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MONTH DAY, 2017

CS 405: Algorithm Analysis II
Homework NUM: TITLE

Each of the exercises below involves a choice among the master theorem templates discussed in lecture. For each, indicate which case applies and specify the asymptotic growth class of the function. If no case applies, simply state that fact; you are not required to attempt a solution when no master theorem case applies.

1. $T(n) = 2T(\lfloor n/4 \rfloor) + n^{1/2}$.
2. $T(n) = 3T(\lfloor n/2 \rfloor) + n \lg n$.
3. $T(n) = 5T(\lfloor n/5 \rfloor) + \frac{n}{\lg n}$.
4. $T(n) = 4T(\lfloor n/2 \rfloor) + n^2\sqrt{n}$.
5. $T(n) = 2T(\lfloor n/2 \rfloor) + n \lg n$.

Solutions.

$a = 3, b = 2$ implies a reference function $g(n) = n^{\log_2 3}$. Converting as follows,

$$\begin{aligned} y &= \log_2 3 \\ 2^y &= 3 \\ y \ln 2 &= \ln 3 \\ y &= \frac{\ln 3}{\ln 2} = 1.585, \end{aligned}$$

we have $g(n) = n^{1.585}$. The "glue" function is $f(n) = n \lg n$. Let $g_\epsilon(n) = n^{1.585-\epsilon}$, for $0 < \epsilon < 0.5$. Since

$$\begin{aligned} \frac{f(n)}{g_\epsilon(n)} &= \frac{n \lg n}{n^{1.585-\epsilon}} = \frac{\lg n}{n^{0.585-\epsilon}} \\ &\leq \frac{\lg n}{n^{0.085}} \to 0 \end{aligned}$$

as $n \to \infty$, we have $f(n) = o(g_\epsilon(n))$, which implies $f(n) = O(g_\epsilon(n))$ and allows case (1) of the master template. Therefore $T(n) = \Theta(g(n)) = \Theta(n^{1.585})$.

Answers to incidental LaTeX question may be found at:

`http://www.tug.org/begin.html`

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