Analysis 1 für Informatikstudien/Analysis T1 10.1.2022

2. Test

Name: [redacted]

Matr.Nr.: [redacted]

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1. **(10)** Führen Sie für die Funktion

$$f(x) = x(x-1)e^{-x} \checkmark$$

folgendes Programm durch:

(a) Definitionsbereich ✓
(b) Nullstellen ✓
(c) Extrema ✓
(d) Wendepunkte ✓
(e) Monotonieintervalle ✓
(f) Krümmungsintervalle ✓
(g) Verhalten für $x \to +\infty$ ✓
(h) Skizze im Intervall $[-1, 10]$. ✓

2. **(6)** Bestimmen Sie das Integral

$$\int_0^1 \frac{x^3 + 7x^2 + 23x + 7}{x^2 + 6x + 5}\,dx. \checkmark$$

3. **(4)** Finden Sie die Lösungen der Gleichung

$$x^2 + (1-i)x - (4-7i) = 0 \checkmark$$

über den komplexen Zahlen.

Sämtliche Zwischenschritte sind anzugeben.

1) $f(x) = x(x-1)e^{-x}$

$= (x^2 - x)e^{-x}$

a) Definiert $\forall\, x \in \mathbb{R}$ ✓

b) $(x_0^2 - x_0)\underbrace{e^{-x_0}}_{\neq 0} = 0$ ✓

$(x_0^2 - x_0) = 0$

$x_{0_{1,2}} = \frac{1}{2} \pm \sqrt{\left(-\frac{1}{2}\right)^2} = \frac{1}{2} \pm \frac{1}{2}$ ✓

$\underline{\underline{x_{0_1} = 1}}$

$\underline{\underline{x_{0_2} = 0}}$

c) $f(x) = (x^2 - x)e^{-x}$

$f'(x) = (2x - 1)e^{-x} - (x^2 - x)e^{-x} =$

$= (-x^2 + 3x - 1)\underbrace{e^{-x}}_{\neq 0}$

$(-x_E^2 + 3x_E - 1) = 0$

$x_E^2 - 3x_E + 1 = 0$

$x_{E_{1,2}} = \frac{3}{2} \pm \sqrt{\left(-\frac{3}{2}\right)^2 - 1} = \frac{3}{2} \pm \sqrt{\frac{5}{4}}$ ✓

$\underline{\underline{x_{E_1} = 2{,}62}}$

$\underline{\underline{x_{E_2} = 0{,}38}}$

$f(x_{E_1}) = 0{,}31$ ✓

$f(x_{E_2}) = -0{,}16$

$$f''(x) = (-2x+3)e^{-x} - (-x^2+3x-1)e^{-x} =$$
$$= (x^2-5x+4)e^{-x}$$

$f''(x_{E_1}) = -0{,}16 < 0 \Rightarrow$ Maximum

$f''(x_{E_2}) = 1{,}53 > 0 \Rightarrow$ Minimum

d) $(x_w^2 - 5x_w + 4) = 0$

$$x_{w1,2} = \frac{5}{2} \pm \sqrt{\left(-\frac{5}{2}\right)^2 - 4} = \frac{5}{2} \pm \sqrt{\frac{25}{4} - \frac{16}{4}} = \frac{5}{2} \pm \frac{3}{2}$$

$\underline{\underline{x_{w_1} = 4}}$

$\underline{\underline{x_{w_2} = 1}}$

$f(x_{w_1}) = 12e^{-4} = 0{,}22$

$f(x_{w_2}) = 0$

$f'(x_{w_1}) = -5e^{-4} = -0{,}09$

$f'(x_{w_2}) = e^{-1} = 0{,}36$

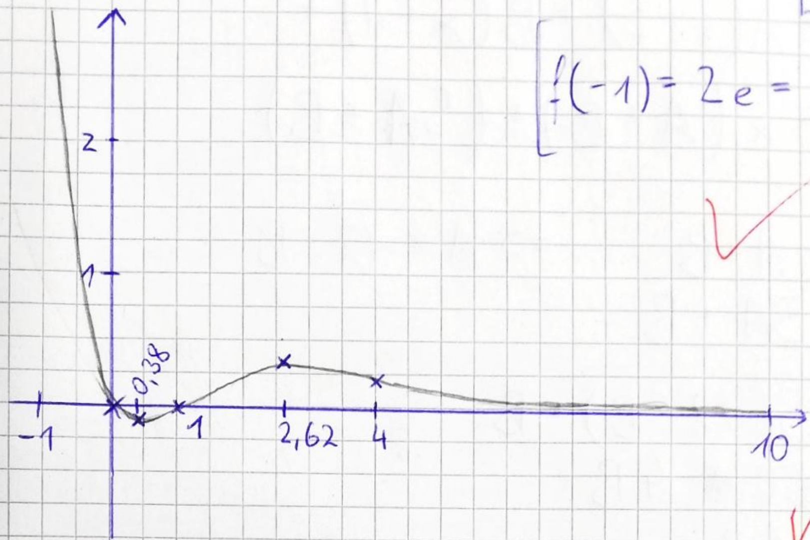
e) $x \in (-\infty; 0{,}38) \Rightarrow f(x) \downarrow$
$x \in (0{,}38; 2{,}62) \Rightarrow f(x) \uparrow$
$x \in (2{,}62; \infty) \Rightarrow f(x) \downarrow$

f) $x \in (-\infty, 1) \Rightarrow f(x)$ konvex
$x \in (1, 4) \Rightarrow f(x)$ konkav
$x \in (4, \infty) \Rightarrow f(x)$ konvex

g) $\lim_{x\to\infty} (x^2-x)e^{-x} = \lim_{x\to\infty} \frac{x^2-x}{e^x} \overset{\text{l'Hospital}}{=}$

$= \lim_{x\to\infty} \frac{2x-1}{e^x} \underset{\text{l'Hospital}}{=} \lim_{x\to\infty} \frac{2}{e^x} = \underline{0}$

h) Skizze:



$\left[f(-1) = 2e = 5{,}44 \right]$

2) $\displaystyle\int_0^1 \frac{x^3+7x^2+23x+7}{x^2+6x+5}\,dx$

$x^2+6x+5=0$

$x_{1,2} = -3 \pm \sqrt{9-5} = \begin{cases} -1 \\ -5 \end{cases}$

$$(x^3+7x^2+23x+7) : (x^2+6x+5) = \underline{\underline{x+1}}$$
$$\underline{-x^3-6x^2-5x}$$
$$x^2+18x+7$$
$$\underline{-x^2-6x-5}$$
$$\underline{\underline{12x+2}}$$

$\Rightarrow \dfrac{x^3+7x^2+23x+7}{x^2+6x+5} = x+1+\dfrac{12x+2}{x^2+6x+5}$

$\dfrac{12x+2}{x^2+6x+5} = \dfrac{A}{x+1} + \dfrac{B}{x+5}$

$12x+2 = A(x+5)+B(x+1)$

$$12x+2 = A(x+5)+B(x+1) =$$

$$= x(A+B)+(5A+B)$$

$$12 = A+B \quad \Rightarrow A = 12-B$$

$$2 = 5A+B$$

$$2 = 5(12-B)+B$$

$$2 = 60-4B$$

$$58 = 4B$$

$$B = \frac{58}{4} = \frac{29}{2}$$

$$12 = A+\frac{29}{2}$$

$$A = \frac{24}{2}-\frac{29}{2} = -\frac{5}{2}$$

$$\rightarrow \int_0^1 \left(x+1-\frac{5}{2(x+1)}+\frac{29}{2(x+5)}\right)dx =$$

$$= \int_0^1 x\,dx + \int_0^1 1\,dx - \frac{5}{2}\int_0^1 \frac{1}{x+1}dx + \frac{29}{2}\int_0^1 \frac{1}{x+5}dx =$$

$$= \frac{1}{2}x^2\Big|_0^1 + x\Big|_0^1 - \frac{5}{2}\ln|x+1|\Big|_0^1 + \frac{29}{2}\ln|x+5|\Big|_0^1 =$$

$$=\left(\frac{1}{2}-0\right)+(1-0)-\left(\frac{5}{2}\ln(2)-\frac{5}{2}\ln(1)\right)+\frac{29}{2}\left(\ln(6)-\ln(5)\right)=$$

$$\approx \underline{2{,}41}$$

[redacted]

3) $x^2 + (1-i)x - (4-7i) = 0$

$$x_{1,2} = -\frac{1-i}{2} \pm \sqrt{\underbrace{\left(\frac{1-i}{2}\right)^2 + 4 - 7i}_{z}}$$

$$z = \left(\frac{1-i}{\boxed{2}}\right)^{\boxed{2}} + 4 - 7i =$$

$$= \frac{\cancel{1}\cancel{-2}i\cancel{-1}}{\boxed{\cancel{2}}} + 4 - 7i =$$

$$= -i + 4 - 7i = 4 - 8i$$ ✓ Hier schon falsch!!

$$\|z\| = \sqrt{4^2 + 8^2} = \sqrt{80}$$ ✓

$$\varphi = \arctan\left(\frac{-8}{4}\right) = -63{,}4^\circ$$ (rechenweg passt)

$$z = \sqrt{80}\, e^{-i 63{,}4^\circ}$$

$$\sqrt{z} = \sqrt{\sqrt{80}}\, e^{-i\frac{63{,}4^\circ}{2}} = \sqrt[4]{80}\, e^{-i 31{,}7^\circ}$$

$$x_{1,2} = -\frac{1}{2} + i\frac{1}{2} \pm \underbrace{\sqrt[4]{80}\, e^{-i 31{,}7^\circ}}$$

[Re] $\sqrt[4]{80} \cos(-31{,}7^\circ) = 2{,}54$

[Im] $\sqrt[4]{80} \sin(-31{,}7^\circ) = -1{,}57$

$$x_{1,2} = -\frac{1}{2} + i\frac{1}{2} \pm (2{,}54 - i\,1{,}57)$$

$$\underline{\underline{x_1 = -1{,}27 - i\,1{,}07}}$$

$$\underline{\underline{x_2 = -3{,}04 + i\,2{,}07}}$$

Anhang: TK2 Bsp. 3 Korrektur

Montag, 31. Januar 2022 14:44

$$\underline{x}^2 + (1-i)\,\underline{x} - (4-7i) = 0$$

$$\underline{x}_{1,2} = -\frac{1-i}{2} \pm \sqrt{\underbrace{\left(\frac{1-i}{2}\right)^2 + 4 - 7i}_{\underline{z}}}$$

$$\underline{z} = \left(\frac{1-i}{2}\right)^2 + 4 - 7i =$$

$$= \frac{\cancel{1} - \cancel{2}i - \cancel{1}}{\cancel{4}\,2} + 4 - 7i =$$

$$= -\frac{15}{2}i + 4$$

$$\|\underline{z}\| = \sqrt{4^2 + \left(\frac{15}{2}\right)^2} = \frac{17}{2}$$

$$\varphi = \arctan\left(-\frac{15}{2\cdot 4}\right) = 1{,}08 \text{ rad}$$

$$\underline{z} = \frac{17}{2} e^{-i\,1{,}08}$$

$$\sqrt{\underline{z}} = \sqrt{\frac{17}{2}}\, e^{-i\frac{1{,}08}{2}}$$

$$\underline{x}_{1,2} = -\frac{1}{2} + i\,\frac{1}{2} \pm \underbrace{\sqrt{\frac{17}{2}}\, e^{-i\frac{1{,}08}{2}}}$$

$$[\mathrm{Re}]\ \sqrt{\frac{17}{2}} \cos\left(-\frac{1{,}08}{2}\right) = \frac{5}{2}$$

$$[\mathrm{Im}]\ \sqrt{\frac{17}{2}} \sin\left(-\frac{1{,}08}{2}\right) = -\frac{3}{2}$$

$$\underline{x}_{1,2} = -\frac{1}{2} + i\,\frac{1}{2} \pm \left(\frac{5}{2} - i\,\frac{3}{2}\right)$$

$$\underline{\underline{\underline{x}_1 = 2 - i}}$$

$$\underline{\underline{\underline{x}_2 = -3 + 2i}}$$