PROLE 2024

Formal specification of the post-quantum signature scheme FALCON in Maude

Víctor García[1] (vicgarval@upv.es)

Santiago Escobar[1] (sescobar@upv.es)

Kazuhiro Ogata[2] (ogata@jaist.ac.jp)

[1]Universitat Politècnica de València (UPV), Camí de Vera, s/n, 46022 València, Valencia, Spain

[2]Japan Advanced Institute of Science and Technology (JAIST), Ishikawa 923–1292, Japan

Contents

UNIVERSITAT POLITÈCNICA DE VALÈNCIA

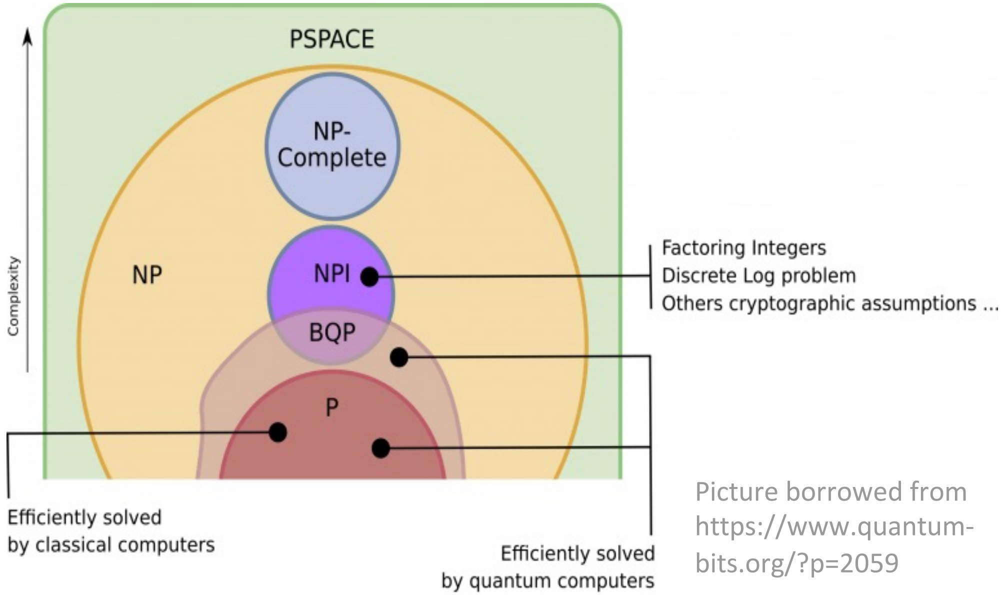
JAIST JAPAN ADVANCED INSTITUTE OF SCIENCE AND TECHNOLOGY 1990

Contents

Motivation

Threat of adversaries with quantum capabilities

- Shor's algorithm to solve the discrete logarithm problem (1994)
- Grover's search algorithm for unique input of a black box function (1996)



Picture borrowed from https://www.quantum-bits.org/?p=2059

Search of solutions by NIST
with the PQC project (round 3)

- Key Encapsulation Mechanisms: CRYSTALS-Kyber
- Digital Signature Schemes: CRYSTALS-Dilithium, **FALCON**, SPHINCS+

Motivation

Types of security analysis

- Computational
  - Mathematical proofs and probabilities
  - Keys, messages,... are bit strings
  - Closer to reality, used by cryptographers
- Symbolic
  - Cryptographic primitives as black boxes
  - Keys, messages,... are symbols
  - Suitable for automation and easier to understand for non-experts of cryptography

Contents

Maude

What is Maude?

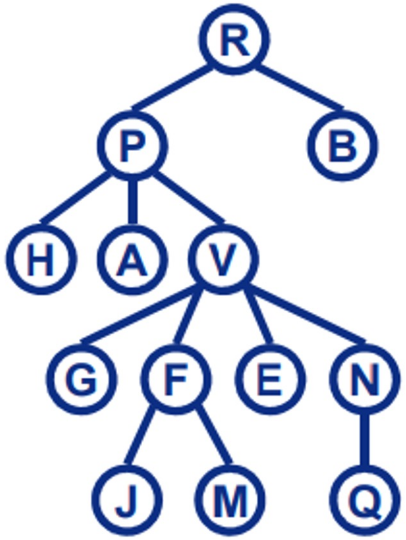
Maude is a modelling, programming and verification language based on rewriting logic.

Why Maude?

Because it allows us to specify and execute systems in a simple and intuitive way.

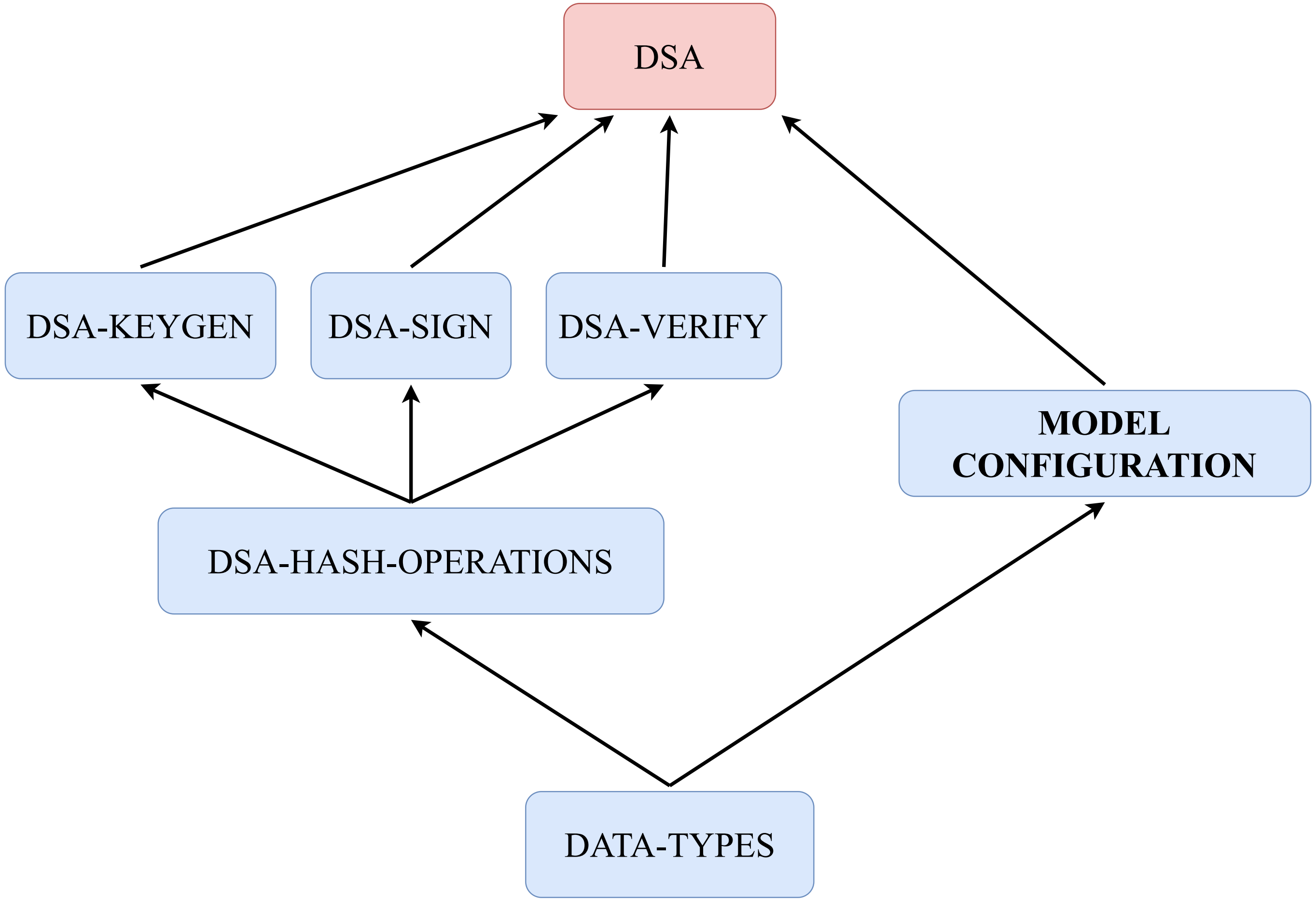
Which verification tools are provided?

Reachability analysis using the *search* command from an initial state to a target state. Moreover, under the assumption of a finite number of reachable states from a given initial state, one can use Maude's LTL model checket to prove any properties with LTL formulas.



Contents

Framework


DSA
DSA-KEYGEN
DSA-SIGN
DSA-VERIFY
MODEL CONFIGURATION
DSA-HASH-OPERATIONS
DATA-TYPES

Contents

1. Motivation
2. Maude
3. Framework
4. **FALCON**
5. Model
6. Experiments
7. Conclusion

FALCON

What is FALCON?

Falcon is a signature scheme based on lattices to sign and verify messages.

ALICE | **BOB**

KeyGen : $(\phi, q) \rightarrow (sk, pk)$

$1 : f, g, F, G \leftarrow NTRUGen(\phi, q)$
$2 : B \leftarrow mat(f, g, F, G)$
$3 : \hat{B} \leftarrow FFT(B)$
$4 : G \leftarrow \hat{B} \times \hat{B}^*$
$5 : T \leftarrow ffLDL^*(G)$
$6 : sk \leftarrow (\hat{B}, T)$
$7 : h \leftarrow gf^{-1} \mod q$
$8 : pk \leftarrow h$

Alice $\xrightarrow{pk}$ Bob

Sign : $(m, sk) \rightarrow sig$

$1 : r \leftarrow \{0,1\}^{320}$
$2 : c \leftarrow HashToPoint(r||m, q, n)$
$3 : t \leftarrow (FFT(c), FFT(0))\hat{B}^{-1}$
$4 : z \leftarrow ffSampling(t, T)$
$5 : s = (t - z)\hat{B}$
$6 : (s_1, s_2) \leftarrow invFFT(s)$
$7 : s \leftarrow Compress(s_2, 8 \cdot sbytelen - 328)$
$8 : sig = (r, s)$

Alice $\xrightarrow{(m, sig)}$ Bob

Verify : $(m, sig, pk) \rightarrow Accept/Reject$

$1 : c \leftarrow HashToPoint(r||m, q, n)$
$2 : s_2 \leftarrow Decompress(s, 8 \cdot sbytelen - 328)$
$3 : s_1 \leftarrow c - s_2 h \mod q$

Contents

MODEL

Main components

Message

```
op msg{(_,_)[_]_} : Identifier Identifier MsgState Content -> Msg .
```

Principal

```
op _[_]_ : Identifier Keys Content -> Principal [ctor] .
```

Environment

```
op {_}<_>net(_) : Content Principals Msgs -> GlobalState
```

MODEL

$$\textbf{KeyGen} : (\phi, q) \rightarrow (sk, pk)$$

$$1 : f, g, F, G \leftarrow NTRUGen(\phi, q)$$
$$2 : B \leftarrow mat(f, g, F, G)$$
$$3 : \hat{B} \leftarrow FFT(B)$$
$$4 : G \leftarrow \hat{B} \times \hat{B}^*$$
$$5 : T \leftarrow ffLDL^*(G)$$
$$6 : sk \leftarrow (\hat{B}, T)$$
$$7 : h \leftarrow gf^{-1} \mod q$$
$$8 : pk \leftarrow h$$

Collapsed

```
crl [KeyGen] :  {phis(SAM1, CONT1), qs(SAM2, CONT2), CONT3}
                <
                 (ID1[emptyK]peer(ID2))
                 PS
                >net(MSGS)
                =>
                {phis(CONT1), qs(CONT2), CONT3}
                <
                 (ID1[publicKey(ID1,PK) ; secretKey(ID1,SK)]
                  qI(ID1,SAM2), phiI(ID1,SAM1), peer(ID2))
                 PS
                >net(MSGS)
                if  L := NTRUGen(SAM1,SAM2) /\
                    SK := ([FFT(mat(L)),ffLDL(FFT(mat(L)) x FFT(mat(L)))]) /\
                    PK := (elem(2,L) p* inv(elem(1,L))) mod SAM2 /\
                    ID1 =/= ID2 .
```

MODEL

UNIVERSITAT POLITÈCNICA DE VALÈNCIA

JAIST JAPAN ADVANCED INSTITUTE OF SCIENCE AND TECHNOLOGY 1990

Sign : $(m, sk) \rightarrow sig$

$1 : r \leftarrow \{0,1\}^{320}$

$2 : c \leftarrow HashToPoint(r||m, q, n)$

$3 : t \leftarrow (FFT(c), FFT(0))\hat{B}^{-1}$

$4 : z \leftarrow ffSampling(t, T)$

$5 : s = (t - z)\hat{B}$

$6 : (s_1, s_2) \leftarrow invFFT(s)$

$7 : s \leftarrow Compress(s_2, 8 \cdot sbytelen - 328)$

$8 : sig = (r, s)$

```
crl [Sign] :    {ms(str(STR), CONT1), rs(SAM1, CONT2), CONT3}
                < (ID1[secretKey(ID1,SK) ; KS1]qI(ID1,Q), CONT4) PS >net(MSGS)
                =>
                {ms(CONT1), rs(CONT2), CONT3}
                < (ID1[KS1]sig(STR,SAM1,Compress(second(Ss),SBYTELEN)) ,
                           mI(ID1,STR), qI(ID1,Q), CONT4) PS >
                net(MSGS)
                if c := HashToPoint(SAM1 || STR, Q, n) /\
                   t := [FFT(c), FFT(0)] p* inv(first(SK)) /\
                   z := ffSampling(t, second(SK)) /\
                   s := (t p- z) p* first(SK) /\
                   Ss := invFFT(s) .
```

MODEL

$$\textbf{Verify} : (m, sig, pk) \rightarrow Accept/Reject$$

$1 : c \leftarrow HashToPoint(r||m, q, n)$

$2 : s_2 \leftarrow Decompress(s, 8 \cdot sbytelen - 328)$

$3 : s_1 \leftarrow c - s_2 h \mod q$

```
crl [Verify] :  {CONT1}
                <
                 (ID1[publicKey(ID2, p mod Q) ; KS1]
                 mI(ID2,STR), sig(STR,R,P), peer(ID2), CONT2) PS >
                net(MSGS)
                =>
                {CONT1}
                < (ID1[KS1]mI(ID2,STR), peer(none), CONT2) PS >
                net(MSGS)
                if  c := HashToPoint(R || STR, Q, n) /\
                    S2 := Decompress(P, SBYTELEN) /\
                    h := p mod Q /\
                    S1 := (c p- (S2 p* h)) mod Q /\
                    (S1 == s1) /\ (S2 == s2) .
```

Contents

Experiments

```
eq init1 =  {phis(phi), qs(q), ms(str(m)), rs(r)}
            <   (Alice[emptyK]peer(Bob))
                (Eve[emptyK]peer(none))
                (Bob[emptyK]peer(Alice))
            >net(emptyM) .
```

Experiments

Executable?

```
==========================================
rewrite in FALCON : init1 .
rewrites: 30 in 0ms cpu (0ms real) (254237 rewrites/second)
result [GlobalState]:
{phis(emptyC),qs(emptyC),ms(emptyC),rs(emptyC)}
<
(Alice[emptyK]peer(none),phiI(Alice, phi),qI(Alice, q))
(Eve[emptyK]peer(none))
Bob[emptyK]peer(none),mI(Alice, m)
>net(msg{(Alice,Bob)[received](g p* inv(f)) mod q}
        msg{(Alice,Bob)[received]str(m),[r,Compress(s2, SBYTELEN)]})
```

Experiments

Search for states where two participants applied the protocol

```
============================================
search in FALCON : init1 =>!
{CONT1}
<
PS
ID1[KS1]peer(none),mI(ID2, STR)
>net(MSGS) such that ID1 =/= ID2 = true .

Solution 1 (state 27)
states: 29  rewrites: 126 in 0ms cpu (0ms real) (320610 rewrites/second)
CONT1 --> phis(emptyC),qs(emptyC),ms(emptyC),rs(emptyC)
PS --> (Alice[emptyK]peer(none),phiI(Alice, phi),qI(Alice, q))
Eve[emptyK]peer(none)
ID1 --> Bob
KS1 --> emptyK
ID2 --> Alice
STR --> m
MSGS --> msg{(Alice,Bob)[received](g p* inv(f)) mod q}
        msg{(Alice,Bob)[received]str(m),[r,Compress(s2, SBYTELEN)]}
```

```
Solution 2 (state 28)
states: 29  rewrites: 127 in 0ms cpu (0ms real) (293981 rewrites/second)
CONT1 --> phis(emptyC),qs(emptyC),ms(emptyC),rs(emptyC)
PS --> (Eve[emptyK]peer(none))
Bob[emptyK]peer(none),phiI(Bob, phi),qI(Bob, q)
ID1 --> Alice
KS1 --> emptyK
ID2 --> Bob
STR --> m
MSGS --> msg{(Bob,Alice)[received](g p* inv(f)) mod q}
        msg{(Bob,Alice)[received]str(m),[r,Compress(s2, SBYTELEN)]}

No more solutions.
states: 29  rewrites: 127 in 0ms cpu (0ms real) (276688 rewrites/second)
Maude>
```

Contents

Conclusion

Concluding remarks

- We specified the digital signature scheme FALCON in Maude using a framework.
- We checked that the specification is executable and asked for final reachable states, capturing the behaviour of FALCON.

Future work

- We are using Maude's LTL Model Checker to verify DS properties: Authentication, Non-Repudiation and Integrity.
- We are adapting the specification to use built-in data structures in the specification.
- We will explore other signature schemes and specify them, so we can compare our models in terms of satisfied properties.
- We could also adapt the specification to use the object oriented notation provided by Maude.

PROLE 2024

Formal specification of the post-quantum signature scheme FALCON in Maude

Víctor García[1] (vicgarval@upv.es)

Santiago Escobar[1] (sescobar@upv.es)

Kazuhiro Ogata[2] (ogata@jaist.ac.jp)

[1]Universitat Politècnica de València (UPV), Camí de Vera, s/n, 46022 València, Valencia, Spain

[2]Japan Advanced Institute of Science and Technology (JAIST), Ishikawa 923–1292, Japan