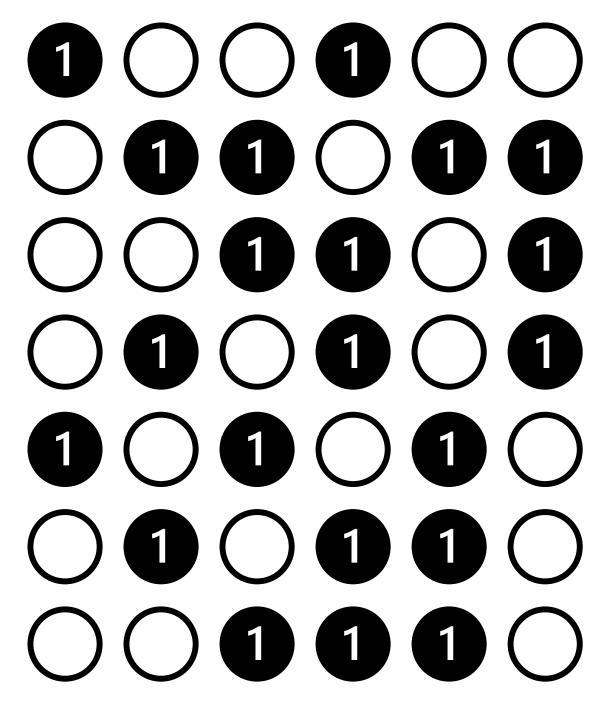
# Licenciatura em Engenharia Informática Algoritmos e Estruturas de Dados



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The Merkle-Hellman Cryptosystem

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# Índice

Introdução	3
Contextualização do Problema	4
Brute Force	5
Clever Brute Force	6
Horowitz and Sahni Technique	7
Schroeppel and Shamir Technique	8
Resultados Obtidos	9
Otimizações	9
Gráficos	9
Conclusão	11
Referências	12
Anexos	13
Código C	13
Código Matlab	19
Masks	

# Introdução

A criptografia é usada na comunicação de dados para proteger mensagens que circulam em canais de comunicação pouco seguros. Só na década de 70, com o aparecimento dos sistemas criptográficos de chave pública ou assimétrica, veio-se resolver o problema do transporte seguro de chaves.

O objetivo deste trabalho é a implementação e estudo de diversos algoritmos de decifragem do sistema criptográfico de Merkle-Hellman.

O problema proposto consiste na identificação de um subconjunto de um dado conjunto de números fornecido a priori, onde a soma dos elementos desse subconjunto corresponde à solução pretendida. O resultado final é apresentado na forma de uma *mask* onde o índice dos bits a 1 indica a posição no conjunto fornecido dos valores pertencentes ao subconjunto.

Para a resolução do problema foram implementados vários algoritmos de modo a mostrar as vantagens e desvantagens de cada um, onde os resultados obtidos vão ser descritos neste relatório.

# Contextualização do Problema

Como referido anteriormente, o problema proposto (*subset sum problem*) consiste num conjunto de n inteiros positivos  $P = (p_0, p_1, ..., p_{n-1})$  e ainda um inteiro positivo x. O objetivo do problema é identificar um subconjunto único  $S \subset P$  onde:

$$x = \sum_{i=0}^{n-1} S_i$$

De modo a apresentar o resultado de uma forma clara, este vai ser representado na forma de uma  $mask\ \pmb{M}=(m_0,m_1,...,m_{n-1})$  onde  $m_i\subset\{0,1\}$  tal que:

$$x = \sum_{i=0}^{n-1} p_i \, m_i$$

O exemplo a seguir demonstra uma solução do problema.

$$P = (234, 429, 769, 835, 858, 874, 998, 1200, 1592, 1655)$$

$$x = 5963$$

$$S = (429, 769, 835, 858, 874, 998, 1200)$$

$$\mathbf{M} = (0,1,1,1,1,1,1,1,0,0)$$

Para alcançar as soluções, foram implementados quatro algoritmos: *Bruteforce, Clever Bruteforce, Horowitz* e Sahni, *Schroeppel* e *Shamir*, que vão ser explicados a seguir.

### **Brute Force**

O método  $Brute\ Force$ , consiste numa implementação recursiva, onde são calculadas todas as combinações possíveis de masks até que M seja encontrado.

Assim, é passado como parâmetros de entrada o vetor P e o seu tamanho n, o nível de recursividade level, inicialmente a 0, a soma obtida até ao momento  $partial\_sum$ , a soma pretendida  $desired\_sum$  e a mask a ser testada mask.

A função começa por avaliar os casos de término, quando *level* é maior que *n*, significa que a solução não foi encontrada e, se *partial\_sum* é igual *desired\_sum*, o que significa que *x* foi encontrado.

O comportamento recursivo da função pode ser representado através de uma árvore onde cada nó representa uma *partial\_sum*. Cada nó pai vai ter dois nós filho, onde  $M_{level}$  vai estar a 0 ou a 1 respetivamente, construindo assim todas as M possíveis. Ao verificar todas as hipóteses possíveis, este método vai ter  $\mathcal{O}(2^n)$  de execução para  $M_n$ .

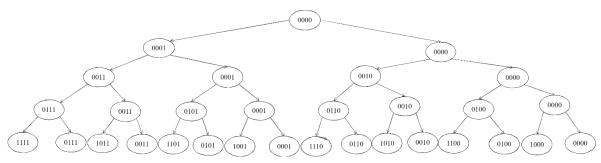


Figura 1- Árvore para n=4

### **Clever Brute Force**

O método *Clever Brute Force*, tal como o nome indica, é uma versão do algoritmo explicado anteriormente, mas com otimizações que removem logo casos improváveis.

São passados como parâmetros os mesmos do método anterior e um vetor sums onde

$$Sums_1 = P_1 e Sums_i = \sum_{j=2}^n Sums_{j-1} + P_j$$

Este método é mais eficiente que o anterior porque é efetuado o *pruning* de certos nós da árvore que nunca iriam resultar em S. É sabido do problema que  $\forall p \in P, p_i < p_{i+1}, i \in N$  então, se  $partial\_sum + Sums_{level} < x$  a mask que está a ser testada já não candidata a solução. Sendo assim, ao invés de testar de forma crescente as máscaras, começando com  $P_0$  começa-se a testar para  $P_{n-1}$  e de forma decrescente para eliminar mais rapidamente masks.

No entanto, este método apesar de ser mais eficiente, continua a ter de complexidade de execução  $\mathcal{O}(2^n)$ , visto que para o pior dos casos vai ter de testar todas as combinações possíveis.

## Horowitz and Sahni Technique

O método *meet-in-the-middle*, idealizado por *E. Horowitz* e *S. Sahni* e tem como intento subdividir o problema. P é dividido em dois subconjuntos de tamanhos aproximadamente iguais, A e B e são geradas e guardadas em dois vetores todas as somas possíveis de cada subconjunto, *sumsA* e *sumsB* com tamanho aproximadamente  $2^{n/2}$ . Os vetores de somas são ordenados com recurso ao algoritmo *quicksort* por ordem crescente.

Sabendo que se  $sumsA_i + sumsB_j = x$  então a solução foi encontrada. Para isto acontecer os dois vetores são percorridos por ordem crescente e decrescente, respetivamente e, caso  $sumsA_i + sumsB_j < x$ , i é incrementado e se  $sumsA_i + sumsB_j > x$ , j é decrementado. Caso  $i > n_{sumsA}$  ou j < 0, a solução não foi encontrada.

Este método, reduz drasticamente a complexidade comparativamente aos métodos apresentados anteriormente tendo  $\mathcal{O}(n2^{n/2})$  na execução, permitindo calcular soluções para n's maiores. Contudo, ao calcular sumsA e sumsB há um grande dispêndio de memória.

<sup>&</sup>lt;sup>1</sup> Ao correr o ficheiro  $subset\_sum\_problem.c$  o cálculo de sumsA e sumsB são efetuados antes de executar o algoritmo, visto que, para cada conjunto de problemas de tamanho igual, P é sempre o mesmo.

## Schroeppel and Shamir Technique

O método de *Schroeppel* e *Shamir* surge com o objetivo de otimizar o método anterior ao reduzir a necessidade de bastante memória. Para isso ao invés de dividir P em 2, divide em 4 subconjuntos de tamanhos aproximadamente iguais, são gerados os vetores de somas sumsA, sumsB, sumsC e sumsD e estes são ordenados recorrendo outra vez ao algoritmo quicksort.

Os vetores *sumsA* e *sumsB* são usados para popular a *minHeap*, onde a raiz é a menor soma possível e *sumsC* e *sumsD* são usados para popular a *maxHeap*, onde a raiz é a maior soma possível. As *heaps* são usadas para gerar as somas entre os pares de subconjuntos *on the fly* para evitar o enorme consumo de memória ao calcular a soma quando essa é necessária.

Seguindo o método anterior, verifica se a soma das raízes das heaps é a soma pretendida. Caso a *partial\_sum* seja maior que *desired\_sum* a raiz da *maxHeap* é eliminada e substituída pelo próximo elemento e, se *partial\_sum* seja menor *desired\_sum* a raíz da *minHeap* é eliminada e substituída pelo próximo elemento.

Este método apresenta complexidade  $\mathcal{O}\left(\frac{n}{4}2^{\frac{n}{2}}\right)$ , na execução e  $\mathcal{O}\left(2^{\frac{n}{4}}\right)$  de memória, conseguindo assim corrigir o problema de memória do algoritmo anterior, conseguindo assim quebrar a barreira dos 64 bits.

### **Resultados Obtidos**

### Otimizações

Ao longo do desenvolvimento dos métodos implementados, foram feitas otimizações de código que melhoraram a performance de cada um. É de se destacar a conversão de ciclos em funções recursivas para o cálculo das somas, o uso de apontadores na divisão de *P* nos dois últimos métodos para evitar alocação de memória desnecessária e ainda a conversão da maioria das operações para operações *bitwise*.

### Gráficos

Em cada um dos gráficos apresentados a seguir vai ser analisada a performance de cada um dos algoritmos consoante o N.

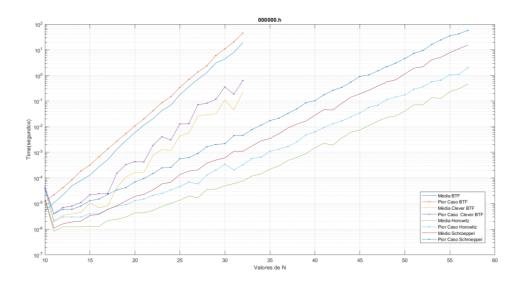


Figura 2- Gráfico do Tempo em função de **N** de 000000.h

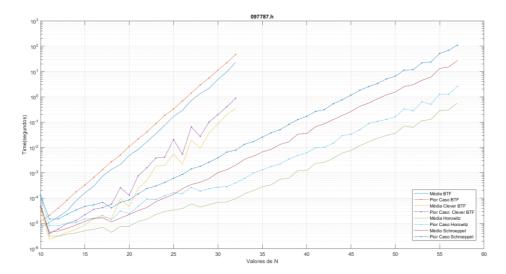


Figura 3 - Gráfico do Tempo em função de **N** de 097787.h

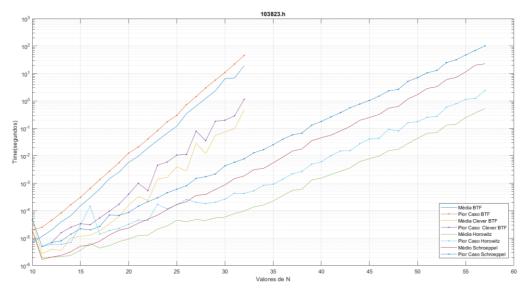


Figura 4- Gráfico do Tempo em função de N de 103823.h

Pode-se aperceber facilmente que os algoritmos tiveram comportamentos semelhantes em todos os ficheiros fornecidos (000000.h, 097787.h e 103823.h), onde se destaca a fraca performance do *Brute Force* com uma diferença acentuada para *o Clever Brute Force*, este que apesar de apresentar melhorias à sua versão não otimizada, continua a ter um crescimento abrupto.

O método *Horowitz e Sahni* para **n**'s relativamente pequenos tem um comportamento inferior aos *Brute Forces*, mas à medida que **n** vai aumentando, é facilmente percetível que consegue superar os métodos anteriores conseguindo alcançar valores de n muito superiores.

O método *Schroeppel e Shamir*, tem um comportamento semelhante ao *Horowitz e Sahni* para **n**'s pequenos, consegue ter uma eficiência muito maior, devido ao facto de não ter de calcular conjuntos de somas tão grandes, nem de as ter de guardar em memória durante toda a sua execução.

Todos os valores usados neste relatório foram obtidos sempre com a mesma máquina com as seguintes caraterísticas: Intel(R) Core (TM) i7-9750H CPU @ 2.60GHz 2.59 GHz; 16,0 GB RAM; Ubuntu 20.04.3 LTS e todos os valores usados para a construção dos gráficos foi incluído o cálculo das somas.

# Conclusão

Com este trabalho, foi possível perceber o quão importante é a procura de várias soluções para o mesmo problema, para perceber as vantagens e desvantagens de cada um deles. Serviu também para aprender mais sobre as técnicas de otimização de código em C e para compreender a necessidade de gestão da memória que o programa precisa para correr.

# Referências

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### **Anexos**

Nesta secção está disponibilizado o código C que foi desenvolvido para as várias soluções apresentadas neste relatório e ainda o código Matlab utilizado para processar os resultados obtidos.

### Código C

```
int bruteforce_iterativo(int n, integer_t p[n], integer_t desired_sum, integer_t * b) {
    for (int mask = 0; mask < 1 << n; mask++) {
       integer_t partial_sum = 0;
for (int bit = 0; bit < n; bit++) {</pre>
          if (mask & (1 << bit))
            partial_sum += p[bit];
       if (partial_sum = desired_sum) {
           * b = mask:
         return 1;
       }
    return 0;
int bruteforce_recursivo(int n, integer_t p[n], int level, integer_t partial_sum, integer_t desired_sum, integer_t
mask, integer_t * b) {
  if (level > n) {
      return 0;
    if (partial_sum = desired_sum) {
       * b = mask;
    if (bruteforce_recursivo(n, p, level + 1, partial_sum + p[level], desired_sum, (mask | (1 << level)), b)) {
       return 1;
    } else {
       return bruteforce_recursivo(n, p, level + 1, partial_sum, desired_sum, mask, b);
}
int bruteforce_recursivo_otimizado(int n, integer_t p[n], int level, integer_t partial_sum, integer_t desired_sum, integer_t mask,
integer_t * b, integer_t * sums) {
  if (partial_sum = desired_sum) {
      * b = mask;
      return 1;
  if (level < 0 || partial_sum > desired_sum || partial_sum + sums[level] < desired_sum) {</pre>
   if (bruteforce_recursivo_otimizado(n, p, level - 1, partial_sum + p[level], desired_sum, (mask | (1 << (level))), b, sums)) {
  } else {
     return bruteforce_recursivo_otimizado(n, p, level - 1, partial_sum, desired_sum, mask, b, sums);
```

```
typedef struct {
    integer_t mask;
    integer_t sum;
mask_data_t;
void swap(mask data t * a, mask data t * b) {
   mask_data_t t = * a;
   * a = * b;
    * b = t;
int partition(mask_data_t arr[], int low, int high) {
   mask_data_t pivot = arr[high]; // pivot
    int i = (low - 1); // Index of smaller element and indicates the right position of pivot found so far
    for (int j = low; j \leq high - 1; j \leftrightarrow) {
       // If current element is smaller than the pivot
       if (arr[j].sum < pivot.sum) {</pre>
          i++; // increment index of smaller element
          swap( & arr[i], & arr[j]);
    }
    swap( & arr[i + 1], & arr[high]);
    return (i + 1);
void quicksort(mask_data_t arr[], int low, int high) {
    if (low < high) {
       /* pi is partitioning index, arr[p] is now
       at right place */
       int pi = partition(arr, low, high);
       // Separately sort elements before
       // partition and after partition
       quicksort(arr, low, pi - 1);
       quicksort(arr, pi + 1, high);
}
void sums_generator(int n, integer_t p[n], mask_data_t result[1 << n], int level, integer_t mask, integer_t subSum, int idx) {</pre>
   if (level = n) {
     result[idx].sum = subSum;
      result[idx].mask = mask;
     return:
   sums_generator(n, p, result, level + 1, mask | (1 << level), subSum + p[level], 2*idx);</pre>
   sums_generator(n, p, result, level + 1, mask, subSum, 2*idx+1);
int horowitz_sahni(int n, integer_t p[], integer_t desired_sum, integer_t * b_result, mask_data_t *sumsA, mask_data_t *sumsB,
int nA, int nB) {
  int i = 0, j = (1 << nB) - 1;
   while (i < (1 << nA) \delta \hat{\sigma} j \geqslant 0) {
     if (sumsA[i].sum + sumsB[j].sum = desired_sum) \{
        *b_result = sumsA[i].mask | (sumsB[j].mask << nA);
         return 1:
     } else if (sumsA[i].sum + sumsB[j].sum < desired_sum) {
      } else {
     j--;
}
   return 0;
typedef struct {
   integer_t mask;
   integer_t sum;
   int i0:
   int i1;
} heapData t;
void minheapInsert(heapData t heap[], heapData t element, int* heapSize)
    int i:
    for (i = \star heapSize; i > 0 \delta heap[(i - 1) / 2].sum > element.sum; i = (i - 1) / 2)
        heap[i] = heap[(i - 1) / 2];
    heap[i] = element;
    (*heapSize)++;
}
```

```
heapData_t deleteMin(heapData_t heap[], int* heapSize)
    int i, son;
   heapData_t element = heap[0];
   (*heapSize)--; for (i = 0; i * 2 + 1 \leq *heapSize; i = son) {
       son = 2 * i + 1;
       if (son < \starheapSize \delta \theta heap[son].sum > heap[son + 1].sum)
           son++;
       if (heap[son].sum < heap[*heapSize].sum)</pre>
           heap[i] = heap[son];
       else
           break;
   heap[i] = heap[*heapSize];
   return element;
void maxheapInsert(heapData_t heap[], heapData_t element, int* heapSize)
    for (i = *heapSize; i > 0 & heap[(i - 1) / 2].sum < element.sum; i = (i - 1) / 2) {
      heap[i] = heap[(i - 1) / 2];
   heap[i] = element;
   (*heapSize)++;
heapData_t deletemax(heapData_t heap[], int* heapSize)
   heapData_t element = heap[0];
    \begin{array}{l} (*\text{heapSize}) --; \\ \text{for (i = 0; i * 2 + 1 } \leqslant *\text{heapSize; i = son) } \{ \\ \text{son = 2 * i + 1;} \\ \text{if (son < *\text{heapSize } \delta G } \text{ heap[son].sum < heap[son + 1].sum)} \end{array} 
          son++;
       if (heap[son].sum > heap[*heapSize].sum)
          heap[i] = heap[son];
          break;
   heap[i] = heap[*heapSize];
   return element;
```

```
int schroeppel_shamir (int n, integer_t p[], integer_t desired_sum, integer_t * b_result, mask_data_t *sumsA,mask_data_t *sumsB, mask_data_t *sumsC, mask_data_t *sumsD,int nA, int nB, int nC, int nD) {
   heapData_t minHeap[1 << nB];</pre>
    heapData_t maxHeap[1 << nC];
    int nHMin = 0;
    int nHMax = 0;
    for (int i = 0; i < (1 << nB); i++) {
        heapData_t sum = {
   .mask = sumsA[0].mask | (sumsB[i].mask << nA),</pre>
            .sum = sumsA[0].sum + sumsB[i].sum,
            .i0 = 0,
            .i1 = i
        minheapInsert(minHeap, sum, &nHMin);
    for (int i = 0; i < (1 << nC); i++) {
        heapData_t sum = {
    .mask = sumsC[i].mask | (sumsD[(1 << nD) - 1].mask << nC),
    .sum = sumsC[i].sum + sumsD[(1 << nD) - 1].sum,
            .i1 = (1 << nD) - 1
        maxheapInsert(maxHeap,sum,&nHMax);
    while (nHMin > 0 & nHMax > 0){
        integer_t partial_sum = maxHeap[0].sum + minHeap[0].sum;
if (partial_sum = desired_sum) {
   *b_result = minHeap[0].mask | (maxHeap[0].mask << (nA+nB));</pre>
            return 1;
        } else if (partial_sum > desired_sum){
            heapData_t old_max = deletemax(maxHeap,&nHMax);
old_max.i1--;
            if (old_max.i1 ≥ 0) {
                heapData_t new = {
   .mask = sumsC[old_max.i0].mask | (sumsD[old_max.i1].mask << nC),
   .sum = sumsC[old_max.i0].sum + sumsD[old_max.i1].sum,</pre>
                     .i0 = old_max.i0,
                    .i1 = old_max.i1
                maxheapInsert(maxHeap,new,&nHMax);
        } else {
            heapData_t old_min = deleteMin(minHeap, &nHMin);
            old_min.i0++;
            if (old_min.i0 < (1 << nA)) {
                heapData_t new = {
   .mask = sumsA[old_min.i0].mask | (sumsB[old_min.i1].mask << nA),</pre>
                     .sum = sumsA[old_min.i0].sum + sumsB[old_min.i1].sum,
                    .i0 = old_min.i0,
.i1 = old_min.i1
                minheapInsert(minHeap, new, &nHMin);
       }
    return 0;
```

```
int main(void) {
    fprintf(stderr, "Program configuration:\n");
    fprintf(stderr, " min_n ..... %d\n", min_n);
    fprintf(stderr, " max_n ..... %d\n", max_n);
    fprintf(stderr, " n_sums .... %d\n", n_sums);
    fprintf(stderr, " n_problems .. %d\n", n_problems);
    fprintf(stderr, " integer_t ... %d bits\n", 8 * (int) sizeof(integer_t));
    ///
    //
// for each n
    //
for (int i = 0; i < n_problems; i++) {
   int n = all_subset_sum_problems[i].n; // the value of n
   if (n > 42)
        continue; // skip large values of n
integer_t * p = all_subset_sum_problems[i].p; // the weights
        //
// for each sum
       double start = cpu_time();
#ifdef BTF_RCR_OTM
   integer_t * sums = (integer_t * ) malloc(n * sizeof(integer_t));
           sums[0] = p[0];
           for(int i=1; i<n; i++) {
    sums[i] = sums[i-1]+p[i];</pre>
        #endif
        #ifdef HW_SN
           int nA = n / 2;
int nB = n - nA;
           integer_t *a,*b;
           a = p;
           b = p + nA;
           mask_data_t * sumsA = malloc((1 << nA) * sizeof(mask_data_t));
mask_data_t * sumsB = malloc((1 << nB) * sizeof(mask_data_t));</pre>
           #endif
#ifdef MIM
    int nA = n / 4;
    int nB = n/2 - nA;
    int nC = nB;
    int nD = n - nA - nB - nC;
    integer_t * a = p;
    integer_t * b = p+nA;
integer_t * c = p+nA+nB;
    integer_t * d = p+nA+nB+nC;
    mask_data_t * sumsA = malloc((1 << nA) * sizeof(mask_data_t));</pre>
    mask_data_t * sumsB = malloc((1 << nB) * sizeof(mask_data_t));</pre>
    mask_data_t * sumsC = malloc((1 << nC) * sizeof(mask_data_t));</pre>
    mask_data_t * sumsD = malloc((1 << nD) * sizeof(mask_data_t));</pre>
    //Para o Array A
    sums_generator(nA, a, sumsA, 0, 0, 0, 0);
    quicksort(sumsA, 0, (1 << nA) - 1);
    //Para o Array B
    sums_generator(nB, b, sumsB, 0, 0, 0, 0);
    quicksort(sumsB, 0, (1 << nB) - 1);
    //Para o Array C
    sums_generator(nC, c, sumsC, 0, 0, 0, 0);
    quicksort(sumsC, 0, (1 \ll nC) - 1);
    //Para o Array D
    sums_generator(nD, d, sumsD, 0, 0, 0, 0);
    quicksort(sumsD, 0, (1 << nD) - 1);
#endif
double end = cpu_time();
printf("%lf\n", end - start);
```

```
for (int j = 0; j < n_sums; j++) {
  integer_t desired_sum = all_subset_sum_problems[i].sums[j]; // the desired sum
  integer_t b = 0; // array to record the solution</pre>
      double start = cpu_time();
#ifdef BTF_ITR
         bruteforce_iterativo(n, p, desired_sum, & b);
       #endif
       #ifdef BTF_RCR_NOTM
         bruteforce_recursivo(n, p, 0, 0, desired_sum, 0, & b);
       #endif
       #ifdef BTF_RCR_OTM
         bruteforce_recursivo_otimizado(n, p, n - 1, 0, desired_sum, 0, &b, sums);
       #endif
       #ifdef HW SN
         horowitz_sahni(n, p, desired_sum, &b, sumsA,sumsB,nA,nB);
       #endif
       #ifdef MIM
          schroeppel_shamir(n,p,desired_sum, &b,sumsA,sumsB,sumsC,sumsD,nA,nB,nC,nD);
       #endif
       double end = cpu_time();
       for (int i = 0; i < n; i++) {
    printf("%s", b & 1 ? "1" : "0");
          b = b >> 1;
       printf(" %lf\n", end - start);
   #ifdef BTF_RCR_OTM
       free(sums);
   #endif
   #ifdef HW_SN
     free(sumsA);
       free(sumsB);
   #endif
   #ifdef MIM
     free(sumsA);
       free(sumsB);
       free(sumsC);
      free(sumsD);
   #endif
return 0;
```

### Código Matlab

```
function Grafico(namefile,x)
%vai buscar o calculo das somas feitas antes do algoritmo
% bruteforce para cada numero mecanografico (1,2,3)
% clever bruteforce para cada numero mecanografico (4,5,6)
% Horowitz para cada numero mecanografico (7,8,9)
% schroppel para cada numero mecanografico (10,11,12)
switch(x)
       case 1
                somas=load("000000\bruteforce notz somas 000000.txt");
                : 2
somas=load("097787\bruteforce_notz_somas_097787.txt");
                somas=load("103823\bruteforce_notz_somas_103823.txt");
       case 0
    somas=load("103823\bruteforce_otz_somas_103823.txt");
case 7
                somas=load("000000\schroeppel_somas_000000.txt");
               somas=load("097787\schroeppel_somas_097787.txt");
                somas=load("103823\schoroeppel_somas_103823.txt");
       case 10 somas=load("000000\schroeppel_somas_000000.txt");
       case 11
    somas=load( '090787\schroeppel_somas_090787.txt*);
case 12
    somas=load("097787\schroeppel_somas_097787.txt*);
case 12
    somas=load("103823\schroeppel_somas_103823.txt*);
               somas=load("097787\schroppel_somas_128_097787.txt");
        T=load(namefile);%Valores do tempo para cada soma
         T= T+somas'
        .- r+somas';
[n,~]=size(T);
N=10:9+n;
       semilogy(N,media)
         hold on
         semilogy(N,ValorMaximoLinha,".-");
         %semilogy(N,ValorMinimoLinha,"•-");
%hold on
 clear
 close all
 %000000
Grafico ("000000\bruteforce_notz_resultados_somas_000000_organizado.txt",1); %bruteforce para 64 bits
Grafico ("000000\bruteforce_otz_resultados_somas_000000_organizado.txt",4); %clever brute force para 64 bits
Grafico ("000000\horowitz_resultados_somas_000000_organizado.txt",7); %horowitz and sahni technique para 64 bits
Grafico ("000000\schroeppel_resultados_somas_000000_organizado.txt",10); %schroeppel and shamir technique para 64 bits
grid on;
title("000000.h")
legend( "Média BTF", "Pior Caso BTF", "Média Clever BTF", "Pior Caso Clever BTF", "Média Horowitz",
"Pior Caso Horowitz", "Médio Schroeppel ", "Pior Caso Schroeppel");
xlabel("Valores de N");
ylabel("Time(segundos)");
hold off:
 %097787
Trgure(2)
Grafico ("097787\bruteforce_notz_resultados_somas_097787_organizado.txt",2); %bruteforce para 64 bits
Grafico ("097787\bruteforce_otz_resultados_somas_097787_organizado.txt",5); %clever brute force para 64 bits
Grafico ("097787\brownitz_resultados_somas_097787_organizado.txt",3); %brorowitz and sahni technique para 64 bits
Grafico ("097787\schroeppel_resultados_somas_097787_organizado.txt",1); %schroeppel and shamir technique para 64 bits
% Grafico ("097787\schroeppel_128_resultados_somas_097787_organizado.txt",13); %schroeppel and shamir technique(128)
 title("097787.h")
legend( "Média BTF", "Pior Caso BTF", "Média Clever BTF", "Pior Caso Clever BTF", "Média Horowitz", "Pior Caso Horowitz", "Médio Schroeppel ", "Pior Caso Schroeppel "); xlabel("Valores de N"); ylabel("Time(segundos)");
 hold off;
 %103823
%103823 figure(3)
Grafico ("103823\bruteforce_notz_resultados_somas_103823_organizado.txt",3); %bruteforce para 64 bits
Grafico ("103823\bruteforce_otz_resultados_somas_103823_organizado.txt",6); %clever brute force para 64 bits
Grafico ("103823\brownitz_resultados_somas_103823_organizado.txt",9); %horowitz and sahni technique para 64 bits
Grafico ("103823\schroeppel_resultados_somas_103823_organizado.txt",12); %schroeppel and shamir technique para 64 bits
grid on;
title("103823.h")
legend( "Média BTF", "Pior Caso BTF", "Média Clever BTF", "Pior Caso Clever BTF", "Média Horowitz",
"Pior Caso Horowitz", "Médio Schroeppel ", "Pior Caso Schroeppel ");
xlabel("Valores de N");
ylabel("Time(segundos)");
 hold off:
```

### Masks

Nesta secção estão dispostas as soluções obtidas para o problema proposto para cada um dos números mecanográficos.

	097787.h	103823.h
10	1100001110 0100010001	010101111 0101111111
	1000011000	1010101100
	0001101101 1100001011	0010111111 0001010010
	1110111100 0000010111	0010100110 0000011010
	1010111110 0011101010	0001111101 1011010000
	0111001100	0010001010
	0101001000 1101000001	01011111100 1100000110
	0111110110 0010001110	1111111010 1111000000
	1100011101	0010001110
	0000101100 0011011100	0011010111 1011000101
	0010100001 1111101100	1001101011 1101111000
	0001111011	1110001111
11	00011101110 01101001110	11100100001 01010001000
	00100101011 10100111011	11101111111 00001110001
	11000011101	10001011001
	00010101100 01110011110	10010100100 10000111010
	10001111000 10111010011	10111001100 00110100010
	11101010000 00100111010	10001000110 10110011011
	01000000111	01001000111
	11010000001 10010001001	01010111101 10010010110
	01111000110 0111111111	00101100111 01110111000
	10001001010	01001001010
	01101110000 1000010101	01100001110 00111011011
10	01001100010 101110010011	11110100110 011001001011
12	0001011111100	101001100000
	111100011111 110000100100	000011011011 001100010010
	001100101100 000100010100	100110111110 111001100100
	000101101011 011010000000	011111000010 111110010001
	010001110100	000010111000
	111000110110 111101101101	110000011111 101001111011
	110101100011 111111011001	001101101010 001011110010
	110000110010 100011101101	100101000110
	001111000111	101100101110 111101100100
	001110010101 110101011101	100011101010 110101101110
	100010111010 010111110111	00001101010 001011111111
13	1101000011100	0111010010110
13	1100101010111 1111101100111	0110010110011 0111011000000
	0000110011011 0011010100010	1001000001011 0101110000001
	1011000110001	0000001100100
	1001101111011 1111100001010	1000111001010 1011001001101
	1001101011111 0001100011101	1000101010001 0010011111010
	1010010110100 0001000010110	1011110010000 0101100100110
	1000111110110	1111110011010
	1100100000111 1110111010000	0101000100111 1010011100011
	0000010101111 10010111111111	1101111110100 0111111001010
	1110010011010 11111111110111	1000001001000 0011010000110
	0111000110100	0001110111110
14	10100111101010 00010001011100	00110110000101 10111010100100
	11011011110111 00010100101101	10111011101110 10011010001001
	00011011101011	00110010101111
	10110101001000 11111110011111	10010110001010 00000011111010
	01000010111001 11110100000000	01011100001101 10010111001011
	01011111011111	1101000101101

	11101111111011	11110001101100
	00111010110110 0100101011110	11001001011000 11011101110001
	101100101111111	11111000010011
	00010000111110	01001101100110
	11011101111101 00001111110010	110001001001111 00100110111001
	11111100100000	10001101110000
	00100101001101	10100110110111
-	01001101111010	10011100010001
15	000111101100111 111101100011010	001111101000010 010011110001001
10	0101010011010	10001010101010
	111010110100011	010111010010001
	001001001111011	001111000100000
	000111110001100 010010011110111	101010111100111 110000011010011
	0110101111111011	011011111000111
	001010001100111	111111100010110
	001100111100111	010101000000100
	111001101001000 100001010001011	011010011100111 110010110010010
	101100011001000	01100000110100
	1110110010111110	110111001010011
	010110000011010	000111000010001
	000100011100101 111011100101101	010011001001000 111101100110100
	000111100001000	110000010001101
	100000011001101	001000011100110
	011110100111100	000101001100011
16	0101000110101110	0110001100111110
	0000100110000101 0100110001111011	100100101111001 1110101001011001
	0011001001001010	1111010101001100
	0100001001010110	1110010010100001
	0100110001100101 1111001001100011	0010000100001001 1100101100001101
	1011110111001101	00110100001101
	1111010110110101	0010001110010100
	0111001111110111	1001110001010001
	0011101000001011 1100110100010110	1111110111000001 1110001101110000
	1111100101110010	1010110111111000
	0110010010101111	0010001100001100
	0110111010100010	0110000010100101
	1100010101101110 0000011001000110	0000101000001010 0100000110000001
	101100011011011	00001000001
	0101111011010010	1000001101110000
	1111010011000010	0110100111010100
17	00100010100011110 10100100101110110	00000010011001110 00110001101011010
- /	1110110110011101	11010010010101010
	11110001111011011	10011011101010101
	10101001110001001	10101001000010111
	00010010111001110 11101010001100010	01111111010000111 10100000101000111
	0001010000100010	00110110100100111
	00010101001001111	01111000000101110
	01001111000111001	11101010011000100
	00111100100110000 01001001001111001	00011010110010000 101111011111100000
	00011010001000011	10011100010000100
	01101001001001101	10000111001011111
	11110101010100011	01101011111101110
	01001011000011100 10011111111101111	11101111110011110 01101101111100111
	00000011110101000	00100111110101011
	00111100100011101	00010010111010011
	01100000010110111	01110000111001100
18	000001111110110111 0011100010111111011	1011101010010111100 010010011110110000
	0100010011111011	100001000011110110
1	001000101100011110	011101111011111111
	111101110001111000	100000100000011
	00000011110010100 100001010101001101	001100000110100010 100110011111110110
	001011001000110011	0010101111111010011
	101011001101101011	00001001011111111
	100001101010011001 101011110010010010	111100110001101100 000010001011010000
1	1001010110010010010	000010001011010000
	000101001111000111	001101110011001110
	011101010011010111	000000110010011011
	110011110101101111 011110001111000101	110101101000001010 000101000101111011
	00010000111000101	1000000101111011
1	001010100001101111	110101001100110010
1	111100011110100101	001101011111010001
	111000011001001000 1000010010100101001	010111100110000010 0110100110011101100
19	11110011011010111110	10011010011101100 10011101100
	1000110111010000010	1001010100011110100
1	1000100011111000000	1010100010011001001
	0110011011001010010 0111001100110110011	00011010101111111000 0101011000000010100
	1010011011011011011 1011 1011 1011011011	1001011000000010100
	10100010011011000001	1011101000011001000
	1100010001110010110	1011011011010011011
	1111110101110100101 0011011010000110011	0010100010010011001 1001011100100101110
	110011111101011111	110101110100101110
1	0111001000010010010	0011111000101001111
•	1001110011011000011	1111011000011011011

	0100110001100001101 100100010111101111	100010110110100000 0011010101100100101
	100100010111101111	0101100100100100101
	1000001111011011111	0001110110111000111
	0001001010011111110	10100111001011110000
-	1001110011111100001 10110000001110100011	0001001001111011001 00111000100111100011
20	11001111001111100110	111111100100011110
	10110101000010101011	01100000111110001100
	01100100101000010000	100011110101011011
	00001101001101111010 0011001011101011011	01011110000111110011 1110000010100011010
	111011111111010011101	00101001101111011101
	01011111101110110001	10000100100111011110
	1101000001000010011	00110000010000000111
	01111011100010010011 00101011101101100111	00100011011001100110 000110011111110100110
	0001001101101011101	01101001100010000010
	10101111111011101110	01110011110001001111
	01111110100010110101	10011010010111110101
	10100000111010011000 01111101010001001101	00111100101010111101 00011000101000001000
	001010001011010100	10001100000101000101
	01010100101101000000	01010010010001010110
	10100101110111100101 10111101010100100010	00001100001111011110 11111010011111000001
	101000011101010111111	01011111010001101101
21	001001110100111111	1110111111111000110011
	111000111110010010000	011000101000001111111
	101011100001011110011	110010110111011010111
	101000101110111100111 010100100110011110110	011101000111100101111 000111110001110000101
	10111011111111101100000	01010111100001110011
	000010001100001110010	111000010010111000001
	110010001100011010001	100010011100010010111
	010100010000001000101 100101011100110101110	010110100011010111010 00011001011110101011
	010101000110010111110	001000010000101110110
	110010011001001101001	110000001001001110101
	100111100001001011011 10001011010110011011	0100111111011001111100
	1000101101100110111	000110101001111101111 101010010000111010100
	111100110010000101001	100110000100010100110
	111001000101000001001	111111110010000100001
	1100110010011011111100 000001001110001101011	010100101011100001001 111000011111101010010
	011011011100111010111	01110110100001111100110
22	1000011000101100001011	10010110000011111100011
	0110100101000001101101	1000000101111010010011
	0110011000011001111011	1111110010110010110001
	11010111111100101000010 1001011111101101	1010001001010001011010 1100101111111111
	11101100110110110100	0010010101111111111101
	10110100100111011111110	0001111111010111011101
	1010011001011111001111	0001000011110011001010
	1111100011011110001101 0100010011101101011101	0001101010111101110110 00001110110100100
	0001010001110000101000	1011111100101101101101
	0011110001011010100111	1000000100011001001110
	1100010000101001100011 111110100101110110	1000100110011001110101 01011111100001100101010
	111101001011101100101	0101111100001100101010
	0100010100110110111000	0011100111011110101110
	0101010001111101011101	1101111000100001000011
	01000011000111111110011 10111011110001110110	1111110011111101101001 1111001111010110001110
	111100101111011000111	0111110101110001110
23	11010101111011100110	1011001011011100101110
	100111100000111111111101	01101000001010001000011
1	0101000101011101011010	111111100001001011000010
1	00000010011000010111110 0110010111110110	10100100010110010110011 00001010100111101111101
1	01000001010111000000100	0001110101101001010001
1	100011110111010001011100	11101001100001011011110
	11100101001010110101110 00110000010010111001010	01111110010111100011111 110100011101110
1	10001000010010111001010	0001001110111101010010
1	01010110010101011000110	10110101110100001101101
1	11110110100110001010110	10000101111000011100010
1	1101101110111010100001 0010011011001100	10011101011110011010100 00001101111111000011011
	01000110011001101111	101011010010111111100
	11010110111011100100101	01101010000110010011111
1	11101110100111001101001	10111101000010010001111
1	11111111100000001100100 0000000101101101	10000111101010001101000 100111010000010001001
24	110010010000011101001111	010111110111100111011101
24	101100011000100000111110	000010111010110100101000
	110111100000010001110110	101111111011101000001100
1	011010100011100011110000 000010001110111110111010	111010110100001011100100 1111110111000011100100
1	00011010111110111010	1111011100001110010001
1	010011100111101100100101	100000101101100011011101
1	000101011111100000110111	001100101111111010010001
1	010110011001011110010100 11111111011101	110011001110111101101001 0001111100100010001001
	0111100001110111111000100	1110110101111100000100110
1	1110010001011011111100110	100101011100011011111110
1	101100001010000101100101	011100100100001000100001
1	011010110101111010100100 010000000111001110011101	111000111000100010011001 100000001110111010010
1	10101110101010011001101	000100100101011010010101
1	101001000101110101011000	101000011011111010011000
	100010101101110100010001	110001010000110011000101

	00011010000101011110101	010110000011010110011111
	11101111110101010011001 100001000010111111	100010000010010110011110 101001111000011010010
25	00010001111100001101001	10011100011101100100011
	01111001101111100000010001	1101111101010011000000101
	1001000011101100010010001	01111010101111111100000110
	0011110001000010000110100 0001100000101110011001110	0010100111011011000111110 010111110010000111010010
	0010010011100000100001101	1011000001110111011010101
	1000000011111011111111111	1000011100011011110100010
	10010010000110111001010	1001100011101111000100110
	1110100100010110111100011 110001100011	1000111100110011100010010 010010110101101
	0101000101000110100101101	1111100000011001011000110
	0110000110011000010110101	0110010111000011001000110
	1100101101101000011111111	1111101001010000101110011 100111110101011001100100
	0001011110110011101010101 1101000101000010100011101	110111110101010101111110
	111110011111100011110100100	1011101000110110101000100
	0010010110010101010010010	1010100001001001101011000
	1011100001010000011000100	1110111000101100100000101
	00101111100110101111110010 110100111100100	0110101001011010000100111 01011010000011100100
26	1110111011011111110000010	1000000111110100001101
	00101110101111001110001000	00000100101001110001100111
	1110010011100000000011110	1001100001000110110110111
	010100001010111111111000011	11111010100100000001110001
	00100110100110011001000100 000101111110110	00000111000011100001011011 10010010011101110110
	00100110110100000110011111	0001000101001110011001100
	11111111001000000001000110	10111110000110101000101000
	11100011100110011011100011	0000010110111100000001111
I	0000011110011001010100001 110110101101	10100011100010010110001010 00111110110011111001110110
I	1101100100100000000001110	11100010011111000110110
	11010011010000100100110000	00101101011101100111100111
	01100001101001101101111100	1111010111110101111111000111
	01110011100000101011100110 0010101110101010101010101	10110110010110010111000100 10010010111110110
	1011000101010101010101	000011010010110110010100
	0011111111111100101011100011	00101101001101011000000100
	10010001101100111010110101	11001001110101010011100111
27	101010100011010100100000100	000011100010111100110100001
- '	000000101001010011001001011 10011111111	1010111101011000111111111101 1000101110001110110
	1011101111111110011000001011	101111110011110100101000001
	100101110111010110011000011	011010011000110011110110101
	1011000100111011011111111100	010100111111010110001100111
	1011110100010010101010101001 001001011000010001001	100001111101011011110111011 01001111110110
	000011010011011100100010100	000110100010100011010000111
	100110111010011100101101100	0100001110110111111101000011
	010111110010000100100111010	101101101001110111100101010
	110011101001100000000011010 101001100010100010001010110	000101110011000111111000000 0000000010110000101101
	100010001100110010001000011	110011001010101010000111010
	001100010111010000011101010	101110000001101001110110010
	010000011101011001001001011	11101111110001000100010011
	001011000000110011101111110 00101101111100001011100100	110000000011100100001010101 0100101000011011
	001111000100100111001110111	100110111011011110110001101
	011101010010011001000101001	1001101111111110000000110110
28	01001010011001010101100101	1100001111001001010000001101
20	00110111111111101010010111100 01001000110001110110	0011011111101000100101101011 0111011110100110101101
	0001101011110100100101101000	10110110100110111100001100100
	11000101001101010101111111101	0000110000001011010101000011
	1110111110010110110110111110	11100100110100000111111110001
	0110011101110111100011110111 110100110101010111110011011	10111000011010001010011111100 000001010000111111
	111111111101110111111111001001	1010100000110000110001010110
I	0100100110001011010100100000	1100010101101110100101111011
	1011100100111110001100100110 0101001101000111000101000011	1001010111101010000110100001 10011101100111110011011
	100110101101000111000101000011	10011101110111111001101111111 10101100111111
I	1101101001110010000010001001	0010010011000000010101100111
I	0001110100000011000010110110	1101101110011100001010001111
I	0001100101001100100101110011 1000010111010111001010000101	0111000010000101111100000001 0111010000111010111001110110
I	1000010111010111001010000101 00111100100	0111010000111010111001110110 000011010101011111001010110
I	0000001000011101100101000011	110111111000000101001111100000
	1110011100101000101111110110	0101010001110110000010011000
29	00011010111110000100111111	100001000000110000101111010
l - ´	11110101110010111011000100011 011000000110011011	11011010010001010000101100111 1010110110
I	000111100000001100110110001001	11001100000101100101000000
	11011111000001111001011011100	111101111111101111110110001100
	10110001101011100011101100101	0011101101010101101101101100
	11111101100000011000011001100 11010111101010001001	11001010101001011000110001100 11111010000101100100
I	010011100110001001111011000	11011001110010010010000001100
I	10100110110001010000010000101	00010101100001010001100001100
I	10010001100001100111110111111	0010000100010000000010001001
	01111110111001111100110100100 11011000011101010110000011000	01011101001010111111100011110 001001011000010111111
I	10010111101101010110000011000	11001011100101111111010111
I	00110111110011011000111001001	10011010100000001001011011000
I	1001100010011011010011110	010011111000111100001110000101
I	01111001100011010011100111001 11000001110001111011110010110	00000001111011110011111001011 011111100100
I	00001001100001100011011000111	11100001100100010011110000101
	110110101001010111100011011101	00100111111111110100011110001011
30	0110011000101111110001010011101	100000111101100101110100000111
50	110010111010010101100001010000	011001001100001100100011000101

_	0.110.00.00.10.10.10.10.10.10.10.10.10.1	4444000444444440004440007
	011101001011111001001001111101 011111111	101100001111111100001010000110 0001001110011011
31	111000100010000010000111110110 00010111101000000	1111110000111001100100001110110 0111100001001
32	0000000001011111010011110101110 1010000101110110	1110000111111110011001000100101 01110001001
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