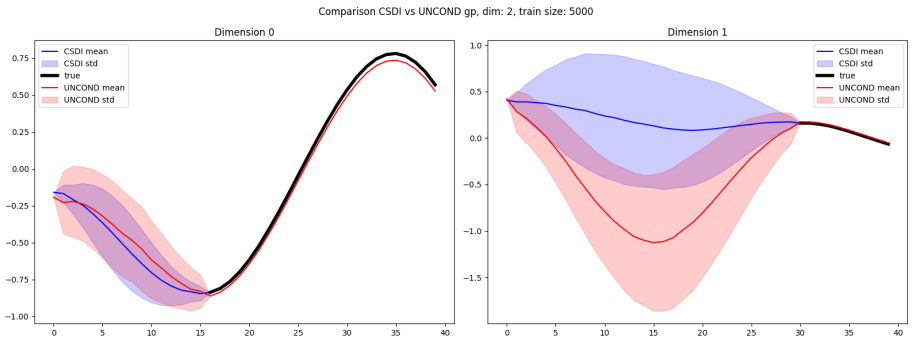


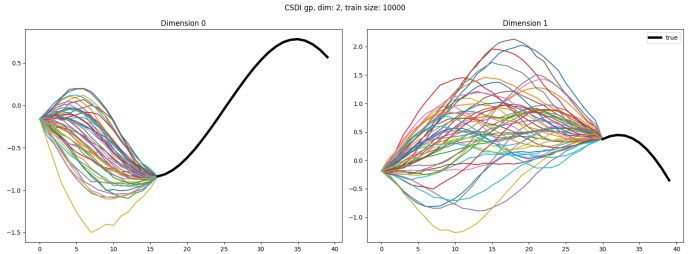
Unconditional Sample

Mean Comparison



Gaussian Process (dim=2), Train Size=10,000

Samples



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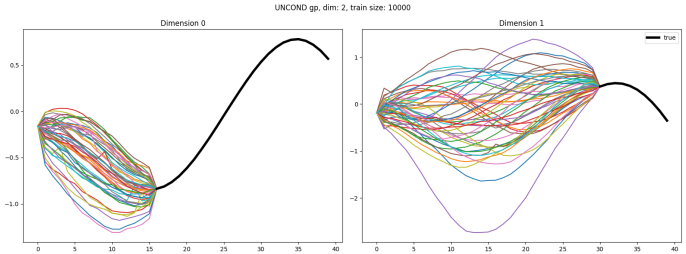
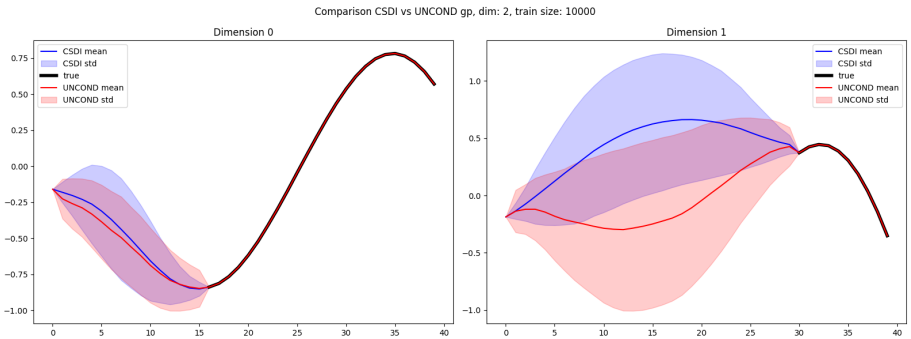


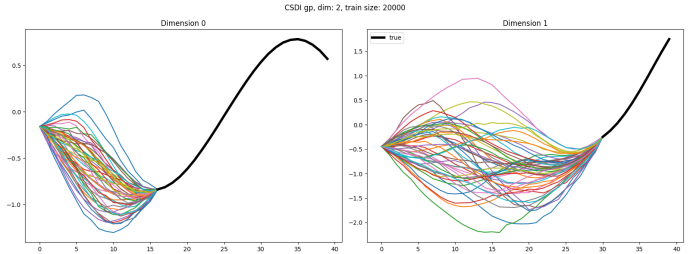
Unconditional Sample

Mean Comparison



Gaussian Process (dim=2), Train Size=20,000

Samples



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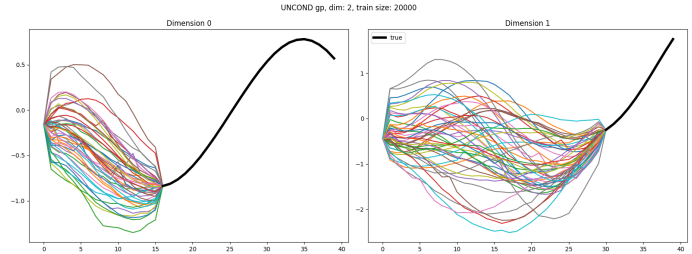

Unconditional Sample

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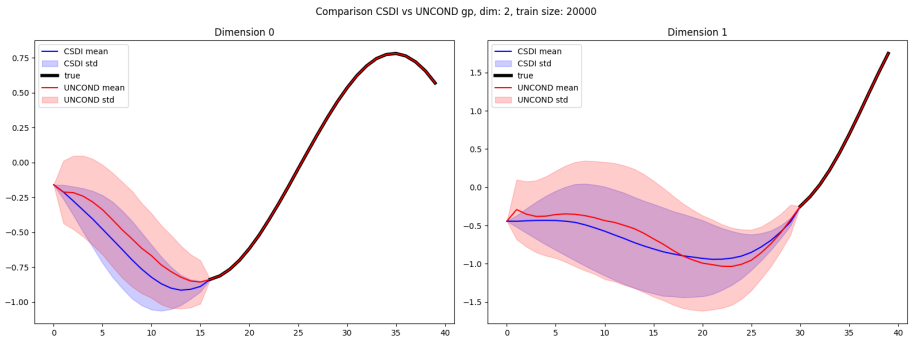

Conclusions and Next Steps

Key Takeaways

- Our tests reveal that current diffusion models often struggle with basic synthetic tasks
- Performance varies significantly between different architectures
- Simple test cases expose limitations not visible in complex real-world benchmarks

Future Work

- Explore additional model architectures
- Conduct more comprehensive testing
- Develop better evaluation approaches for fundamental properties

Bibliography

Xinyu Yuan and Yan Qiao, Diffusion-TS: Interpretable Diffusion for General Time Series Generation, The Twelfth International Conference on Learning Representations, 2024.

Yusuke Tashiro, Jiaming Song, Yang Song and Stefano Ermon, CSDI: Conditional Score-based Diffusion Models for Probabilistic Time Series Imputation, Advances in Neural Information Processing Systems, 2021.